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A New Technique of The q-Homotopy Analysis Method for Solving Non-Linear Initial Value Problems

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Abstract.

In this paper, a new procedure of the q-homotopy analysis technique (NTq-HAM) was submitted for solving non-linear initial value problems. The NTq-HAM contains just a single convergence control parameter α. To show the dependability and proficiency of the technique, this approach is applied to solve two non-linear IVPs, and the outcomes uncover that the NTq-HAM is more general of the He's homotopy perturbation technique (HPM) [27] and the He's HPM is only special case of the NTq-HAM when $\alpha = 1$.

Keywords: q-Homotopy analysis technique, Initial value problem, Convergence control parameter.

1 Introduction

The homotopy analysis technique (HAM) is created in 1992 by Liao [19-26] . It is an analytical approach to get the series solutions of linear and nonlinear problems. The distinction with the other perturbation technique is that this method is free of small/large physical parameters. It likewise gives a simple way to guarantee the convergence of series solution [3]. This technique has been effectively connected to solve numerous linear and nonlinear partial differential equations in different fields of science and engineering by numerous authors [1-3, 6,7,14 ,19-26,28,30]. The homotopy analysis technique is valuable and proficient for obtaining both analytical and numerical. approximations of linear or nonlinear differential equations. El-Tawil M.and Huseen Sh. [4] developed a procedure namely q- Homotopy Analysis Method (q-HAM) which is a more general of Liaos Homotopy analysis method, the q-HAM contains an assistant parameter $\alpha \geq 1$ as well as h with the end goal that the instance of $\alpha = 1$ the Liaos Homotopy analysis method can be come to . The q-HAM has been effectively applied to various problems in science and engineering [4,5,8-13,15-18]. In this paper, we introduced a new technique of the q-homotopy analysis method (NTq-HAM) which is contains only one assistant parameter $\alpha \neq 0$ for solving non-linear IVPs and the outcome uncover that the NTq-HAM is more general of the homotopy perturbation technique (HPM) and the HPM is just a special case of the NTq-HAM when $\alpha = 1$.

2 The New Technique of The q-Homotopy Analysis Method (NTq-HAM)

Consider the following differential equation

$$N[w(x,t)] - g(x,t) = 0 \tag{2.1}$$

where N is a nonlinear operator , (x,t) means independent variables, $g(x,t)$ is a known function and $w(x,t)$ unknown function. Give us a chance to develop the supposed zero-order deformation equation

$$(1-\alpha q)\underline{L}[\mu(x,t:q) - w_0(x,t)] + q(N[\mu(x,t:q) - g(x,t)) = 0 \tag{2.2}$$

where $\alpha \neq 0$, q varies from 0 to $\frac{1}{\alpha}$,signifies the so - called inserted parameter , $\underline{L}$ is an auxiliary linear operator with the property $\underline{L}[g] = 0$ when $g = 0$. It is evident that when $q = 0$ and $q = \frac{1}{\alpha}$

equation (2.2) progresses toward becomes:

$$\mu(x,t;0)=w_0(x,t), \qquad \mu(x,t;\frac{1}{\alpha})=w(x,t) \tag{2.3}$$

Respectively. In this way as q increases from 0 to $1/\alpha$, the solution $\mu(x,t:q)$ changes from the initial guess $w_0(x,t)$ to the solution $w(x,t)$. Having the freedom to choose $w_0(x,t)$, $\underline{L}$, we can expect that every one of them can be legitimately picked with the goal that the solution $\mu(x,t:q)$ of equation (2) exists for $q=\frac{1}{\alpha}$ Expanding $\mu(x,t:q)$ in Taylor series, one has:

$$\mu(x,t;q)=w_0(x,t)+\sum_{m=1}^{+\infty} w_m(x,t)q^m, \tag{2.4}$$

where

$$w_m(x,t)=\frac{1}{m!}\frac{\partial^m \mu(x,t;0)}{\partial q^m}|q=0 \tag{2.5}$$

Assume that $\underline{L}$, $w_0(x,t)$ are so legitimately picked with the end goal that the series (4) converges at $q=\frac{1}{\alpha}$ and

$$w(x,t)=\mu(x,t;\frac{1}{\alpha})=w(x,t)+\sum_{m=1}^{+\infty} w_m(x,t)(\frac{1}{\alpha})^m, \tag{2.6}$$

Defining the vector $w_r(x,t) = \{w_0(x,t)\ ,\ w_1(x,t)\ ,\ w_2(x,t)\ ,\ \ldots,\ w_r(x,t)\ \}$. Differentiating equation (2) m times for q and afterward setting $q=0$ and lastly dividing them by $m!$ we have the so-called m^{th} order deformation equation

$$\underline{L}[w_m(x,t)-\mathcal{C}_m w_{m-1}(x,t)]=-\delta_m(w_{m-1}(x,t)) \tag{2.7}$$

Where

$$\delta_m(w_{m-1})=\frac{1}{(m-1)!}\frac{\partial^{m-1}(N[\mu(x,t;q)])}{\delta q^{m-1}})|(q=0) \tag{2.8}$$

and

$$C_m=\begin{cases} 0, & m\leq 1; \\ n, & m\geq 2. \end{cases} \tag{2.9}$$

It ought to be underscored that $w_m(x,t)$ for $m\geq 1$ is administered by the linear equation (7) with linear boundary conditions that come from the original problem. It ought to be noticed that the cases of $(\alpha=1)$ in equation (2), the HPM can be come to.

3 Applications

3.1 Example 1

Consider the Helmholtz equation [29]

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - w = 0, \tag{3.1}$$

with the initial conditions

$$w(0, y) = y, w_x(0, y) = y + coshy \tag{3.2}$$

The exact solution of this problem is

$$w(x, y) = ye^x + xcosh(y), \tag{3.3}$$

The problem (3.1-3.2) solved by HPM [27].To solve the problem by NTq-HAM we select the linear operator

$$\underline{L}[\mu(x, y; q)] = \frac{\partial^2 \mu(x, y; q)}{\partial y^2} \tag{3.4}$$

with the property $\underline{L}[d_1] = 0$, where d_1 is constant.
Utilizing initial approximation $w_0(x, y) = y(1+x)+xcoshy$ coshy we define a nonlinear operator as

$$N[\mu(x, y; q)] = \frac{\partial^2 \mu(x, y; q)}{\partial x^2} + \frac{\partial^2 \mu(x, y; q)}{\partial y^2} - \mu(x, y; q) \tag{3.5}$$

Let We define the zeroth-order deformation equation as follows

$$(1 - \alpha q)\underline{L}[\mu(x, y; q) - w_0(x, y)] + qN[\mu(x, y; q)] = 0$$

then , the m^{th} order deformation equation is

$$\underline{L}[w_m(x, y) - C_m w_{m-1}(x, y)] = -\delta_m(w_{m-1}(x, y)) \tag{3.6}$$

and the initial conditions for $m \geq 1$

$$w_m(x,0) = 0 \tag{3.7}$$

Such that C_m accordingly (2.9) and

$$\delta_m(w_{m-1}(x,y)) = \frac{\partial^2 w_{m-1}(x,y)}{\partial x^2} + \frac{\partial^2 w_{m-1}(x,y)}{\partial y^2} - w_{m-1}(x,y)$$

Presently the solution of problem (3.1-3.2) for $m \geq 1$ becomes

$$w_m(x,y) = C_m w_{m-1}(x,y) - \underline{L}^{-1}[\delta_m(w_{m-1})]$$

Then, the NTq-HAM components solution are

$w_1(x,y) = \frac{1}{6}x^2(3+x)y$

$w_2(x,y) = \frac{1}{6}\alpha x^2(3+x)y + \frac{1}{120}x^2(-60-20x+5x^2+x^3)y$

$w_3(x,y) = \frac{(x^2(2520+840x-420x^2-84x^3+7x^4+x^5+42n(-60-20x+5x^2+x^3))y)}{5040} + \alpha(\frac{1}{6}\alpha x^2(3+x)y + \frac{1}{120}x^2(-60 - 20x + 5x^2 + x^3)y)$

$w_m(x,y)$,$(m = 4,5,)$ can be calculated similarly. As special case if $\alpha = 1$, then we get a similar outcome got by HPM [27].Now the series solution expression by NTq-HAM can be composed in the form

$$w(x,y) \cong W_m(x,y;\alpha) = \sum_{i=0}^{m} w_i(x,y;\alpha)(\frac{1}{\alpha})^i \tag{3.8}$$

Equation (3.8) is the series solution to (3.1-3.2) in terms of the parameter α .To find the useful region of α, the α-curve given by the 10^{th} order NTq-HAM solution at specific values of x , y is drawn in figure (1).(a): This figure demonstrates the region of α where the value of $W_5(x,y)$ is constant at specific values of x and y .(b):demonstrates the region of α where the value of $W_{10}(x,y)$ is constant at specific values of x and y. Figure (2).(a):demonstrates the 10^{th} order solution of $(NTq - HAM; \alpha = 1)$ for problem(3.1-3.2) at $-1 \leq x \leq 1$, $-1 \leq y \leq 1$.(b):demonstrates the exact solution for problem(3.1-3.2) at $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. Table (1) demonstrates the Comparison between the 5^{th} order approximations of NTq-HAM at various values of α with the exact solution of (3.1-3.2). Table (2)demonstrates the comparison between the 10^{th} order approximations of errors of NTq-HAM at various values of α with the exact solution of (3.1-3.2). Table (3)demonstrates the absolute errors of W_5 of NTq-HAM at various values of α for problem (3.1-3.2) .Table (4) demonstrates the absolute errors of U_10 of NTq-HAM at various values of α For problem (3.1-3.2).

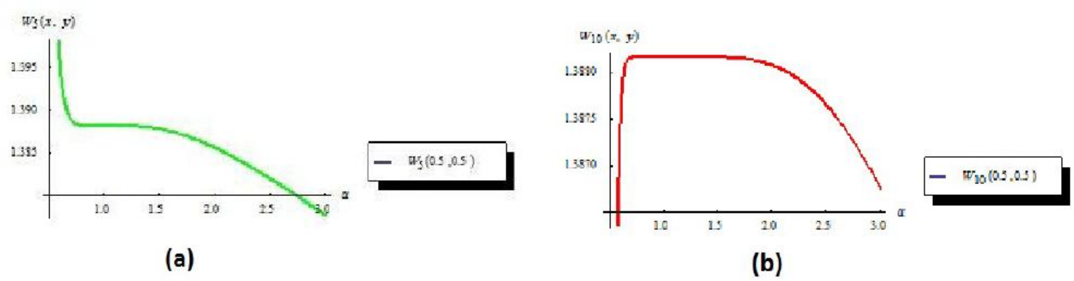

Figure 1: a: α - curve of W_5 approximate solution of (NTq-HAM) of problem (3.1-3.2) at different values of x and y, b: α - curve of W_{10} approximate solution of (NTq-HAM) of problem (3.1-3.2) at different values of x and y.

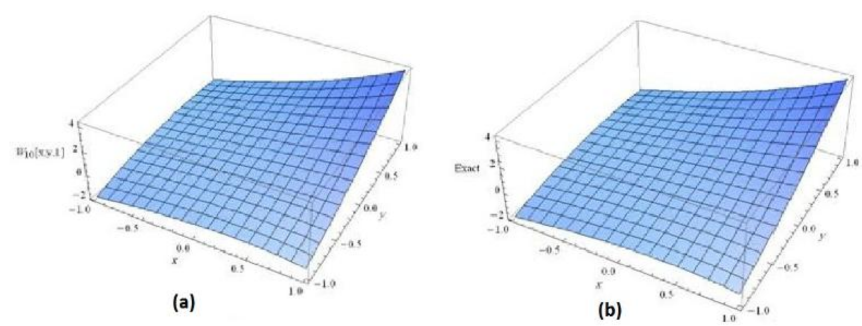

Figure 2: a: The 10^{th} order approximate solution of (NTq-HAM; $\alpha = 1$) for problem(3.1-3.2) at $-1 \leq x \leq 1$, $-1 \leq y \leq 1$ and b: The exact solution of problem (3.1-3.2) at $-1 \leq x \leq 1$,$-1 \leq y \leq 1$.

Table 1: Comparison between the 5^{th} order approximations of NTq-HAM at different values of α with the exact solution of (3.1-3.2).

x	y	Exact solution	W_5 NTq-HAM $\alpha = 1$	W_5 NTq-HAM $\alpha = 0.9$	W_5 NTq-HAM $\alpha = 1.1$
-1	-1	-1.910960075	-1.910960074	-1.910954027	-1.910938072
-0.8	-0.8	-1.429411128	-1.429411128	-1.429408496	-1.429404430
-0.6	-0.6	-1.0405661136	-1.040566113	-1.040565747	-1.040564425
-0.4	-0.4	-0.700556967	-0.700556967	-0.700557145	-0.700556656
-0.2	-0.2	-0.367759502	-0.367759502	-0.367759555	-0.367759474
-0.0	-0.0	0.0	0.0	0.0	0.0
0.2	0.2	0.448293903	0.448293903	0.448293964	0.448293872
0.4	0.4	1.029158828	1.0291588278	1.029159097	1.029158441
0.6	0.6	1.804550411	1.804550411	1.804550234	1.804548163
0.8	0.8	2.850380700	2.850380700	2.850377738	2.850371295
1	1	4.261362463	4.261362461	4.261353809	4.261330125

Table 2: Comparison between the 10^{th} order approximations of NTq-HAM at different values of α with the exact solution of (3.1-3.2).

x	y	Exact solution	W_{10} NTq-HAM $\alpha = 1$	W_{10} NTq-HAM $\alpha = 0.9$	W_{10} NTq-HAM $\alpha = 1.1$
-1	-1	-1.9109600760	-1.9109600759	-1.9109600759	-1.9109600755
-0.8	-0.8	-1.4294111280	-1.4294111283	-1.4294111283	-1.4294111282
-0.6	-0.6	-1.0405661130	-1.0405661126	-1.0405661126	-1.0405661125
-0.4	-0.4	-0.700556970	-0.7005569671	-0.7005569671	-0.7005569671
-0.2	-0.2	-0.367755010	-0.3677595017	-0.3677595017	-0.3677595017
-0.0	-0.0	0.0	0.0	0.0	0.0
0.2	0.2	0.4482939020	0.4482939027	0.4482939027	0.4482939027
0.4	0.4	1.0291588280	1.0291588277	1.0291588277	1.0291588277
0.6	0.6	1.8045504110	1.8045504111	1.8045504112	1.8045504111
0.8	0.8	2.850380700	2.8503806998	2.8503806998	2.8503806996
1	1	4.2613624630	4.2613624632	4.2613624632	4.2613624626

Table 3: The absolute errors the 5^{th} order approximations of NTq-HAM at different values of α with the exact solution of (3.1-3.2).

x	y	A.E ($\alpha = 1$)	A.E ($\alpha = 0.9$)	A.E ($\alpha = 1.1$)
-1	-1	1.9378E-9	6.0493E-6	2.2004E-5
-0.8	-0.8	1.0809E-10	2.6325E-6	6.6976E-6
-0.6	-0.6	2.6059E-12	3.6539E-7	1.6877E-6
-0.4	-0.4	1.3656E-14	3.7755E-7	3.1094E-7
-0.2	-0.2	0.0	5.2879E-8	2.7348E-8
-0.0	-0.0	0.0	0.0	0.0
0.2	0.2	5.5511E-17	6.1042E-8	3.100E-8
0.4	0.4	1.4655E-14	2.6951E-7	3.8728E-7
0.6	0.6	2.8579E-12	1.7674E-7	2.2480E-6
0.8	0.8	1.2226E-10	2.9620E-6	9.4053E-6
1	1	2.2606E-9	8.6538E-6	3.2339E-5

Table 4: The absolute errors of the 10^{th} order approximations of NTq-HAM at different values of α with the exact solution of (3.1-3.2).

x	y	A.E ($\alpha = 1$)	A.E ($\alpha = 0.9$)	A.E ($\alpha = 1.1$)
-1	-1	0.0	5.1191E-11	4.8336E-10
-0.8	-0.8	8.8818E-16	3.4923E-11	1.1423E-10
-0.6	-0.6	0.0	1.8520E-11	2.1944E-11
-0.4	-0.4	0.0	6.8301E-13	3.0093E-12
-0.2	-0.2	5.5511E-17	7.2870E-13	1.9690E-13
-0.0	-0.0	0.0	0.0	0.0
0.2	0.2	5.5511E-17	8.5265E-13	2.2188E-13
0.4	0.4	0.0	1.3567E-13	3.6533E-12
0.6	0.6	4.4409E-16	2.0841E-11	2.7989E-11
0.8	0.8	4.4409E-16	5.3536E-11	1.5147E-10
1	1	0.0	1.8180E-11	6.6263E-10

3.2 Example 2

Consider the Fisher's equation

$$\frac{\partial w}{\partial t} - \frac{\partial^2 w}{\partial x^2} - 6w(1-w) = 0, \tag{3.9}$$

with the initial conditions

$$w(x,0) = \frac{1}{(1+e^x)^2}, \tag{3.10}$$

The exact solution of this problem is

$$w(x,0) = \frac{1}{(1+e^{x-5t})^2}, \tag{3.11}$$

The problem(3.9-3.10) solved by HPM [27]. To solve the problem by NTq- HAM we select the linear operator

$$\underline{L}[\mu(x,t;q)] = \frac{\partial \mu(x,t;q)}{\partial t} \tag{3.12}$$

with the property $\underline{L}[d_1] = 0$, where d_1 is constant.
Utilizing initial approximation $w_0(x,y) = \frac{1}{(1+e^x)^2}$ we define a nonlinear operator as

$$N[\mu(x,t;q)] = \frac{\partial \mu(x,t;q)}{\partial t} - \frac{\partial^2 \mu(x,t;q)}{\partial x^2} - 6\mu(x,t;q) + \mu^2(x,t;q)$$

Let We define the zeroth-order deformation equation as follows

$$(1-\alpha q)\underline{L}[\mu(x,t;q) - w_0(x,t)] + qN[\mu(x,t;q)] = 0$$

then ,the m^{th} order deformation equation is

$$\underline{L}[w_m(x,t) - C_m w_{m-1}(x,t)] = -\delta_m(w_{m-1}(x,t)), \tag{3.13}$$

with the initial conditions for $m \geq 1$

$$w_m(x,0) = 0 \tag{3.14}$$

Such that C_m accordingly (2.9) and

$$\delta_m(w_{m-1}(x,t)) = \frac{\partial w_{m-1}(x,t))}{\partial t} - \frac{\partial^2 w_{m-1}(x,t))}{\partial x^2} - 6w_{m-1}(x,t) + \sum_{i=1}^{m-1} w_i(x,t)w_{m-1-i}(x,t)$$

Presently the solution of problem (3.9-3.10)for $m \geq 1$ becomes

$$w_m(x,t) = C_m w_{m-1}(x,t) - \underline{L}^{-1}[\delta_m(w_{m-1})]$$

Then, the NTq- HAM components solution are

$w_1(x,t) = \{\{-(\frac{6}{(1+e^x)^4} - \frac{6e^{2x}}{(1+e^x)^4} + \frac{(2e^x)}{(1+e^x)^3} - \frac{6}{(1+e^x)^2})t\}\}$

$w_2(x,t) = \{\{\{-(\frac{6}{(1+e^x)^4} - \frac{(6e^{2x})}{(1+e^x)^4} + \frac{(2e^x)}{(1+e^x)^3} - \frac{6}{(1+e^x)^2})\alpha t + \frac{(10e^x(-t-e^x t-\frac{(5t^2)}{2}+5e^x t^2))}{(1+e^x)^4}\}\}\}$

$w_3(x,t) = \{\{\{\{\alpha(-(\frac{6}{(1+e^x)^4} - \frac{(6e^2x)}{(1+e^x)^4} + \frac{(2e^x)}{(1+e^x)^3} - \frac{6}{(1+e^x)^2})\alpha t + \frac{(10e^x(-t-e^x t-\frac{(5t^2)}{2}+5e^x t^2))}{(1+e^x)^4}) + \frac{1}{3(1+e^x)^5})5e^x t(6-30t+25t^2-3\alpha(2+5t)+e^x(12-30t-175t^2+3\alpha(-4+5t))+2e^2x(3-30t+50t^2+3\alpha(-1+5t)))\}\}\}\}$

$\vdots$

$uw_m(x,y)$,$(m = 4, 5,)$ can be calculated similarly. As special case if $\alpha = 1$, then we get a similar outcome got by HPM [27]. Now the series solution expression by NTq- HAM can be composed in the form

$$w(x,t) \cong W_m(x,t;\alpha) = \sum_{i=0}^{m} w_i(x,t;\alpha)(\frac{1}{\alpha})^i \tag{3.15}$$

Equation (3.15) is the series solution to (3.9-3.10) in terms of the parameter α .To find the useful region of α , the α -curves given by the $(10)^{th}$ order NTq-HAM solution at specific values of x , t is drawn in Figure(3).(a): This figure demonstrates the region of α at where the value of $W_{10}(x,t)$ is constant at specific values of x and t .(b): demonstrates the $(10)^{th}$ order solution of (NTq-HAM;$\alpha = 1$) for problem(3.9-3.10) at$0 \leq x \leq 1$,$0.2 \leq t \leq 0.4$.Figure(4).(a): demonstrates the exact solution for problem(3.9-3.10) at $0 \leq x \leq 1$, $0.2 \leq t \leq 0.4$.(b):demonstrates the exact solution for problem(3.9-3.10)at $0 \leq x \leq 1$, $0.2 \leq t \leq 0.4$.Table (5)demonstrates the Comparison between the 5^{th} order approximations of NTq-HAM at various values of α with the exact solution of (3.9-3.10). Table (6)demonstrates the Comparison between the 10^{th} order approximations of NTq-HAM at various values of α with the exact solution of (3.9-3.10).Table (7) demonstrates the absolute errors of W_5 of NTq-HAM at various values of α For problem (3.9-3.10). Table (8)demonstrates the absolute errors of W_{10} of NTq-HAM at different values of α For problem (3.9-3.10).

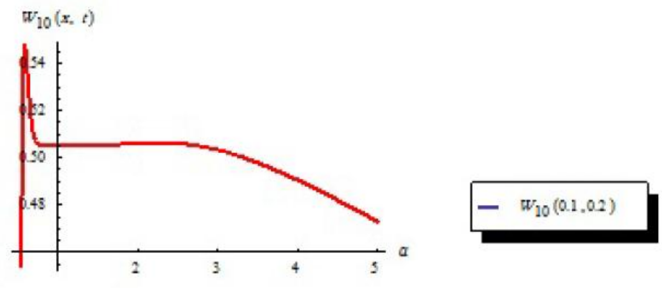

Figure 3: α - curve of W_{10} approximate solution of (NTq-HAM) of problem (3.9-3.10) at different values of x and t .

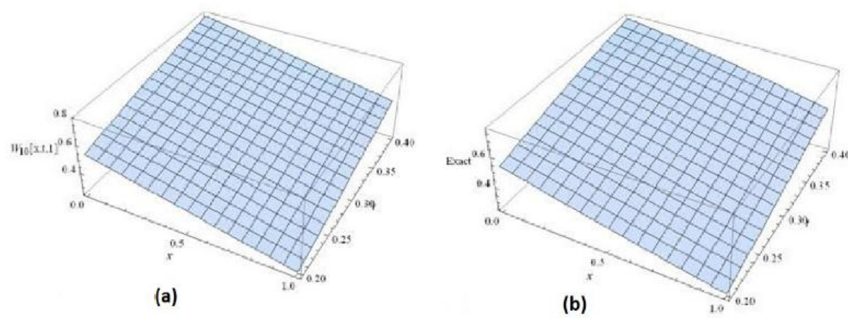

Figure 4: a: The $(10)^{th}$ order approximate solution of (NTq-HAM; $\alpha = 1$) for problem(3.9-3.10) at $0 \leq x \leq 1$, $0.2 \leq t \leq 0.4$ and b: The exact solution of problem (3.9-3.10) at $0 \leq x \leq 1$, $0.2 \leq t \leq 0.4$.

Table 5: Comparison between the 5^{th} order approximations of NTq-HAM at different values of α with the exact solution of (3.9-3.10).

x	t	W_5NTq-HAM $\alpha = 1$	W_5NTq-HAM $\alpha = 1.1$	W_5NTq-HAM $\alpha = 0.9$	Exact solution
0	0.2	0.533333333	0.533372969	0.535991295	0.534446646
0.2		0.474601536	0.475364531	0.476282907	0.476064785
0.4		0.415457157	0.416756783	0.416005274	0.416872066
0.6		0.357402715	0.358935506	0.356974830	0.358426914
0.8		0.301852414	0.303306749	0.300800990	0.302317425
1		0.250065549	0.251212791	0.248781371	0.25
0	0.4	0.733333333	0.720503133	0.802224988	0.775803493
0.2		0.664456614	0.674979181	0.696045199	0.736419595
0.4		0.609573982	0.639450515	0.603103936	0.692254593
0.6		0.570614520	0.611486926	0.534960086	0.643498991
0.8		0.542439779	0.584828027	0.491458616	0.590630343
1		0.516276064	0.552553864	0.463510566	0.534446645

Table 6: Comparison between the 10^{th} order approximations of NTq-HAM at different values of α with the exact solution of (3.9-3.10)

x	t	W_{10}NTq-HAM $\alpha = 1$	W_{10}NTq-HAM $\alpha = 1.1$	W_{10}NTq-HAM $\alpha = 0.9$	Exact solution
0	0.2	0.534451	0.534443	0.534438	0.534447
0.2		0.476063	0.476064	0.476078	0.476065
0.4		0.416866	0.416876	0.416892	0.416872
0.6		0.358421	0.358434	0.358441	0.358427
0.8		0.302315	0.302323	0.302321	0.302317
1		0.250001	0.250002	0.249998	0.25
0	0.4	0.786896	0.772285	0.809223	0.775803
0.2		0.737311	0.729524	0.776915	0.73642
0.4		0.683109	0.68612	0.715244	0.692255
0.6		0.631297	0.641865	0.641808	0.643499
0.8		0.582835	0.593994	0.574287	0.590630
1		0.533733	0.540167	0.517799	0.534447

Table 7: the absolute errors 5^{th} order approximations of NTq-HAM at different values of α with the exact solution of (3.9-3.10).

x	t	A.E ($\alpha = 1$)	A.E ($\alpha = 1.2$)	A.E ($\alpha = 0.9$)
0	0.2	1.1133E-3	1.0737E-3	1.5447E-3
0.2		1.4633E-3	7.0025E-4	2.1812E-4
0.4		1.4149E-3	1.1528E-4	8.6679E-4
0.6		1.0242E-3	5.0859E-4	1.4521E-3
0.8		4.6501E-4	9.8932E-4	1.5164E-3
1		6.5549E-5	1.2128E-3	1.2186E-3
0	0.4	4.2470E-2	5.5300E-2	2.6422E-2
0.2		7.1963E-2	6.1440E-2	4.0374E-2
0.4		8.2681E-2	5.2804E-2	8.9151E-2
0.6		7.2885E-2	32012E-2	1.0854E-1
0.8		4.8191E-2	5.8023E-3	9.9172E-2
1		1.8171E-2	1.8107E-2	7.0936E-2

Table 8: the absolute errors 10^{th} order approximations of NTq-HAM at different values of α with the exact solution of (3.9-3.10).

t	β	A.E ($\alpha = 1$)	A.E ($\alpha = 1.2$)	A.E ($\alpha = 0.9$)
0	0.2	4.5162E-6	4.1020E-6	8.6170E-6
0.2		1.8010E-6	9.8378E-7	1.3349E-5
0.4		6.3840E-6	3.6275E-6	2.0425E-5
0.6		6.3030E-6	6.5988E-6	1.4162E-5
0.8		2.7080E-6	5.8939E-6	4.0700E-6
1		1.1090E-6	2.2329E-6	2.4900E-6
0	0.4	1.1093E-2	3.5181E-3	3.3420E-2
0.2		8.9111E-4	6.8958E-3	4.0495E-2
0.4		9.1450E-3	6.1349E-3	2.2989E-2
0.6		1.2202E-2	1.6339E-3	1.6913E-3
0.8		7.7951E-3	3.3638E-3	1.6343E-2
1		7.1402E-4	5.7204E-3	1.6648E-2

4 Conclusion

In this paper, new strategy of the q-homotopy analysis method (NTq-HAM) proposed for solving linear and nonlinear IVPs. To show the dependability and productivity of the technique, this approach is applied to solve two IVPs. The accomplishment of this approach lies in the fact that the NTq-HAM provides a non zero convergence-control parameter α which can be utilized to adjust and control the convergence region and rate of the series solutions obtained. The illustrative examples recommend that NTq-HAM is a great technique for non-linear problems in science and engineering.

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