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Estimating sample size to approximate some sampling distributions by information measures

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Abstract

B-entropy measure, Fisher information measures and Akaike information criterion are considered as three different types of information measures, entropy, parametric and statistical measures respectively. The main objective of this paper is to estimate the optimal sample size under which a random variable belonging to Gamma or Poisson distribution can be approximated by a random variable following the normal distribution in the sense of the central limit theorem, based on the concept of the percentage relative error in information due to approximation. The idea is to determining the sample size for which the percentage relative error in information measure is less than a given accuracy level $100\epsilon\%$ for small $\epsilon > 0$.

Keywords: B-entropy measure; Fisher information measures; Akaike information criterion; approximate distribution; percentage relative error.

2000 Mathematics Subject Classification: 94A17.

1. INTRODUCTION

Measures of information appear in several contexts in probability and statistics [1], [2], [3], [4], [5] and [6]. Those information measures have a long history since the papers of Fisher [7] , Shannon[8] and Kullback -Liebler [9]. One important measure is the B-entropy measure [10] defined by Azzam and Awad which is based on the maximum likelihood function, and is classified as entropy measure, which is defined by:

$$B_n = -E\left(log\frac{L(\underline{X};\theta)}{L(\underline{X};\hat{\theta})}\right),$$

where, L is the likelihood function, provided that the maximum likelihood estimator (MLE) $\hat{\theta}$ exists and is unique .

Another information measure that will be considered in this paper is the Fisher information measure [7], which is classified as parametric measure and is defined by:

$$F(\theta) = -E\left(\frac{\partial^2\{logf(X;\theta)\}}{\partial\theta^2}\right)$$

where $f(X;\theta)$ is the probability density function (p.d.f) .

The third information measure is Akaike information criterion (AIC) which is classified as a statistical measure and is defined by :

$$AIC = -2log(max\ L) + 2k$$

Where k is the number of free parameters in the vector θ, and $max\ L$ is the MLE of the likelihood function of the model under consideration.

We will study the information embedded in a random variable X_n whose distribution for large n is approximated by the distribution of Y_n according to the central limit theorem. The questions that arise here are that how large n should be for this approximation to be acceptable? To answer these questions, we follow the sequential authors papers [11], [12] and [13].

The idea of this paper is as follows. Let I be an information measure, we would like to select the optimal sample size n such that the percentage relative error (PRE) is defined by

$$PRE = \left(1 - \frac{I(X_n)}{I(Y_n)}\right) 100\%$$

in each of the considered information measures is less than some given number 100ϵ % for small $\epsilon > 0$.

2. PRELIMINARY RESULTS

This section provides some results about gamma and digamma functions [10], [14] which will be used in the sequel .

2.1 Theorem

If $X \sim Gamma(\alpha, 1)$,then

$$E(logX) = \int_0^\infty \log x \cdot \frac{e^{-x} x^{\alpha-1}}{\Gamma(\alpha)} dx = \Psi(\alpha),$$

Where $\Psi(\alpha)$ is the digamma function

2.2 Remark

Let $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, $\alpha > 0$ and $\Psi(\alpha) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)}$ denote the gamma and digamma, respectively. For the computation of the digamma function one might use the following known result.

$(i)\Psi(1) = -\gamma \approx -0.577215665$

$(ii)\Psi(n) = -\gamma + \sum_{k=1}^{n-1} \frac{1}{k}$ if n is an integer ; $n \geq 2$

2.3 Corollary

If $X \sim Gamma(\alpha, \beta)$, with p.d.f

$f(x;\theta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} e^{-x/\beta} x^{\alpha-1}$ if $x > 0$ and zero otherwise,

$\theta = (\alpha, \beta)$

then $X/\beta \sim Gamma(\alpha, \beta)$ and so $E(logX) = log\beta + \Psi(\alpha)$.

2.4 Corollary

Let $Y \sim Gamma(\alpha, 1)$ denotes a gamma random variable with parameters $(\alpha, 1)$ then

$(i)\Psi(\alpha) = E\big(log(Y)\big)$

$(ii)\Psi'(\alpha) = Var\big(log(Y)\big)$

3. METHODOLOGY

3.1 Approximations by the B-entropy measure

We measure information by using B-entropy measure assuming that the observations of this units are to be independent and identically distributed (i.i.d) as a random variables X whose $p.d.f$ is $f(x, \theta)$.

3.1.1 Normal Approximation to Gamma Distribution

Let $Y = \overline{X}$, then $Y \sim G\left(n, \frac{\theta}{n}\right)$ then p.d.f of Y is

$$f_1(x;\theta) = \begin{cases} \frac{1}{\Gamma(n)\left(\frac{\theta}{n}\right)^\alpha} e^{-yn/\theta} y^{n-1} & y > 0 \\ 0 & otherwise \end{cases} \qquad (1)$$

To find the MLE for θ:

Note $logf_1(x;\theta) = -ln\Gamma(n) - nln\theta + nln(n) - \frac{yn}{\theta} + (n-1)lny$

$$\frac{\partial}{\partial\theta}\big(logf_1(y,\theta)\big) = -\frac{n}{\theta} + \frac{ny}{\theta^2} = 0 \quad \Rightarrow$$

$$\hat{\theta} = y \tag{2}$$

Now to find B-entropy for equation (1) :

$$B_G(\theta) = -E\left(log\frac{f_1(\underline{X};\,\theta)}{f_1(\underline{X}\,;\,\hat{\theta})}\right) = -E\left(log\big(\theta^{-n}e^{-yn/\theta}\big) - log\big(\hat{\theta}^{-n}e^{-yn/\hat{\theta}}\big)\right)$$

$$= -E\left(nlog\left(\frac{y}{\theta}\right) - \frac{yn}{\theta} + n\right)$$

$$B_G(\theta) = n\big(logn - \Psi(n)\big) \tag{3}$$

from which we notice that $B_G(\theta)$ is free of θ

Let Y be approximated by $X{:}\,N(\theta,\frac{\theta^2}{n})$; $\theta > 0$ with a $p.d.f$

$$f_2(y,\theta) = \begin{cases}\left(\frac{n}{n\pi\theta^2}\right)^{1/2} exp(-n(x-\theta)^2/(2\theta^2)) & x > 0, \\ 0 & otherwise\end{cases} \tag{4}$$

To find the MLE for θ, note that:

$$logf_2(x,\theta) = (1/2)logn - (1/2)log(2\pi) - log(\theta) - n(x-\theta)^2/(2\theta^2)$$

From which, $\frac{\partial}{\partial\theta}logf_2(x,\theta) = -\frac{1}{\theta} + \frac{nx}{\theta^2}(x/\theta - 1)^2/(2n) = 0 \;\Rightarrow$

$$\hat{\theta}^2 - nx\hat{\theta} - nx^2 = 0$$

Hence,

$$\hat{\theta} = Cx \tag{5}$$

where$C = \frac{n}{2}\left\{\sqrt{1+\frac{4}{n}} - 1\right\}$ $\quad ,x > 0$

To find the B-entropy for equation (4)

$$B_N(\theta) = -E\left(log\frac{f_2(\underline{X};\,\theta)}{f_2(\underline{X}\,;\,\hat{\theta})}\right) = -E\left(-log\theta - \frac{n}{2\theta^2}(x-\theta)^2 + log\hat{\theta} + \frac{n}{2\hat{\theta}^2}\big(x-\hat{\theta}\big)^2\right)$$

$$B_N(\theta) = log\theta + \frac{1}{2} + \frac{nx}{2}\left(\frac{1}{C} - 1\right)^2 + logC + logx \tag{6}$$

Therefore, the percentage relative error in the B-entropy due to this approximation is:

$$PRE = \left(1 - \frac{B_G(\theta)}{B_N(\theta)}\right)100\% \tag{7}$$

3.1.2 Normal Approximation to Poisson Distribution

Let$Y_1, \dots, Y_n$ be i.i.d from $P(\theta)$, let $X = \sum_{i=1}^{n} Y_i : P(n\theta)$. The p.d.f of X is

$$f_3(x;\theta) = \begin{cases} e^{-n\theta}(n\theta)^x/x! & if \quad x = 0,1,2,\dots \\ 0 & otherwise\end{cases} \tag{8}$$

Since$E(X) = n\theta$

To find the MLE for :

$logf_3(x;\theta) = -n\theta + xln(n\theta) - lny$

$$\frac{\partial log f_3(x;\theta)}{\partial \theta} = -n + \frac{x}{\theta} = 0 \;\Rightarrow$$

$$\hat{\theta} = \frac{x}{n} \tag{9}$$

Using Equation (8), the B-entropy is given by:

$$B_P^*(\theta) = -E\left(log\frac{f_3(\underline{X};\,\theta)}{f_3(\underline{X}\,;\,\hat{\theta})}\right) = -E\left(-n\theta + xln(n\theta) + n\hat{\theta} - xln(n\hat{\theta})\right)$$

$$B_P^*(\theta) = -n\theta ln(n\theta) + E(xlnx) \tag{10}$$

LetX be approximated by $Y{:}\,N(n\theta, n\theta)$ with a $p.d.f$

$$f_4(y,\theta) = \begin{cases}\{1/\sqrt{2\pi n\theta}\}exp(-(y-n\theta)^2/(2n\theta)) & y>0\\ 0 & otherwise\end{cases}, \tag{11}$$

To find the MLE for θ, we note that:

$$log f_4(y,\theta) = -(1/2)log(2n\pi) - (1/2)log(\theta) - (y-n\theta)^2/(2n\theta)$$

From which, $\frac{\partial}{\partial\theta}log f_4(y,\theta) = -(1/(2\theta)) + y/\theta - n + (y/\theta - n)^2/(2n)$

$$\frac{\partial}{\partial\theta}log f_4(y,\theta) = 0 \;\Rightarrow 2n^2\hat{\theta}^2 + 2n\hat{\theta} - 2y^2 = 0$$

Hence,

$$\hat{\theta} = \frac{-1+\sqrt{1+4y^2}}{2n} \tag{12}$$

Also, we use Equation (4) to find the B-entropy, which is given by:

$$B_N^*(\theta) = -E\left(log\frac{f_4(\underline{X};\,\theta)}{f_4(\underline{X}\,;\,\hat{\theta})}\right)$$

$$B_N^*(\theta) = \frac{1}{2}log\theta + \frac{1}{2} - \frac{1}{2}E\left[\sqrt{1+4y^2} - 1\right] + \frac{1}{2}log2 + \frac{1}{2}logn - \frac{1}{2}E\left[\sqrt{1+4y^2}\right]$$
$$+n\theta$$

Therefore, the percentage relative error in the $B - entropy$due to this approximation is:

$$PRE = \left(1 - \frac{B_P^*(\theta)}{B_N^*(\theta)}\right)100\% \tag{14}$$

3.2 Approximations by Fisher information measure

In this section, we approximate the Gamma and Poisson distributions by the normal distribution, under the Fisher information measure.

3.2.1 Normal Approximation to Gamma Distribution

Let$Y = \overline{X}$and$Y \sim G\left(n, \frac{\theta}{n}\right)$, then the p.d.f of Y is as given in Equation (1)

Note that $\quad \frac{\partial^2(log\, f_1(y;\theta))}{\partial\theta^2} = \frac{n}{\theta^2} - \frac{2ny}{\theta^3}$

then,

$$F_G(\theta) = -E\left(\frac{\partial^2(log\, f_1(y;\theta))}{\partial\theta^2}\right) = \frac{n}{\theta^2} \tag{15}$$

Let Y be approximated by $X{:}\,N(\theta, \frac{\theta^2}{n})$; $\theta > 0$ with a $p.d.f$ as in Equation (4), note that

$$\frac{\partial^2(\log f_2(x;\theta))}{\partial\theta^2} = \frac{1-n}{\theta^2} + \frac{3n(x-\theta)^2}{\theta^4}$$

Then,

$$F_N(\theta) = -E\left(\frac{\partial^2(\log f_2(x;\theta))}{\partial\theta^2}\right) = \frac{n-4}{\theta^2} \quad (16)$$

Therefore, the percentage relative error in the Fisher information due to this approximation is

$$PRE = \left(1 - \frac{F_G(\theta)}{F_N(\theta)}\right)100\% \quad (17)$$

3.2.2 Normal Approximation to the Poisson Distribution

Let $Y_1, \dots, Y_n$ be i.i.d from $P(\theta)$, then $X = \sum_{i=1}^{n} Y_i : P(n\theta)$ with a $p.d.f$ as in Equation (8). To find the Fisher information of X :

Note that: $\log(g(x;\theta)) = -n\theta + x\log(n\theta) - \log(x!)$

From which, $\frac{\partial^2\{\log g(x;\theta)\}}{\partial\theta^2} = -\frac{x}{\theta^2}$

Hence the Fisher information in X about θ is

$$F_P^*(\theta) = -E\left(\frac{\partial^2\{\log g(x;\theta)\}}{\partial\theta^2}\right) = n/\theta \quad (18)$$

Let X be approximated by $Y: N(n\theta, n\theta)$ with a $p.d.f$ as in Equation (11)

Note that:

$$\log\big(f_4(y,\theta)\big) = -(1/2)\log(2n\pi) - (1/2)\log(\theta) - (y-n\theta)^2/2n\theta$$

From which,

$$\frac{\partial^2 \log f_4(y,\theta)}{\partial\theta^2} = 1/2\theta^2 - y^2/n\theta^3$$

Hence, the Fisher information in Y about θ is

$$F_N^*(\theta) = -1/2\theta^2 - E\,y^2/n\theta^3$$

But, $EY^2 = var(Y) + \big(E(Y)\big)^2 = n\theta + n^2\theta^2 = n\theta(1+n\theta)$

Hence,

$$F_N^*(\theta) = -1/2\theta^2 + n\theta(1+n\theta)/n\theta^3 = (1+n\theta)/\theta^2 - 1/2\theta^2$$

$$F_N^*(\theta) = (1+2n\theta)/2\theta^2 \quad (19)$$

Therefore, the percentage relative error in the Fisher information due to this approximation is:

$$PRE = \left(1 - \frac{F_P^*(\theta)}{F_N^*(\theta)}\right)100\% \quad (20)$$

3.3 Approximation by AIC

3.3.1 Normal Approximation to Gamma Distribution

Let$Y = \overline{X}$and$Y \sim G\left(n, \frac{\theta}{n}\right)$, then p.d.f of Y as in Equation (1)

The log-likelihood is:

$$\log f_1(y;\theta) = -\ln\Gamma(n) - n\ln\theta + n\ln(n) - \frac{yn}{\theta} + (n-1)\ln y$$

The MLE for θ is given by Equation (2),then the AIC for the gamma model is:

$$AIC(1) = -2[-\ln\Gamma(n) - n\ln y + n\ln(n) - n + (n-1)\ln y] + 2 \quad (21)$$

LetY be approximated by $X: N(\theta, \frac{\theta^2}{n})$; $\theta > 0$ with a$p.d.f$ as in Equation (4), the log-likelihood is:

$$logf_2(x,\theta) = (1/2)logn - (1/2)log(2\pi) - log(\theta) - n(x-\theta)^2/(2\theta^2)$$

The MLE for θ is given by Equation (5)

$$AIC(2) = -2\left[(1/2)logn - (1/2)log(2\pi) - log(\hat{\theta}) - n(x-\hat{\theta})^2/(2\hat{\theta}^2)\right] + 2 \quad (22)$$

Therefore, the percentage relative error in the AIC due to this approximation is:

$$PRE = \left(1 - \frac{AIC(1)}{AIC(2)}\right) 100\% \quad (23)$$

3.3.2 Normal Approximation to Poisson distribution

Let $X{:}\,P(n\theta)$ with a $p.d.f$ as in Equation (8), then the log-likelihood is

$$L_1^*(x;\theta) = -n\theta + xlog(n\theta) - log(x!)$$

The MLE for θ is given by Equation (9). Then the AIC for the Poisson model is:

$$AIC^*(1) = -2[-n(x/n) + xlog\{n(x/n)\} - \log(x!)] + 2$$

$$= -2[-x + xlog(x) - \log(x!)] + 2 \quad (24)$$

Let X be approximated by $Y{:}\,N(n\theta, n\theta)$ with a $p.d.f$ as in equation (11), then the log-likelihood is

$$L^*{}_2(y,\theta) = -(1/2)log(2n\pi) - (1/2)log(\theta) - (x-n\theta)^2/2n\theta.$$

The MLE for θ is given by equation (12), then

$$AIC^*(2) = -2\left[(-1/2)log2\pi n - (1/2)log\hat{\theta} - (x-n\hat{\theta})^2/2n\hat{\theta}\right] + 2 \quad (25)$$

Therefore, the percentage relative error in the AIC due to this approximation is

$$PRE = \left(1 - \frac{AIC^*(1)}{AIC^*(2)}\right) 100\% \quad (26)$$

4. RESULTS

Throughout this section, the Matlab8.1 program was used to perform the computations.

4.1 The Approximation by the B-entropy Measure

Equation (7) is used to estimate the sample size when approximating the Gamma distribution by the normal distribution within $\theta = 0.1$, the result is shown in table 4.1.1.

Table 4.1.1: sample size for approximating Gamma distribution by the normal distribution using PRE in B-entropy measure

n	$B_G(\theta)$	$B_N(\theta)$	PRE
15	0.505553	0.535970	5.675069
17	0.504900	0.531297	4.968273
19	0.504385	0.527715	4.421039
21	0.503967	0.524884	3.985034
23	0.503623	0.522588	3.629185
25	0.503333	0.520687	3.332927
27	0.503086	0.519085	3.082207
29	0.502873	0.517717	2.867112
31	0.502688	0.516533	2.680448
33	0.502525	0.515499	2.516858
35	0.502381	0.514588	2.372269
37	0.502252	0.513779	2.243521
39	0.502137	0.513055	2.128126
41	0.502032	0.512404	2.024095
43	0.501938	0.511815	1.929817
45	0.501852	0.511280	1.843975
47	0.501773	0.510791	1.765479
49	0.501701	0.510343	1.693421
51	0.501634	0.509931	1.627037
53	0.501572	0.509550	1.565680
55	0.501515	0.509198	1.508797
57	0.501462	0.508871	1.455914
59	0.501412	0.508566	1.406624
61	0.501366	0.508282	1.360570

For the approximation of the Poisson distribution by the normal distribution, Equation (14) was used to compute the values of the PRE versus the sample size n within $\theta = 0.03$. In table 4.1.2 we show the results.

Table 4.1.2: sample size for approximating the Poisson distribution by the normal distribution using the PRE in the B-entropy measure

n	$B_P^*(\theta)$	$B_N^*(\theta)$	PR
20	0.528282	0.401314	31.637900
21	0.534246	0.422470	26.457639
22	0.539641	0.441717	22.168894
23	0.544518	0.459142	18.594633
24	0.548924	0.474834	15.603239
25	0.552898	0.488883	13.094085
26	0.556477	0.501384	10.988212
27	0.559695	0.512437	9.222128
28	0.562581	0.522148	7.743573
29	0.565163	0.530627	6.508561
30	0.567467	0.537989	5.479282
31	0.569513	0.544350	4.622584
32	0.571323	0.549831	3.908873
33	0.572917	0.554554	3.3113090
34	0.574310	0.558639	2.805223
35	0.575520	0.562209	2.367703
36	0.576560	0.565381	1.977320
37	0.577445	0.568273	1.613947
38	0.578185	0.570998	1.258676
39	0.578793	0.573666	0.893799
40	0.579280	0.576382	0.502841

4.2 The Approximation by the Fisher Information Measure.

Equation (17) was used to evaluate the PRE, within parameter $\theta = 0.5$ to approximate the Gamma distribution by the normal distribution. The results are shown in Table 4.2.1.

Table 4.2.1: Sample size for approximating the Gamma distribution by the normal distribution using PRE in Fisher information measure.

n	$F_G(\theta)$	$F_N(\theta)$	PRE
20	80.000	64.000	25.000000
22	88.000	72.000	22.222222
24	96.000	80.000	20.000000
26	104.000	88.000	18.181818
28	112.000	96.000	16.666667
30	120.000	104.000	15.384615
32	128.000	112.000	14.285714
34	136.000	120.000	13.333333
36	144.000	128.000	12.500000
38	152.000	136.000	11.764706
40	160.000	144.000	11.111111
42	168.000	152.000	10.526316
44	176.000	160.000	10.000000
46	184.000	168.000	9.523810
48	192.000	176.000	9.090909
50	200.000	184.000	8.695652
52	208.000	192.000	8.333333
54	216.000	200.000	8.000000
56	224.000	208.000	7.692308
58	232.000	216.000	7.407407
60	240.000	224.000	7.142857
62	248.000	232.000	6.896552
64	256.000	240.000	6.666667
66	264.000	248.000	6.451613
68	272.000	256.000	6.250000
70	280.000	264.000	6.060606
72	288.000	272.000	5.882353
74	296.000	280.000	5.714286
76	304.000	288.000	5.555556
78	312.000	296.000	5.405405
80	320.000	304.000	5.263158

To approximate the Poisson distribution by the normal distribution, Equation (20) was used with $\theta = 0.5$, the gives results was shown in Table 4.2.2

Table 4.2.2: Sample size for approximating the Poisson distribution by the normal distribution using PRE in Fisher information measure.

n	$F_P^*(\theta)$	$F_N^*(\theta)$	PRE
10	20.000000	22.000000	9.090909
12	24.000000	26.000000	7.692308
14	28.000000	30.000000	6.666667
16	32.000000	34.000000	5.882353
18	36.000000	38.000000	5.263158
20	40.000000	42.000000	4.761905
22	44.000000	46.000000	4.347826
24	48.000000	50.000000	4.000000
26	52.000000	54.000000	3.703704
28	56.000000	58.000000	3.448276
30	60.000000	62.000000	3.225806
32	64.000000	66.000000	3.030303
34	68.000000	70.000000	2.857143
36	72.000000	74.000000	2.702703
38	76.000000	78.000000	2.564103
40	80.000000	82.000000	2.439024
42	84.000000	86.000000	2.325581
44	88.000000	90.000000	2.222222
46	92.000000	94.000000	2.127660
48	96.000000	98.000000	2.040816
50	100.000000	102.000000	1.960784
52	104.000000	106.000000	1.886792
54	108.000000	110.000000	1.818182
56	112.000000	114.000000	1.754386

4.3 The Approximation by the AIC.

Equation (23) was used to evaluate the PRE, within x = 40 to approximate the Gamma distribution by the normal distribution. Table 4.3.1 shows the results.

Table 4.3.1: Sample size for approximating the Gamma distribution by the normal distribution using the PRE in AIC

n	AIC(1)	AIC(2)	PRE
5	9.639487	9.436687	-2.149067
7	9.293519	9.143510	-1.640609
9	9.036922	8.917783	-1.335979
11	8.832888	8.734041	-1.131748
13	8.663505	8.579026	-0.984714
15	8.518695	8.444929	-0.873492
17	8.392225	8.326757	-0.786243
19	8.279968	8.221116	-0.715861
21	8.179049	8.125598	-0.657812
23	8.087388	8.038428	-0.609068
25	8.003426	7.958262	-0.567521
27	7.925972	7.884055	-0.531661
29	7.854087	7.814983	-0.500377
31	7.787025	7.750379	-0.472829
33	7.724179	7.689700	-0.448376
35	7.665050	7.632496	-0.426513
37	7.609222	7.578390	-0.406842
39	7.556348	7.527064	-0.389045
41	7.506129	7.478246	-0.372859
43	7.458312	7.431701	-0.358072
45	7.412677	7.387228	-0.344507
47	7.369034	7.344649	-0.332015
49	7.327217	7.303810	-0.320471

To approximate the Poisson distribution by the normal distribution using the PRE, Equation (26) was used to evaluate PRE. The results obtained are explained in Table 4.3.2.

Table 4.3.2: Sample size for approximating the Poisson distribution by the normal distribution using the PRE in AIC

n	AIC*(1)	AIC*(2)	PRE
5	5.480604	5.397357	-1.542382
7	5.807581	5.748088	-1.034997
9	6.053613	6.007331	-0.770418
11	6.250920	6.213049	-0.609535
13	6.415644	6.383598	-0.502011
15	6.557037	6.529262	-0.425386
17	6.680893	6.656386	-0.368182
19	6.791087	6.769159	-0.323944
21	6.890335	6.870495	-0.288773
23	6.980617	6.962502	-0.260180
25	7.063419	7.046753	-0.236506
27	7.139886	7.124455	-0.216600
29	7.210920	7.196552	-0.199642
31	7.277240	7.263800	-0.185034
33	7.339435	7.326809	-0.172326
35	7.397987	7.386082	-0.161175
37	7.453299	7.442038	-0.151317
39	7.505712	7.495029	-0.142542
41	7.555514	7.545352	-0.134685
43	7.602953	7.593263	-0.127610
45	7.648243	7.638984	-0.121209
47	7.691571	7.682706	-0.115391
49	7.733099	7.724595	-0.110081

5. CONCLUSIONS

This paper discussed the approximations of both Gamma and Poisson distributions by a normal distribution using the percentage relative error in B-entropy measure, Fisher information measures and Akaike information criterion.

One may note that for all cases which are considered the PRE is decreasing when simple size n increasing.

In the B-entropy measure if fixed $\epsilon < 0.03$, the approximation of Gamma distribution by normal distribution is shown in table 4.1.1, gives that the sample size is greater than or equal 27 within $\theta = 0.1$ since the PRE is less than 3.082207 ,and for approximation the Poisson distribution by the normal distribution table 4.1.2 shown that sample size is less than or equal 35 within $\theta = 0.03$.

In the Fisher information measure, it is clear from the table 4.2.1 that the sample size is greater than or equal 30 when $\epsilon < 0.15$ within $\theta = 0.5$ for approximating Gamma distribution by normal distribution andfor approximation the Poisson distribution by the normal distribution for the same value of θ table 4.2.2 shown that sample size is greater than or equal 32 when $\epsilon < 0.03$.

In the Akaike information criterion, tables4.3.1 and4.3.2 shown that the PRE is almost closed to zero in both approximation of Gamma and Poisson distributions by normal distribution which is given that the sample size for those two approximation is small enough .

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References.

[1] A. M. Awad, and A.Abu-Taleb, Information relationship measures, Journal of mathematical control and information , 4 ,1,(1987),13-23.

[2] A. M. Awad, The Shannon entropy of generalized gamma and of related distributions: Proc. First Jordanian mathematics Conf. Yarmouk university, Jordan(1991),13-27 .

[3] M.M.Azzam and A . M. Awad, Application Of the Shannon entropy to type II censoring in life testing experiments, Journal of fractional calculus Vol.11, May(1997) , 103-112.

[4] V.L.Hartley , Transmission of information , *Bell System Tech.* J. Vol. 7, (1928) 535-563 .

[5] A. Renyi, On measures of entropy and information, Proc. 4th BerkelySymp. Stat. and Pro. ,Vol.1(1960) :547-561 .

[6] A.D.Zighed, G.Ritschard and S. Marcellin, Asymmetric and sample size sensitive entropy measures for supervised learning ,Advances in intelligent information systems Vol. 265,(2010), 27-42.

[7] R.A.Fisher ,Theory of statistical estimation, Proc. Combridge Philos., Soc.,Vol.22,(1925)pp.700-725 .

[8] C. A. Shannon, Mathematical theory of communication, Bellsystem Tech. J., Vol.27(3)(1948) ,379-423.

[9] S.Kullback and R.A.Leibler , On information and sufficiency, Ann. Math, Statist. ,Vol.22(1951) pp.79-86.

[10] M.M.Azzam,andA.M. Awad, Entropy measures and some distribution approximations, Microelectron, Reliab. Vol.36(10) (1996) ,1569-1580.

[11] M.E.Gobar,,E.B.Bashier and A.M.Awad, Information measures and some distribution approximations, Journal of advances in mathematics, Vol.6,No3,(2014), 1008-1014.

[12] M. E. Gobar and E.B.Bashier,Akaike information criterion and some distribution approximations, International J. of Math. Sci. and Engg. Appls. (IJMSEA),Vol.8, No II,(2014) PP.1-9.

[13] M. E. Gobar ,E. B.Bashier and A.M.Awad, The best sample size in approximating some sampling distributions , International J. of math. Sci. and Engg. Appls. (IJMSEA) , Vol.8, No3, (2014) ,179-192.

[14] M.A.Hammad , A.M. Awad, Distribution of Shannon statistic from Normal sample, International Journal of Statistics , vol.3 ,(2007) 259-275.