Journal of Progressive Research in Mathematics(JPRM)
ISSN: 2395-0218

SCITECH
RESEARCH ORGANISATION|

Volume 4, Issue 1
May 20, 2015|

Journal of Progressive Research in Mathematics
www.scitecresearch.com

Stabilization of Discrete Nonlinear Systems with Continuous Feedback Law

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Abstract

This paper we studied the stabilization of discrete nonlinear systems, a sufficient and necessary condition for a quadratic function to be a control Lyapunov function is given. And a continuous state feedback law is constructed.

1 Introduction

The most powerful tool is the approach of control Lyapunov function which has been employed to address various issues to nonlinear control systems, and the stabilization problem of nonlinear control systems has attracted much attention . As we know, Artstein given an important theorem [1] which proved that the control system exists a control Lyapunov function if and only if there is a stabilizing relaxed feedback. Of course the existence of a smooth Lyapunov function fails for a nonlinear system in general. The key result used in most of the feedback stabilizers is a well known theorem due to Sontag[2] and Brockett [3].

The stabilization of discrete systems is studied by means of this paper. Cai Xiushan[4] ,Tang Fengjun [5] also studied the same systems , but the control Lyapunov function and control law which they given are not easy to get, we will give another way to get continuous control law.

2 Main result

This paper is concerned with stabilization problem for standard discrete nonlinear control system of the form

$$x_{k+1} = f(x_k) + u_k g(x_k), \tag{2.1}$$

where f and g are continuous on R^n and $f(0) = 0$. We say that the system (2.1) is globally asymptotically stable at the origin, if there exists a map $x \to u(x)$ such that the resulting system

$$x_{k+1} = f(x_k) + u(k) g(x_k), \tag{2.2}$$

is globally asymptotically stable at $0 \in R^n$.

Definition 1 A smooth, proper and positive definite function $V : R^n \to R$ is said to be a control Lyapunov function for the discrete nonlinear system (2.1) if and only if

$$\inf_{u_k \in R} (\Delta V(x_k, u_k) = V(x_{k+1} - V(x_k)) < 0, \tag{2.3}$$

for $\forall x \in R^n \setminus 0$.

In the following, we will assume that $V(x)$ is a quadratic control Lyapunov function, that is to say there exists a positive definite matrix P such that $V(x)=x^T Px$.

Theorem 1 $V(x)$ is a quadratic control Lyapunov function for the discrete time system (2.1), if and only if $f^T Pf < x^T Px$, $\forall x \in \Omega_0$ and $(f^T Pg)^2 + g^T Pgx^T Px > f^T Pfg^T Pg$, $\forall x \in \Omega_1$.

Proof (Necessity) For any $(x,u) \in R^n \times R,$ let

$$h_x(u) = u^2 g^T Pg + 2uf^T Pg + F^T Pf - x^T Px, \tag{2.4}$$

and $\Omega_0 = \{x \in R^n \setminus \{0\} \mid g(x)=0\}$, $\Omega_1 = \{x \in R^n \setminus \{0\} \mid g(x) \neq 0\}$.

It is easy for us to get: for a given control law u_k, the difference $\Delta V = h_x(u)$. If $x_k \in \Omega_0$, the difference does not depend on u_k, so the function V is a control Lyapunov function for the system (2.1).

If $x_k \in \Omega_1$, $g(x_k)^T Pg(x_k) > 0$, there exists $u_k \in R$ such that $h_{x_k}(u_k) < 0$, thus $(f^T Pg)^2 + g^T Pgx^T Px > f^T Pfg^T Pg$.

(Sufficiency) If $f(x_k)^T Pf(x_k) < x_k^T Px_k$, $\forall x_k \in \Omega_0$, then $h_{x_k}(u_k) < 0$ for any $u_k \in R.$ Since $(f(x_k)^T Pg(x_k))^2 + g(x_k)^T Pg(x_k)x_k^T Px_k > f(x_k)^T Pf(x_k)g(x_k)^T Pg(x_k)$, $\forall x_k \in \Omega_1$, and then there exists $u_k = -\dfrac{f^T(x_k)Pg(x_k)}{g(x_k)Pg(x_k)}$ such that $h_{x_k}(u_k) < 0.$ So $V(x) = x^T Px$ is a quadratic control Lyapunov function for the system (2.1). The proof is end.

Proposition 1 If $V(x)$ is a quadratic control Lyapunov function for the discrete nonlinear system (2.1), then the control

$$u = \begin{cases} -\dfrac{f^T(x)Pg(x)}{g^T(x)Pg(x)}, & x \in \Omega_1, \\ 0, & x \in \Omega_0. \end{cases} \tag{2.5}$$

globally asymptotically stabilizes the equilibrium $x=0$ of the system (2.1)

Proposition 2 Suppose $g(x) \neq 0$ for any $0 \neq x \in R^n$. $V(x)$ is a control Lyapunov function for the nonlinear system (2.1), then the control law (2.5) is smooth and globally asymptotically stabilizes the equilibrium $x=0$ of the system (2.1).

If $\Omega_0 \neq \phi$, the control law (2.5) maybe discontinuous. We can modify the control law u that make it be continuous.

Proposition 3 Let

$$u_1(x) = \frac{-f^T(x)Pg(x) - \sqrt{(F^T(x)Pg(x))^2 - g^T(x)Pg(x)f^T(x)Pf(x) + g^T(x)Pg(x)x^T Px}}{g^T(x)Pg(x)},$$

$$u_2(x) = \frac{-f^T(x)Pg(x) + \sqrt{(F^T(x)Pg(x))^2 - g^T(x)Pg(x)f^T(x)Pf(x) + g^T(x)Pg(x)x^T Px}}{g^T(x)Pg(x)},$$

then

$$\lim_{x\to\Omega_0} u_1(x) = -\infty;\ \lim_{x\to\Omega_0} u_2(x) = +\infty.$$

Proposition 4 Let $F: R^n \to R$,

$$F(x) = \begin{cases} (u_1(x), u_2(x)), & x \in \Omega_1, \\ (-\infty, +\infty), & x \in \Omega_0. \end{cases}$$

then $(x, F(x))$ is a convex open set, and there exists continuous function $u(x) \in F(x)$ such that: $u(0) = 0, h_x(u(x)) < 0$.

Theorem 2 If there exists a quadratic control Lyapunov function for the discrete time system (2.1), there exist infinite continuous stabilizing feedback $k: R^n \to R$, and $k(0) = 0$.

Next, we will use two methods to prove there exist continuous stabilizing feedback law.

Proof (1) Let $\Omega_\varepsilon = \Omega_0 + \varepsilon B$, obviously Ω_ε is a convex open set, so for any $x \in \Omega_\varepsilon$, there exist $x_\varepsilon \in \partial\Omega_\varepsilon$, $x_0 \in \partial\Omega_0$ and $0 < \lambda < 1$ such that $x = \lambda x_\varepsilon + (1-\lambda)x_0$.

Let $u(x) = \dfrac{u_1(x) + u_2(x)}{2}, x \notin \Omega_\varepsilon$,

$\Gamma(x) = \lambda u(x_\varepsilon), x = \lambda x_\varepsilon + (1-\lambda)x_0 \in \Omega_\varepsilon, x_\varepsilon \in \partial\Omega_\varepsilon, x_0 \in \partial\Omega_0$. Then $u(x), \Gamma(x)$ are all continuous function, let

$$k(x) = \begin{cases} u(x), & x \notin \Omega_\varepsilon, \\ \Gamma(x), & x \in \Omega_\varepsilon. \end{cases} \tag{2.6}$$

thus $k(x)$ is a continuous stabilizing feedback law.

(2) Take two continuous functions

$$\psi(t) = \begin{cases} 0, & t \le 0, \\ t, & 0 < t < 1, \\ 1, & t \ge 1. \end{cases} \tag{2.7}$$

for any $c > 1$, and

$$\psi(t) = \begin{cases} 0, & t \le 0, \\ t, & 0 < t < 1, \\ 1, & t \ge 1. \end{cases} \tag{2.8}$$

Then, for $x \in \Omega_1$, we let

$$\alpha(x) = (u_1(x)+1)\psi(u_1(x)+1) + (u_2(x)-1)(1-\psi(u_2(x)-1)). \tag{2.9}$$

$$\beta(x) = \varphi(u_2(x) - u_1(x)), \tag{2.10}$$

$$u(x) = \frac{u_1(x) + u_2(x)}{2}. \tag{2.11}$$

Using the same way of [4], we can prove

$$k(x) = \begin{cases} 0, & x \in \Omega_0, \\ \alpha(x)\beta(x) + (1-\beta(x))u(x), & x \in \Omega_1. \end{cases} \tag{2.12}$$

is a continuous stabilizing feedback law.

3 Example

Consider a discrete nonlinear system

$$x_{k+1} = ax_k^2 + (x_k + b)u_k, \tag{3.1}$$

where $0 < a < 1, b > 0$. Let $V(x) = x^2$, then we can get

$$H_x(u) = (x+b)^2u^2 + 2ax^2(x+b)u + a^2x^4 - x^2,$$

$$u_{1,2} = \frac{-ax^2 \pm |x| sgn(x+b)}{x+b}, \tag{3.2}$$

By the Theorem 2, the control law

$$u(x) = \begin{cases} -\dfrac{2ax^2}{x+b}, & x \notin (-b-\varepsilon, -b+\varepsilon), \\ -\lambda\dfrac{2a(\varepsilon-b)^2}{\varepsilon}, & x = -b+\lambda\varepsilon, 0 < \lambda \le 1, \\ \dfrac{2a\lambda(b+\varepsilon)^2}{\varepsilon}, & x = -\lambda\varepsilon - b, 0 < \lambda \le 1. \end{cases} \tag{3.3}$$

is continuous and globally asymptotically stabilizes the equilibrium $x = 0$ of the system (3.1).

Acknowledgements: This paper supported by the project 14B110005 of Education Department of Henan Province

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