Journal of Progressive Research in Mathematics(JPRM)
ISSN: 2395-0218

SCITECH
RESEARCH ORGANISATION

Volume 14, Issue 1
Published online: August 25, 2018|

Journal of Progressive Research in Mathematics
www.scitecresearch.com/journals

An economic growth system based on cause-n-effect

M.M. Khoshyaran

Abstract. In this paper an economic system is represented as a Causal Dynamical Network, (CDN). Each link of CDN exists due to cause-n-effect. each node is either a demand or supply node. The link between demand, and supply exists due to the existence of causality, here is taken as consumer/producer surplus which is a function of preference manifold. Growth is the existence of entropy in CDN. Entropy is measured as a metric probability, which measures change in local equilibrium. Local equilibrium is equilibrium on disordering locality links.

ACM: 52C45

Key words: Growth; Cause-n-effect; utility manifold; equilibrium; Causal Dynamical Network; disordering locality; dialectic complexity; dialectic causality; entropy.

1 Introduction

The objective of this paper is to build a systematic approach to the analysis of economic growth. The idea is that economic growth is a function of cause-n-effect. A cause is an stimuli such as demand, utility, and surplus. An effect is the consequence of the existence of an stimuli; which in this case could be supply, prices, and choice of products. Though other cause-n-effect factors can be considered, in the present paper only the above mentioned elements are taken into account. An economic system is considered to be similar to a network consisting of nodes and links. Each node is either a cause or an effect. A link exists if there is a causal relationship between an entry nodes and an exit node. The network that is the most appropriate to represent this concept is the Causal Dynamical Network (CDN), [1], [2], [3], [4], [5] in relation to dialectic complexity. CDN is a connected graph. Connectivity is the result of dynamical interaction of cause-n-effect. Therefore CDN is time independent. Connection is turned “on" and “off" based on the existence of causality. One particularity of this system is the occurrence of disordering locality links, [7]. Disordering localities occur when there exists random cause-n-effect. CDN is called a complex system if there exists dialectic causality, [6]. Dialectic causality occurs when there are several causes for an effect and these causes seem to be contradictory from each other. In addition, there exists a certain dynamics among these causes. The same can be said about the effects. Each cause can have many effects, and these effects may seem to be contradictory from each other. In addition, there exists a certain dynamics among the effects.

In this paper cases are considered where there is one or several causes for an effect.

Multiple causes are either compatible or dialectic causes. Each cause is considered to be a tensor, and each effect is in a tensor field, [8]. The number and thus the stability of the links of a CDN depend on the existence of dialectic causality. It is this dialectic causality that defines the level of complexity of a CDN. In this context dialectic causality is synonymous with dialectic complexity. The degree of the complexity of a CDN varies depending on the existence of direct and dialectic causality in regular and disordering locality links. The higher the dialectic complexity, the higher the entropy of a CDN. Dialectic complexity can vary from high to medium to low levels. Dialectic complexity can be measured by calculating the entropy, [9] of a CDN. The entropy of a CDN is measured using metric probability in the Universal Probability Space, (UPS) introduced in [10], and [11].

Entropy in the context of an economic system is an indicator of economic growth, [12].[13], [14], [15], [16]. Entropy is at maximum if there exists a stable general equilibrium,[17], [18] where consumer and producer utility functions are fixed and are not differentiable any longer. Entropy decreases as a function of dialectic causality and its representation, the disordering locality links. Dialectic causality corresponds to evolutionary consumer and producer utility manifolds. Utility is considered to be a manifold as it is a function of a multitude of factors such as consumer/producer surplus, quantity of goods produced, and preferences. Consumer/producer preferences are functions of interpretation and it's derivatives which are the elements adjacent to it, [19]. Since interpretation is prone to unexpected or random changes, it is therefore, considered to be the main factor in producing dialectic causality, and in consequence disordering locality links. It is stated that dialectic causality produces non-locality or disordering locality links. In resume, it is stated that growth occurs when disordering locality links are present in a CDN. The more the number of disordering locality links the higher the growth level. This is proven in the next section. The schematics of the dynamics of dialectic complexity and entropy are shown in Figure 1. In Figure 1, the existence of dialectic causality leads to positive entropy $(H \geq 0)$ which leads to an evolution in a CDN. If dialectic complexity is low, then eventually, the network can reach a local equilibrium, meaning that no new change is occurring locally in the network. This could eventually break down, and lead to a new entropy and the cycle continues. Or on the other hand, local equilibrium can spread to a general equilibrium, $(H = 0)$. General equilibrium leads to a static state, which is a static network that stays fixed. On the other hand accidental or unexpected dialectic causality changes the state of the network back to an entropic state and a new cycle begins. The overall objective is to explain economic growth as a function of dialectic complexity in the context of a CDN, and to analyse its' characteristics, [20].

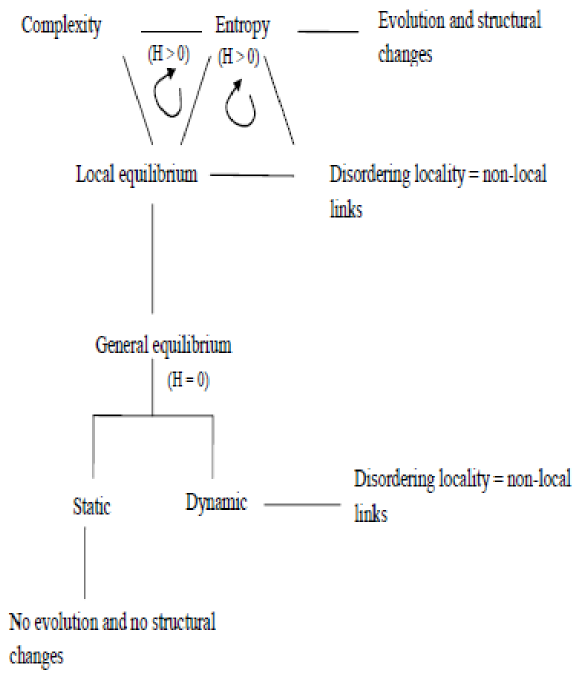

Figure 1. Dialectic Complexity

2 Analysis of growth based CDN

In order to understand the growth based CDN, it is compared to a standard representation of growth by a network. A standard representation of growth based network is given in Figure 2. Each link of this network consists of two nodes. An entry node is denoted by (D_i), where (D) represents demand, and (i) is a node number. The exit node is denoted by (S_i), where (S) represents supply. The link between every supply-demand node exists through utility as a function of price,$(u_i(p_i))$. For a core CDN, utility is chosen as a causality. Given that utility is chosen to be the function of price, there is a direct (simple) causality between cause and effect. This is chosen intentionally in order to constitute a core CDN, with no disordering locality links. The link between every supply-demand nodes is depicted as a solid curved arrow on the graph. Some fraction of demand (D_i) goes to supply $(S_j; j \neq i)$, meaning that some residual demand exists for other products supplied. Let this fraction be denoted by (δ_i). The residual demand is denoted by$(\delta_i \times D_i)$. If the network is at equilibrium, then for each supply (S_i), there exists, $(S_i = \sum \delta_i \times D_i)$, an equal demand. This equilibrium is Pareto optimal if the equilibrium is stable. An stable equilibrium implies that the utility function can not be differentiated any longer. In other words, no change in the network through coalition can improve the status of the network. At an stable equilibrium, this network constitutes a core network. Once the network reaches a core status, entropy is at maximum, (H=1). For growth to occur, it is necessary and sufficient to introduce disordering locality links. These links are not time dependent, but rather occur as a result of cause-n-effect, and create connections that can be superimposed on the core network. The cause is the existence of a new form of utility that is a function of consumer surplus. Consumer surplus itself is defined in terms of preference and the quantity consumed. Preference is ranking the desirability of a product. preference is the result of (knowledge accumulation, interaction, expectation, prediction, and interpretation), all of which are external to economic factors such as price, and capital which constitute an economic system. An example of preference creating method is a marketing strategy aiming at introducing a new product, or a novel communication method that is based on interaction with consumers.

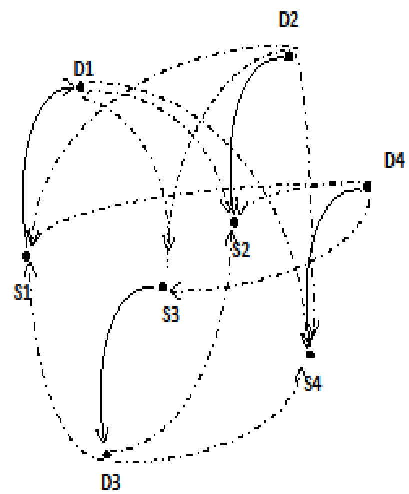

Figure 2. A standard representation of growth based network

Consumer surplus as a function of preference and quantity consumed is shown in Figure 3. In Figure 3, consumer surplus is the area, (Δ) under the demand (D), $(\Delta(0aq^c_{max}))$. In Figure 3, demand is a function of quantity consumed, (q^c_i), and preferences, (ρ^i), (i) stands for product number. Consumer surplus, $(\Delta(0aq^c_{max}) = \frac{1}{2} \times \rho^i_{max} \times q^c_{max})$ is equal to (1/2) times preference level times quantity consumed at the corresponding level of preference. In general this area is equal to $(\Delta(0aq^c_i) = \frac{1}{2} \times \rho^i \times q^c_i)$, where (q^c_i) is the level of consumption that corresponds to the level of preference. Since preference is a multi-dimensional element, involving components such as (knowledge accumulation, interaction, expectation, prediction, and interpretation), then consumer surplus should be considered as a manifold. The (5) elements mentioned are not exclusive, many others can be identified and included. Each of the elements of preference are tensors. This makes consumer utility function also a manifold. It is assumed that consumer surplus, is equal to consumer utility, (u^c_i) which is formulated as $(u^c_i = \frac{1}{2} \times \rho^i \times q^c_i)$. It is stated earlier that demand is a function of consumer surplus, $(D_i = D(u^c_i) = \frac{1}{2} \times \rho^i \times q^c_i)$. Let (Γ) be the space of all preferences such that $(\rho^i \in \Gamma)$, and let (Ω) be the space of all possible levels of consumption such that $(q^c_i \in \Omega)$,then $(D(u^c_i) \subset (\Gamma \times \Omega))$ is a sub-manifold in the space of all possible preference-consumption manifolds $(\Gamma \times \Omega)$. Producer surplus is a mirror image of consumer surplus, and is equal to $(\Delta(0a\rho^d_{max}))$, as is shown in Figure 3. In general, this area is equal to $(\Delta(0a\rho^i))$, where (ρ^i) represent a preference level for product (i). The same preference level (ρ^i) is assumed for producer as well. Producer surplus ($u_i^s = \frac{1}{2} \times \rho^i \times q_i^s$), where ($q_i^s = q_i^c$), the quantity produced is equal to the quantity consumed. The subscript (s) is used for formulation use only. Supply ($S_i = S(u_i^s) = \frac{1}{2} \times \rho^i \times q_i^s$) is a function of producer surplus. Therefore supply is a sub-manifold in the preference-consumption space ($\Gamma \times \Omega$), ($S(u_i^s) \subset (\Gamma \times \Omega)$). [21]

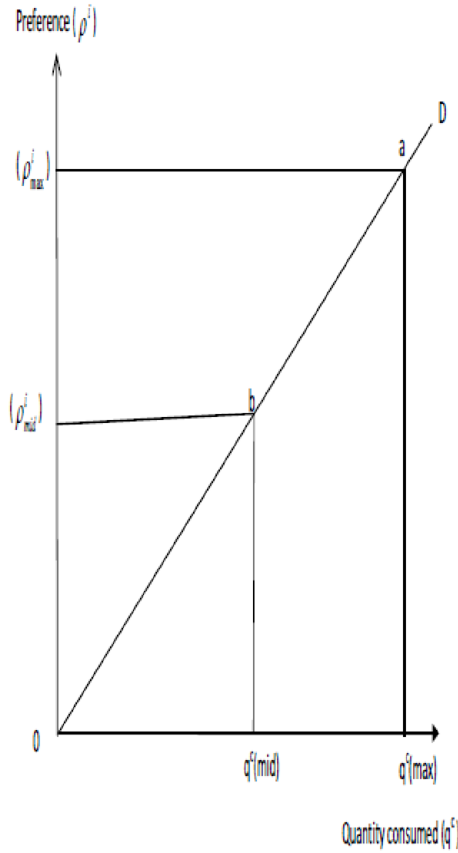

Figure 3. Consumer/producer surplus based on preferences

Formulating consumer/producer surplus as a function of preference allows for the introduction of disordering locality links in the core network shown in Figure 2. An example of disordering locality links is given in Figure 4. In Figure 4, all the bold dashed links (D_1, S^*), (D_2, S^*), (D_3, S^*), (D_4, S^*) are disordering locality links. These links appear due to the existence of cause-n-effect. The location of a new supply node, (S^*) is not random. The location is a function of the nature of product supplied. If the product is complementary, then the new supply (S^*) is close to its' complementary counterpart, otherwise, if the product is competitive, then the new supply node is a distance from its' competition. Cause is an evolution or change in consumer/producer surplus, and the effect is a new supply (S^*). It is the correlation between the consumer surplus and the producer surplus through preferences that creates a link between demand for product (i), (D_i), and a new supply (S^*). As a result of the existence of disordering locality links, the fraction of demand (δ_i) is modified and is different from the ones' of the core network. The existence of disordering locality links, means that the entropy is lower than the one of the core network,$(0 < H < 1)$. The higher the number of disordering locality links the lower the entropy level.

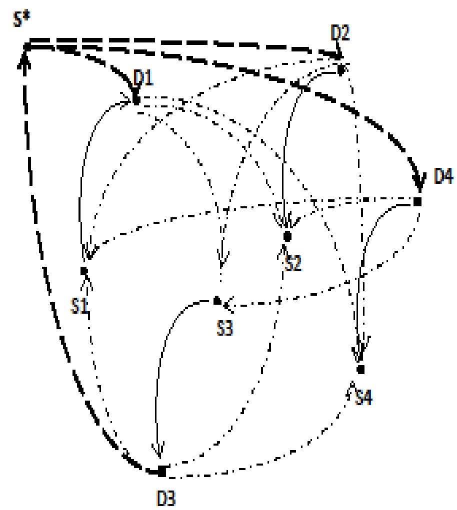

Figure 4. Example of a CDN with disordering locality links

In a CDN representation of economic growth, it is entropy that signals the existence or absence of growth. It is said that general equilibrium is maximum entropy in a network. If equilibrium is based on pricing, and welfare theorems, [22], [23] , then (2) possibilities exist: equilibrium is either unique, and thus stable, or there exists multiple equilibrium solutions, that are unstable, [24]. Since disordering locality links are constructed based on consumer/producer manifolds, equilibrium becomes equilibrium manifolds. Given the nature of consumer/producer surplus, equilibrium manifolds are not smooth manifolds, and thus become unstable. The advantage of this property is that it allows for change in supply and demand. It will be shown that in this case local equilibriums occur, but they are not unique, and are unstable.

Theorem 2.1. *Given that CDN contains disordering locality links, then the equilibrium manifolds of this network are open sub-manifolds of preference-consumption $(\Gamma \times \Omega)$ space.*

Proof. At equilibrium, $(S_j^* = D_i; j \neq i)$, where (S_j^*) is a new supply, the occurrence of which is due to dialectic causality. Equilibrium occurs when there is a strong connection between cause-n-effect. Since cause in this case is a dialectic element, then it is possible that many variations of cause would also be strongly related to effect. Let (ρ) be a sub-manifold of preferences in space $(\Gamma \times \Omega)$, $(\rho \subset \Gamma \times \Omega)$, if there exists another sub-manifold of preferences (ρ'), then this sub-manifold is homeomorphic to (ρ), which makes any equilibrium based on (ρ'), $(S_j^*(\rho') = D_i(\rho'); j \neq i)$ also homeomorphic to $(S_j^*(\rho) = D_i(\rho); j \neq i)$. Both $(S_j^*(\rho) = D_i(\rho) \subset \Gamma \times \Omega)$, and $(S_j^*(\rho') = D_i(\rho') \subset \Gamma \times \Omega)$ are sub-manifolds of space $(\Gamma \times \Omega)$. Thus the intersection of $((S_j^*(\rho)) = D_i(\rho) \cap S_j^*(\rho') = D_i(\rho'))) \neq 0)$ is a non-empty sub-manifold. Therefore the sub-manifolds of preference-consumption space $(\Gamma \times \Omega)$ are open sub-manifolds. □

Corollary 2.2. *Let $(\pi = \rho \to \rho')$ be a mapping from one preference manifold, (ρ) to another manifold, (ρ') is a preference manifold result of a mapping (π). Then any differential of (π) denoted by $(d(\pi) : T_p(\rho) \to T_p(\rho'))$ is a differentiable map that is onto. (p) signifies projection.*

Proof. Given theorem 2.1, since all sub-manifolds of preference-consumption space $(\Gamma \times \Omega)$ are open sub-manifolds, then the rank of any differentiable mapping is the same as the rank of the mapping itself for any set of preferences. Therefore, any differentiable mapping is an onto mapping. □

Let $(\mathfrak{B}(\rho))$ denote a particular property of one of the elements of preference : knowledge accumulation, (κ),interaction, (i_τ),expectation, (e_τ),prediction,(p_τ) and interpretation, (ι),that is satisfied at equilibrium. Each of the elements of preference are tensor sets. If $(\mathfrak{B}(\rho))$ exists for any disordering locality link, then the link is at equilibrium, and this equilibrium is a local equilibrium. Local equilibrium is an equilibrium that exists for a specific disordering locality link, and satisfies a particular property, $(\mathfrak{B}(\rho))$, and does not affect any other disordering locality links. The following Theorem demonstrates that local equilibrium for any disordering locality link constitutes a closed covering of the differentiable mapping. This means that local equilibriums are not stable, and they are infinite in nature. In general, the preference here given as a cause evolves in two ways: horizontal projection, $(T_p(\rho))$, where (p) stands for projection, and vertical projection, $(V_p(\rho))$. Let (ρ) be a preference manifold, and (ρ') be a mapping such that $(\mathfrak{B}(\rho \backslash \iota))$ is a particular form of interpretation that is adopted. Horizontal projection, $(T_p(\rho))$, is a diffeomorphism such that there exists an open covering of $(\mathfrak{B}(\rho \backslash \iota) \in \rho')$ that corresponds to the same point in (ρ). Horizontal transformation in this case is preference tensor (ρ) that is multiplied by a vector of constants. Thus horizontal transformation is an increase in the magnitude of preferences, $(\frac{\partial \rho}{\partial \iota}) = (\mathfrak{B}(\rho) \backslash \iota) \otimes \rho)$, where $(\mathfrak{B}(\rho \backslash \iota))$ is a vector of constants. In general, vertical mapping is the differential of the preference manifold given a particular property of one of its' elements, (κ), (ι), (i_τ), (e_τ), and (p_τ). In this example, it is the particular property of interaction (ι) that causes local equilibrium. Vertical mapping is then defined as $(\frac{\partial \rho}{\partial \iota} = \frac{\partial((\mathfrak{B}(\rho \backslash \iota))}{\partial \iota} = 0)$, and thus is equal to zero. By Corollary 2.2, this transformation is an onto transformation, and thus vertical mapping, $(V_p(\rho))$ is a closed covering.

Theorem 2.3. *Any vertial mapping of preference manifold is a closed covering.*

Proof. Let the elements that make up preference manifold be denoted by a tensor set, $(\rho(x_i^j) = (x^1, x^2, \cdots, x^n; j = 1, \cdots, n)$, where each $(x_i^j; j = 1, \cdots, n)$ contains a number of elements of the preference manifold. It is shown that for a preference manifold with a particular property $(\mathfrak{B}(\rho(x_i^l)); l \in j)$, vertical mapping is the differential of $(\rho(x_i^j))$ with respect to $(\mathfrak{B}(\rho(x_i^l)))$, and is equal to zero. This can be generalized. Each preference $(\rho(x_i^j))$ is a tensor set such that the intersection of this tensor set with another tensor set, $(\rho(x_i^{j'}))$, $(\rho(x_i^j) \cap \rho(x_i^{j'}) = \mathfrak{B}(\rho(x_i^l)))$ is a non-empty set since by Corollary 2.2, both sets, $(\rho(x_i^j))$, and $(\rho(x_i^{j'}))$ are isomorphic tensor sets. For any specific $(\rho(x_i^j)))$, there exists a specific $(\rho(x_i^{j'}))$. Therefore, vertical mapping, $(\frac{\partial\rho(x_i^j)}{\partial(x_i^l)} = \frac{\partial\rho(x_i^{j'})}{\partial(x_i^l)} = \frac{\partial(\mathfrak{B}(\rho(x_i^l)))}{\partial(x_i^l)} = 0)$, is a closed covering. □

Theorem 2.4. *Both horizontal mapping (open covering), $(T_p(\rho))$, and vertical mapping, (closed covering), $(V_p(\rho))$, of disordering locality links at local equilibrium are infinite sets.*

Proof. By Theorem 2.1, horizontal mapping $(T_p(\rho))$ is in the preference-consumption space, $(\Gamma \times \Omega)$, $(T_p(\rho) \in (\Gamma \times \Omega))$, since it is a projection of an equilibrium manifold. Thus horizontal mapping, $(T_p(\rho))$ is an open covering. An open covering is not compact and discrete. By Corollary 2.2, both horizontal and vertical mappings, $(T_p(\rho))$,$(V_p(\rho))$ are isomorphic. By Theorem 2.3, vertical mapping $(V_p(\rho))$ is a closed isomorphic covering such that the intersection at equilibrium of horizontal and vertical mappings is $(T_p(\rho) \cap V_p(\rho) = Z)$, where (Z) contains sets of zeros and constants corresponding to particular characteristics of one or several elements of preference manifold (ρ). Given the structure of preference manifolds, the preferences are either discrete rankings, or can be extended to continuous rankings based on the nature of tensor sets. Therefore there exists an infinite number of sets (Z), and thus both horizontal and vertical mappings, $(T_p(\rho))$, and $(V_p(\rho))$, are infinite tensor sets. □

At this point it must be shown that given disordering locality links, the number of local equilibria, is not constant, it depends on horizontal and vertical coverings. There exists variable local equilibrium points. The advantage of variable number of local equilibria for any disordering locality link, over standard number of local equilibria based on pricing and individual endowment (budget), is that local equilibrium evolves. This signifies that an economy can grow in a continuous manner without any interruptions, or discontinuities. Constant number of local equilibria based on pricing and budget constraint implies that there must be interruptions in an economy and this is structural. In the classical approach, the constant number is proven based on pairwise disjoint union of connected components of a set of prices, and a set of endowments. For each pairwise connected component, there exists a fixed number of equilibria. By contrast, the number of local equilibria for each disordering locality link depends on the causality which is taken to be preference manifolds. Preference manifold must for one or more of its' elements possess a particular characteristic, $(\mathfrak{B}(\rho))$ at local equilibrium. There exists an infinite number of these particular characteristics, $(\mathfrak{B}(\rho))$, and by Theorem 2.4, there exists infinite set of horizontal and vertical mappings which are differentials, then there exists an infinite number of changes in causality (preference). Therefore, there exists a infinite number of local equilibria for each disordering locality link.

Theorem 2.5. *The number of local equilibria for each disordering locality link is variable.*

Proof. Let $(N(\rho^*))$ be the number of local equilibria of a disordering locality link. $(N(\rho^*))$ is a set of integers, $(N(\rho^*) \subset \aleph)$, where $(\aleph)$ represents all integers. (ρ^*) represents all preference manifolds that induce equilibrium. By Theorem 2.4, there exists an infinite number of horizontal, and vertical coverings, $(T_p(\rho^*))$, and $(V_p(\rho^*))$.

These coverings represent sets that contain one or several equilibrium points. Constant number of local equilibria $(N(\rho^*))$ is in contradiction with Theorem 2.4, thus the number of local equilibria, $(N(\rho^*))$ is variable. □

The implication of Theorem 2.5 is significant since it implies that economic cycles can be avoided through the introduction of disordering locality links and thus dialectic causality. An economic cycle is defined to be an economic situation that jumps between two economic conditions of growth and stagnation. Dialectic causality allows for variable local equilibria, which in turn induces the emergence of new supply and partial shift in demand towards the new supply. This movement allows an economic system to reach a new local equilibrium. Based on Theorem 2.5, this process could continue indefinitely without interruptions, and therefore, would avoid jumps or discontinuities. Another pertinent topic is the Pareto optimality of local equilibria. Local equilibrium of a disordering locality link is Pareto optimal. This statement is proven in the next Theorem.

Theorem 2.6. *Local equilibrium of a disordering locality link is Pareto optimal.*

Proof. Both consumer surplus,$(u_i^c > u_t(c))$, and producer surplus $(u_i^s > u_t(s))$, where (u_t) represents utility function, are quantities that are greater than their corresponding utility functions. This is given by the definition of surplus. Surplus is the satisfaction obtained beyond the utility of either consumption or production. Utility function is used as the basis of allocation of resources in either demand or supply. There exists no other allocation that is greater than surplus based allocation. Therefore, local equilibrium as a function of consumer/producer surplus is Pareto optimal. □

One of the advantages of modelling growth based on consumer/producer surplus as a function of preferences here chosen to be cause (demand) for effect (supply) is that growth becomes predictable in the presence of disordering locality links. The predictability of growth on disordering locality links can be proven if local equilibria are homeomorphic, and diffeomorphic to each other. For example, let one local equilibrium be denoted by $(D_j(u_j^c(\rho^1)), S_j(u_j^s(\rho^1)))$, and another equilibrium be denoted by $(D_j(u_j^c(\rho^2)), S_j(u_j^s(\rho^2)))$, where (ρ^1), and (ρ^2) are two seperate preference manifolds. By Corollary 2.2, it is shown that the second local equilibrium is homeomorphic to the first local equilibrium. This due to mapping from $(D_j(u_j^c(\rho^1)), S_j(u_j^s(\rho^1)))$ to $(D_j(u_j^c(\rho^2)), S_j(u_j^s(\rho^2)))$. By Corollary 2.2, this mapping is onto, and therefore homeomorphic. Both preference manifolds, (ρ^1), and (ρ^2) possess horizontal and vertical mappings, then by Corollary 2.2, the homeomorphism is a diffeomorphism. The mapping, $(D_j(u_j^c(\rho^2)), S_j(u_j^s(\rho^2)))$ is a diffeomorphism of the first local equilibrium, $(D_j(u_j^c(\rho^1)), S_j(u_j^s(\rho^1)))$.

Theorem 2.7. *All elements of preference manifolds are either simply connected, (simple metric connection), or pathconnected,(connected by a union of several simple metric connections).*

Proof. simple metric connection: Let each local equilibrium have its' corresponding preference tensor denoted by $(\rho^i; i = 1, \cdots, n)$, where (i) represents the number of the components of a preference tensor. let (λ^i) be the Eigenvalues of the preference tensor (ρ^i). Given that each equilibrium is reached for certain values of the preference tensor, and Eigenvalues represent these certain quantities, then all Eigenvalues are positive definite, $(\lambda^i \geq 0)$. For a simple connection, a metric is defined as the magnitude of the difference between two consecutive Eigenvalues, $(ds =\mid \lambda^i - \lambda^{i-1} \mid= \sqrt{(\lambda^i - \lambda^{i-1})^2})$. Path connectedness occurs when there exists $(j = 1, \cdots, N)$ local equilibria. Let the corresponding preference tensor be denoted by (ρ^i_j). In this case, there exists (j) number of metrics each denoted by $(ds_j =\mid \lambda^i_j - \lambda^{i-1}_j \mid= \sqrt{(\lambda^i_j - \lambda^{i-1}_j)^2}; \forall i)$. Preferences are path connected if there exists a sum of simple metric connections, $(ds = \sum_{j=1}^{N} ds_j)$. Simple metric connection applies when preferences are manifolds, and path connected metric applies when there exists a multitude of preference manifolds. Based on Theorem 2.1, all preference manifolds are open sub-manifolds of the preference-consumption space $(\Gamma \times \Omega)$ at equilibrium. The intersection of these sub-manifolds are non-empty. Therefore, it is possible to find a connection made by the union of several simple metric connections. □

Theorem 2.8. *Local equilibria are homeomorphic.*

Proof. Let $(D_j(\rho^1), S^*_j(\rho^1))$, and $(D_j(\rho^2), S^*_j(\rho^2))$ be two equilibrium points. Each local equilibrium point $(D_j(\rho^1), S^*_j(\rho^1) \subset (\Gamma \times \Omega))$,$(D_j(\rho^2), S^*_j(\rho^2) \subset (\Gamma \times \Omega))$, is in the preference-consumption space, $(\Gamma \times \Omega)$. By Theorem 2.7, all elements of preferences (ρ^1), and (ρ^2) are simply connected. Both equilibrium points depend on preference manifolds, (ρ^1), and (ρ^2). Therefore, they are simply connected. Thus $(D_j(\rho^2), S^*_j(\rho^2))$ is homeomorphic to $(D_j(\rho^1), S^*_j(\rho^1))$. □

Theorem 2.9. *Local equilibria are diffeomorphic.*

Proof. From Corollary 2.2, for any local equilibrium point $(D_j(\rho^1), S^*_j(\rho^1))$, if there exists a differential mapping $(d(\rho^1) = T_{\rho^1}(D_j(\rho^1), S^*_j(\rho^1)) = D_j(\rho^2), S^*_j(\rho^2))$, then this mapping is onto. Given, Theorems 2.7, and 2.8, this differential mapping is simply connected, and therefore homeomorphic. Thus any local equilibrium is diffeomorphic. □

It is shown that disordering locality links can reach equilibrium. This is a local equilibrium. Some properties of local equilibria in the context of CDN are discussed such as: 1) predictability through connectedness, and homeomorphism, 2) changeability due to diffeomorphism, and dialectic causality, and 3) Pareto Optimality. In general, local equilibria are not stable due to their causal nature. They are pareto optimal when they occur. They are predictable if they occur due to simple causality, (a cause for an effect), or (multiple compatible causes for an effect). If the causality is more complex, meaning there exists dialectic causality, then local equilibria are predictable due to connectedness, homeomorphic, and diffeomorphic properties proven in earlier Theorems. Given that due to causality local equilibria are liable to change, it is important to measure this change. Change in equilibria represents economic growth, since it implies changes in demand and supply. It is assumed that economic growth can be compared to entropy, and entropy can be measured. In the next section the measurement of economic growth through entropy is discussed.

3 Entropy as a measure of economic growth

So far, an economic system is modelled as a network of nodes connected by links. The entry node of each link represents demand and the exit node of each link represents supply. demand and supply are connected by a causal relationship. Cause is considered to be preference, and it is demand/supply surplus as a function of preference that makes a connection between the entry and exit points. These links are called disordering locality links. Local equilibrium occurs when demand is equal to supply. Equilibrium on the network is an indication of maximum entropy, (H=1). Growth occurs when there is change in entropy. Growth occurs in multiple forms. If there exists a cause for an effect or there exists several compatible causes for an effect, then growth can occur as a horizontal mapping. In this case, growth is comparable to topological entropy. Here, growth can be considered, as a numerical invariant or topological entropy similar to an orbital growth in a dynamical system. $(T_P(\rho) = \rho \times c)$, where $(T_P(\rho))$ is a horizontal mapping of any local equilibrium point as a function of preference, (ρ), and (c) represents a constant, $(c \in \Re)$. $(\Re)$ is the space of all real numbers. Horizontal growth, is a generalized linear projection. Growth can be a change in entropy due to vertical mapping. In this case, growth is the differential of the existing local equilibrium. This type of growth is isomorphic to local equilibrium point. In case of the existence of dialectic causality, where there exists many competitive causes for an effect, growth or entropy is a fibre bundle. In the following section all different types of growth are analysed in details, and their entropy measures are given.

Orbital growth or topological entropy is the simplest form of growth. Any change in any of the components of preference manifold leads to a change in consumer/producer surplus. This in turn causes change in demand and supply. Thus if a disordering locality link is at local equilibrium, it holds no longer. This change is considered to be growth since by definition growth is any change in local equilibrium which affects the entropy of a CDN. Let orbital growth or topological entropy be denoted by (τ_o). Since (τ_o) is the result of a horizontal mapping, then $(\tau_o = T_{\tau_o}(\rho))$, it is dependent on preference manifold. Based on Theorem 2.7 there exists a simple metric connection, then metric is denoted as (d_{τ_o}). Let the metric $(d_{\tau_o} = d_{\tau_o}(\rho^i, \rho^{i+1}))$, be the distance between an existing local equilibrium, and a new disequilibrium created by change in preference manifold. (ρ^i), is the preference manifold at local equilibrium. (ρ^{i+1}), is the preference manifold after causal disturbance. Given that orbital growth (τ_o) is a horizontal mapping, then by Theorem 2.8, due to homeomorphism of horizontal mapping, there exists a sequence of metrics, $(d^i_{\tau_o}; i = 1, \cdots, N)$. Let each preference manifold be a tensor of general linear group type $(GL(n, \Re))$. Let $(\lambda^i_j; (j = 1 \cdots, n))$, be the Eigenvalues of the preference tensor for each (i). By Theorem 2.7, the metric is denoted by, $(d^i_{\tau_o}(\rho^i_j, \rho^{i+1}_j) = d^i_{\tau_o}(\tau_o(\rho^i_j), \tau_o(\rho^{i+1}_j))$, where $(\tau_o(\rho^i_j))$, and $(\tau_o(\rho^{i+1}_j))$ are horizontal mappings. Simple metric is calculated as a function of the Eigenvalues, $(d^i_{\tau_o}(\rho^i_j, \rho^{i+1}_j)) = d^i_{\tau_o}(\tau_o(\rho^i_j), \tau_o(\rho^{i+1}_j)) =\mid \lambda^{i+1}_j - \lambda^i_j \mid= \sqrt{(\lambda^{i+1}_j - \lambda^i_j)^2})$. To measure entropy, let $(\wp^i_{\tau_o})$ be the metric probability that measures change with respect to prior local equilibria points. metric probability, $(\wp^i_{\tau_o})$ can be formulated as $(\wp^i_{\tau_o} = \frac{d^i_{\tau_o}(\tau_o(\rho^i_j), \tau_o(\rho^{i+1}_j))}{\sum_{i=1}^N d^i_{\tau_o}(\tau_o(\rho^i_j), \tau_o(\rho^{i+1}_j))}; \forall j)$. The first occurrence of growth or entropy on a disordering locality link, can be measured by $(\wp_{\tau_o}^{init} = \frac{d_{\tau_o}^{init}(\lambda_j^{init}, \lambda_j^i)}{\|\lambda_j^{init}\|}; \forall j; i = 2, \cdots, N)$, where $(\| \lambda_j^{init} \|)$ is the magnitude of the initial growth Eigenvalues. Metric probability $(\wp_{\tau_o})$ by definition is different from a standard probability. Standard probability is defined based on a process of observations or experiments. The outcome of an experiment is random meaning that it is not predictable. The probability of an outcome is defined as the number of times this outcome is observed when an experiment is repeated a number of times. The notion of randomness or uncertainty is fundamental in probability measure. Entropy in this context is a quantity that measures the magnitude of uncertainty. The quantity $(H = -\sum_{i=1}^{n} p_i \times log(p_i))$, where (p) would be a standard probability based on the occurrence of random events, is a suitable measure of this uncertainty. Metric probability, on the other hand as the name suggests deals with metrics or distances that are measurements, and in no case are random occurrences. The best measure of entropy when using metric probabilities is the probability itself. The higher the metric probability, the higher the entropy. This is true since in the context of a CDN, this indicates a longer distance from a point of reference which by itself is a local equilibrium. The more distant the equilibriums points are from each other the more stable they are. Stability in this context is equivalent to high entropy. Smaller distances imply lower entropy, since these distances orbit around a local equilibrium point, and constitute perturbations or oscillations that lower the stability of the equilibrium point, since any of these orbits is as acceptable as the local equilibrium point itself.

If there exists several compatible causes for an effect, then some properties of topological entropy can apply as well. In a special case where there is a direct relationship between cause and effect, it is possible to find a generator or a minimum horizontal covering. In economics, the minimum horizontal covering translates into a change in local equilibrium that is predictable. A generator exists as long as the cause. Given that there exists a horizontal covering of preference manifold, $((c \times \rho) = T_p(\rho))$, for any constant (c), and any preference manifold (ρ), then it is possible to find a sub-covering $((c_o \times \rho^o) = T_p(\rho^o))$, $(T_p(\rho^o) \subset T_p(\rho))$, that generates other sub-coverings $(T_p(\rho^i))$ such that $(T_p(\rho^i) = f^i(T_p(\rho^o)))$ all sub-coverings are functions of the generator. The whole space of horizontal covering, $(T_p(\rho))$ can be defined as $(lim_{N\to\infty} \cup_{i=1}^{N} f^i(T_p(\rho^o) = T_p(\rho))$, the limit as (i) goes to infinity of the union of all sub-coverings $(T_p(\rho^i))$ is equal to the horizontal covering space of the preference manifold, $(T_p(\rho))$.

Theorem 3.1. *If cause-n-effect are simply connected, then there exists a horizontal covering that is a generator such that $(lim_{N\to\infty} \cup_{i=1}^{N} f^i(T_p(\rho^o) = T_p(\rho))$, the limit exists and is equal to the horizontal covering space of the preference manifold, $(T_p(\rho))$.*

Proof. $(T_p(\rho^o))$ is a linear operator such that $(T_p(\rho^i) = f^i(T_p(\rho^o))$, any horizontal covering is generated by it. Let (λ_j^o) be the Eigenvalues of the operator $(T_p(\rho^o)$. Let $(\lambda_j^i; i = 1, \cdots, N)$ be the Eigenvalues of any other horizontal covering (i) generated by the operator $(T_p(\rho^o)$. For the operator $(T_p(\rho^o)$ to be a generator, then the condition $(\lambda_j^i \in \Re, \forall(i,j) : \lambda_j^i I - \lambda_j^o \subset T_p(\rho))$ must be true. Here (I) is the identity operator. If this condition exists, then the horizontal covering $(T_p(\rho^o)$ is a closed covering. Given that $(T_p(\rho^o)$ is a closed horizontal covering, then the inequality $(\| \lambda_j^i I - \lambda_j^o \|^{-1} \leq \| \lambda_j^o \|)$ must hold. This implies that $(\| \lambda_j^i I - \lambda_j^o \| \neq 0)$ is strictly positive. Therefore, the operator, $(T_p(\rho^o)$ is a generator, and if the left hand side of the inequality is summed over (i), then the sum must be less than or equal to the horizontal covering space $(T_p(\rho))$, $(\sum_{i=1}^{N} \| \lambda_j^i I - \lambda_j^o \|^{-1} \leq T_p(\rho))$. This shows that the inverse of the left hand side of the inequality summed over (i) at the limit, must be equal to the horizontal covering space, $(T_p(\rho))$. Therefore, the limit $(lim_{N\to\infty} \cup_{i=1}^{N} \| \lambda_j^i I - \lambda_j^o \| = T_p(\rho))$ exists, is finite, and is equal to $(lim_{N\to\infty} \cup_{i=1}^{N} f^i(T_p(\rho^o) = T_p(\rho))$, since $(f^i(T_p(\rho^o) = \| \lambda_j^i I - \lambda_j^o \|)$. □

In the case of multiple causality, where there exists many compatible causes for an effect, it is not possible to find a general minimum covering span. Let (ρ_j^i) represent multiple compatible causality manifolds, where $(i = 1, \cdots, N)$, and $(j = 1, \cdots, n)$. Let each causality manifold have both horizontal and vertical mappings denoted by $(T_p^i(\rho_j^i))$, and $(V_p^i(\rho_j^i))$, respectively. A minimum spanning generator exists, if for any two horizontal mappings, the intersection is a shared region of the two coverings, $(T_p^i(\rho_j^i) \cap V_p^i(\rho_j^i)) \neq 0)$ that is non-empty. It is possible to have a minimum spanning generator for these regions only. The implication for the economy and in particular in terms of economic growth is that, growth is predictable in the presence of the condition that causality manifolds have some common attributes. One particular attribute is that in the shared region, the Eigenvalues are linearly dependent.

When causality is dialectic, then each causality manifold is the result of both horizontal and vertical mappings, $(\pi^i(\rho_j^i) = T_p^i(\rho_j^i) \oplus V_p^i(\rho_j^i))$, where $(\pi^i(\rho_j^i))$ is dialectic causality manifold (i), which is the sum of the horizontal and vertical mappings. In this case, no minimum spanning generator exists, but it is possible to predict growth, by calculating all possible occurrences in terms of their locations in a CDN. The growth that is most likely to occur in the future, is the one that has the highest entropy given different metric probabilities. This idea is explained in more details. For demonstration purposes, let's consider a (2) dimensional case. Let growth on a disordering locality link be represented by a vector with horizontal and vertical projections, as is shown in Figure 5. In Figure 5, local growth is a function of preferences. preference is a plain consisting of (2) elements, (x_1, x_2).

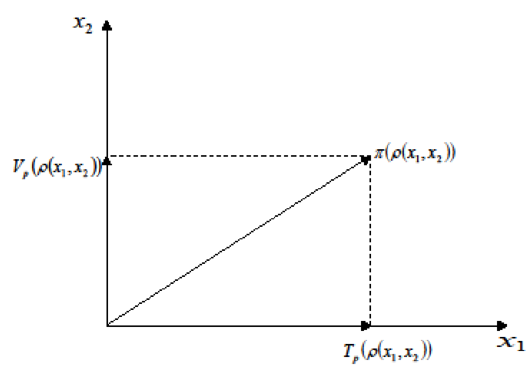

Figure 5. Representation in (2D) of the growth vector

Growth vector $(\pi(\rho(x_1, x_2))$ is the sum of the horizontal and vertical projections, $(T_p(\rho(x_1, x_2))$, and $(V_p(\rho(x_1, x_2))$. The location of the future local growth given that a disordering locality link exits is determined by calculating the differentials of the horizontal and vertical projections, with respect to change in the elements of preference, (x_1, x_2). These differentials are denoted by $(\frac{\partial T_p(\rho(x_1,x_2)}{\partial x_1})$, and $(\frac{\partial V_p(\rho(x_1,x_2)}{\partial x_2})$. Let $(\frac{\partial T_p(\rho(x_1,x_2)}{\partial x_1} = c_1)$, and $(\frac{\partial V_p(\rho(x_1,x_2)}{\partial x_2} = c_2)$, where (c_1), and (c_2), are two constants given that preference contains only two variables. In general when dialectic cause is a manifold, the solution to the differential mappings based on the change of one element is a matrix of Eigenvalues of size $(n \times n)$, here denoted by $(\lambda_j^{i,T}; j = 1 \cdots, n)$, and $(\lambda_j^{i,V}; j = 1 \cdots, n)$. Once the location of the horizontal and vertical mapping is identified, then the growth vector is the sum of the two horizontal and vertical mappings. Given the dialectic nature of the elements of cause (here taken to be preference), many horizontal and vertical mappings are possible. A metric probability can be estimated for each version of the likely local growth. The most likely future local growth is the one that presents a high entropy value. The metric probability in the two variables case is formulated as: $(\wp = \frac{\|(c_1' - c_1)\| + \|(c_2' - c_2)\|}{\|c_1 + c_2\|})$, in the two variable case, where (c_1'), and (c_2'), are the solutions to the previous horizontal, and vertical projections. In the case of the cause being a manifold, the metric probability for each possible future local growth is formulated as $(\wp^i = \frac{\|\lambda_j^{i,T} - \lambda_j^{i-1,T}\| + \|\lambda_j^{i,V} - \lambda_j^{i-1,V}\|}{\sum_{i=1}^{n} \|\lambda_j^{i,T} \oplus \lambda_j^{i,V}\|})$, where $(\lambda_j^{i-1,T})$, and $(\lambda_j^{i-1,V})$ are previous solutions of differential horizontal and vertical mappings. Given that all elements of a cause manifold can change, then the most likely local economic growth is the one with the highest entropy.

4 Conclusion

The objective of this paper is to provide a sustainable systematic method of analysing economic growth. Economic growth is defined not in terms of general equilibrium but rather in terms of the level of entropy in a network consisting of nodes and links, where each node represents either a demand or a supply for a category of product, and each link represents the relationship between supply and demand. The relationship between supply and demand nodes is called cause-n-effect. The cause is identified to be due to consumer/producer surplus. The effect is the occurrence of a link. The network constructed this way is called Causal Dynamical Network. The causal links are referred to as disordering locality links, since their location in a CDN is due to causality. The randomness is due to cause-n-effect characteristic. Cause is defined to be equivalent of consumer/producer surplus which is formulated as a function of preferences and quantity consumed/produced. This is different from the standard definition of consumer/producer surplus which is a function of price. Preference occur because it depends on factors such as (knowledge accumulation, interaction, expectation, prediction, and interpretation). Since every factor evolves due to consumer/producer experiences, level of adaptability, and differential fitness of interpretation to interaction, then preference is a dialectic element that allows for the existence of disordering locality links. The characteristic that defines these links is that they occur at random at any location in a CDN. Entropy depends on the existence of disordering locality links. The higher the number of disordering locality links the lower the entropy level, and thus the higher the economic growth. Disordering locality links can reach equilibrium, but given the dialectic nature of these links, equilibrium is local and usually not stable. These two properties are proven in the paper. Entropy for a CDN network is calculated. A new method of calculating entropy in the context of a CDN is proposed.

The advantage of the method proposed in this paper is that it provides an alternative to the standard method of economic growth estimation and is adapted to recent economic structures and new technologies and data accessibilities.

References

[1] J. Ambjorn et al., *Emergence of a 4-D world from Causal Quantum Gravity*, Phys. Rev. Lett. 93, 2004.

[2] J. Ambjorn et al., *Nonpurterbative Quantum Gravity*, arXiv:1203.3591V1, [hep-ph], 2012.

[3] F. Markopoulou, *Space Does Not Exist, So Time Can*, arXiv:0909.1861V1, [gr-qc], 2009.

[4] T. Konopka, F. Markopoulou, L. Smolin, *Quantum Graphity*, arXiv:hp-th/0611197V1, 2006.

[5] T. Konopka, F. Markopoulou, L. Smolin, *Quantum Graphity: A Model of Emergent Locality*, arXiv:0801.0861V2, 2008.

[6] A. Ferrarin, *The powers of pure reason: Kant and the idea of cosmic philosophy*, The University of Chicago Press, Chicago, 2015.

[7] L. Smolin, *Time Reborn*, Mariner Books, Houghton-Mifflin-Harcourt Publisher, Boston, 2013.

[8] D. Bleecker, *Gauge Theory and Variational Principals*, Dover Publication Inc., N.Y., 1981.

[9] A.I. Khinchin, *Mathematical Foundation of Information Theory*, Dover Publication Inc., N.Y., 1957.

[10] M.M. Khoshyaran, *On a Class of Universal Probability Spaces*, Advances in Research,vol.6, no.5, 1-7, 2016.

[11] M.M. Khoshyaran, *On a Class of Universal Probability Spaces: Case of Complex Fields*, Advances in Research, vol.8, no.2, 1-12, 2016.

[12] P. A. Samuelson, *The collected scientific papers of Paul A. Samuelson*, vol. 1, The MIT Press, Cambridge, Massachusetts, 1966.

[13] J. E. Meade, *The growing economy*, Aldine Publishing Company, Chicago, 1968.

[14] E. Helpman, *The mystery of economic growth*, Harvard University Press, Harvard, Massachusetts, 2004.

[15] H. J. Johnson, *Money trade and economic growth*, Harvard University Press, Harvard, Massachusetts, 1962.

[16] J. Mokyr, *Culture of growth: the origin of the modern economy*, Princeton University Press, Princeton, 2017.

[17] K. B. Athreya, *Big ideas in macroeconomics: A non-technical view*, The MIT Press, Cambridge, Massachusetts, 2013.

[18] K. J. Arrow, G. Debreu, *Existence of a Competative Equilibrium for a Competitive Economy*, Econometrica,vol.22, no.3, 265-290, 1954.

[19] R. J. Bogdan, *Interpreting minds*, The MIT Press, Cambridge, Massachusetts, 2003.

[20] M.M. Khoshyaran, *Causal Dynamical Networks: dialectic complexity*, Poster Presentation 29 at BIFI International Conference: Complexity, Networks and Collective Behaviour Zaragoza, Spain , 2018.

[21] Y. Balasko, *The Equilibrium manifold*, MIT Publication Inc., Boston, MA., 2009.

[22] A. M. Feldman, R. Serrano, *Welfare Economics and Social Choice Theory*, Springer, Boston, New York, 2006.

[23] J. B. DeLong, *productivity Growth, Convergence, and Welfare: Comment*, American Economic Review, vol. 78, no. 5, 1138-1154, 2005.

[24] G. Debreu, H. Scarf, *A limit theorem on the core of an economy*, International Economic Review, vol. 4, 235-246, 1963.