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SOME EXAMPLES OF ARMENDARIZ RINGS

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Abstract

We construct various examples of Armendariz and related rings by reviewing and extending some results concerning the structure of monoid *M*. In particular, we give some examples of Armendariz rings related to a monoid. We prove that, if *M* be a strictly totally ordered monoid with $|M| \geq 2$. Then, *R* is linear *M*-Armendariz and reduced if and only if *T*(*R*) is linear *M*-Armendariz. It is also shown that, *R* is a *PP*-ring (Baer ring) if and only if *R*[*M*] is a *PP*-ring (Baer ring, respectively) and those of the monoid ring *R*[*M*] in case *R* is linear *M*-Armendariz ring.

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1. Introduction

Throughout this paper, R and M denote an associative ring with identity and a monoid, respectively. Given a ring R, the polynomial ring over R is denoted by $R[x]$. Rege and Chhawchharia[12] introduced the notion of an Armendariz ring. A ring R is called Armendariz if whenever polynomials $f(x) = a_0 + a_1x + \cdots + a_nx^n$, $g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j = 0$ for each i, j. (The converse is always true.) The name "Armendariz ring" was chosen because Armendariz [5, Lemma 1] had shown that a reduced ring (i.e., a ring without nonzero nilpotent elements) satisfies this condition. The study of Armendariz rings, which is related to polynomial rings, was initiated by Armendariz [5] and Rege and Chhawchharia [12]. Some properties of Armendariz rings have been studied in Rege and Chhawchharia [12], Armendariz [5], Anderson and Camillo [3], and Kim and Lee[14]. Due to Lee and Wong [15], a ring R is called linear Armendariz if for given $f(x) = a_0 + a_1x$ and $g(x) = b_0 + b_1x \in R[x]$ such that $f(x)g(x) = 0$ then $a_ib_j = 0$ for all i, j. In [11], Zhongkui studied a generalization of Armendariz rings, which are called M-Armendariz rings, where M is a monoid. A ring R is called M-Armendariz if whenever elements $\alpha = a_1g_1 + a_2g_2 + \cdots + a_ng_n$, $\beta = b_1h_1 + b_2h_2 + \cdots + b_mh_m \in R[M]$, satisfy $\alpha\beta = 0$, then $a_ib_j = 0$, for each i, j. Let $M = (N \cup \{0\}, +)$. Then the ring R is M-Armendariz if and only if R is Armendariz. Reduced rings are Armendariz by [5, Lemma 1] and subrings of Armendariz rings are also Armendariz ring. It is obviously that Armendariz rings are linear Armendariz and that subrings of linear Armendariz rings are still linear Armendariz ring. There is linear Armendariz ring but not Armendariz by [15, Example 3.2]. A ring is called abelian if every idempotent is central. Linear Armendariz rings are abelian by [15, Lemma 3.4(3)]. In [16]. A ring R is called linear M-Armendariz (linear Armendariz ring relative to monoid M), if whenever elements $\alpha = a_1g_1 + a_2g_2, \beta = b_1h_1 + b_2h_2 \in R[M]$ satisfy $\alpha\beta = 0$, then $a_ib_j = 0$ for all i, j. If $M = \{e\}$, then every ring is linear M-Armendariz. Let $M = (\mathbb{N} \cup \{0\}, +)$. Then the ring R is linear M-Armendariz if and only if R is linear Armendariz. If R is reduced and M-Armendariz ring, then R is linear M-Armendariz.

Recall that a monoid M is called u.p. monoid (unique product monoid) if for any two non-empty finite subsets $A, B \subseteq M$ there exists an element $g \in M$ uniquely presented in the form ab where $a \in A$ and $b \in B$. The class of u.p.monoids is quite large and important (see Birkenmeier and Park [6] and Passman [4]). For example, this class includes the right or left ordered monoids, submonoids of a free group, and torsion-free nilpotent groups. Every u.p. monoid M has non unity element of finite order. In the following, e will always stand for the identity of M.

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Motivated by results in Lee and Wong [15], Jeon et al. [8], Zhongkui [11], and Dixit and Singh [16], we have some examples of linear M-Armendariz ring which is a generalization of M-Armendariz ring.

2. Linear M-Armendariz ring

Every M-Armendariz ring is linear M-Armendariz, but the converse is not true. See [16, Example 2.1].

Let $(M, \leq)$ be an ordered monoid. If for any $g, g', h \in M, g < g'$ implies that $gh < g'h$ and $hg < hg'$, then $(M, \leq)$ is called a strictly ordered monoid.

Lemma 2.1. *Let M be a strictly totally ordered monoid and R a reduced ring. Then $R[M]$ is reduced.*

Proof. Suppose that $\alpha = a_1g_1 + \cdots + a_ng_n \in R[M]$ is such that $\alpha^2 = 0$. Not loss the generality we assume that $g_1 < g_2 < \cdots < g_n$. Then from $\alpha^2 = 0$ it follows that

$$a_1^2g_1^2 + a_1a_2g_1g_2 + a_2a_1g_2g_1 + \cdots + a_n^2g_ng_n = 0.$$

Suppose $g_1g_1 = g_ig_j$ for some $1 \leq i, j \leq n$. Then $g_1 \leq g_i$, $g_1 \leq g_j$. If $g_1 < g_i$, then $g_1g_1 < g_ig_1 \leq g_ig_j = g_1g_1$, a contradiction. Thus, $g_1 = g_i$. Similarly $g_1 = g_j$. Thus, we have $a_1^2 = 0$ and so $a_1 = 0$ since R is reduced. Now $\alpha = a_2g_2 + \cdots + a_ng_n$. By analogy with above proof, we have $a_2 = 0, \cdots$, $a_n = 0$. Thus $\alpha = 0$ This means that $R[M]$ is reduced. □

Proposition 2.2. *Let M be a strictly totally ordered monoid and I an ideal of R. If I is reduced and R/I is linear M-Armendariz, then R is linear M-Armendariz.*

Proof. Let $\alpha = a_1g_1 + a_2g_2$ and $\beta = b_1h_1 + b_2h_2 \in R[M]$ be such that $\alpha\beta = 0$ and $g_1 < g_2$, $h_1 < h_2$. Note that in $(R/I)[M]$, $(\bar{a}_1g_1 + \bar{a}_2g_2)(\bar{b}_1h_1 + \bar{b}_2h_2) = 0$. Since R/I is linear M-Armendariz, $a_ib_j \in I$, for all i, j. If there exist $1 \leq i, j \leq 2$ such that $g_ih_j = g_1h_1$, then $g_1 \leq g_i$, and $h_1 \leq h_i$. If $g_1 < g_i$, then $g_1h_1 < g_ih_1 \leq g_ih_j = g_1h_1$, a contradiction. Thus $g_1 = g_i$. Similarly $h_1 = h_j$. Hence $a_1b_1 = 0$ and so, $(b_1Ia_1)^2 = b_1Ia_1b_1Ia_1 = 0$. Thus, $b_1Ia_1 = 0$. If there exist $1 \leq i, j \leq 2$ such that $g_ih_j = g_2h_2$, then $g_i \leq g_2$, $h_j \leq h_2$. If $g_i < g_2$, then $g_ih_j < g_2h_j \leq g_2h_2 = g_ih_j$, a contradiction. Thus, $g_i = g_2$. Similarly $h_j = h_2$. Hence $a_2b_2 = 0$. If $g_1h_2 \neq g_2h_1$, then from $\alpha\beta = 0$, it follows that $a_1b_2 = 0$, $a_2b_1 = 0$. Now suppose that $g_1h_2 = g_2h_1$. Then from $\alpha\beta = 0$ it follows that $a_1b_2 + a_2b_1 = 0$. Thus, $(a_1b_2)^3 + a_2b_1(a_1b_2)^2 = 0$. But $a_2b_1(a_1b_2)^2 = a_2b_1a_1b_2a_1b_2 \in a_2b_1Ia_1b_2 = 0$. Thus, $(a_1b_2)^3 = 0$. On the other hand, $a_1b_2 \in I$. Thus $a_1b_2 = 0$, since I is reduced. Hence $a_2b_1 = 0$. Thus shows that R is linear M-Armendariz. □

Recall that a monoid M is called torsion-free if the following property holds: if $g, h \in M$ and $k \geq 1$ are such that $g^k = h^k$, then $g = h$.

Let R be a ring. Define a ring $T(R)$ as follows

$$T(R) = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \mid a, b, d \in R \right\}$$

Proposition 2.3. *Let M be a strictly totally ordered monoid with $|M| \geq 2$. Then the following conditions are equivalent:*

(1) *R is linear M-Armendariz and reduced.*
(2) *$T(R)$ is linear M-Armendariz.*

Proof. (1)⇒(2) We complete the proof of by adapting the proof of [2. Proposition 17]. It is easy to see that there exists an isomorphism of rings $T(R)[M]\to T(R[M])$ define by:

$$\sum_{i=1}^{n}\begin{pmatrix} a_i & b_i & c_i \\ 0 & a_i & d_i \\ 0 & 0 & a_i \end{pmatrix} g_i \to \begin{pmatrix} \Sigma_{i=1}^n a_i g_i & \Sigma_{i=1}^n b_i g_i & \Sigma_{i=1}^n c_i g_i \\ 0 & \Sigma_{i=1}^n a_i g_i & \Sigma_{i=1}^n d_i g_i \\ 0 & 0 & \Sigma_{i=1}^n a_i g_i \end{pmatrix}.$$

Suppose that $\alpha = A_1g_1 + A_2g_2$, and $\beta = B_1h_1 + B_2h_2 \in T(R)[M]$ are such that $\alpha\beta = 0$, where $A_i, B_j \in T(R)$. We claim $A_iB_j = 0$ for each $1 \leq i, j \leq 2$. Assume that

$$A_i = \begin{pmatrix} a_i & b_i & c_i \\ 0 & a_i & d_i \\ 0 & 0 & a_i \end{pmatrix}, B_j = \begin{pmatrix} a'_j & b'_j & c'_j \\ 0 & a'_j & d_j \\ 0 & 0 & a'_j \end{pmatrix}.$$

Then we have

$$\begin{pmatrix} \Sigma_{i=1}^2 a_i g_i & \Sigma_{i=1}^2 b_i g_i & \Sigma_{i=1}^2 c_i g_i \\ 0 & \Sigma_{i=1}^2 a_i g_i & \Sigma_{i=1}^2 d_i g_i \\ 0 & 0 & \Sigma_{i=1}^2 a_i g_i \end{pmatrix} \times \begin{pmatrix} \Sigma_{j=1}^2 a'_j h_j & \Sigma_{j=1}^2 b'_j h_j & \Sigma_{j=1}^2 c'_j h_j \\ 0 & \Sigma_{j=1}^2 a'_j h_j & \Sigma_{j=1}^2 d'_j h_j \\ 0 & 0 & \Sigma_{j=1}^2 a'_j h_j \end{pmatrix} = 0.$$

Thus

$$(\Sigma_{i=1}^2 a_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0,$$

$$(\Sigma_{i=1}^2 a_i g_i)(\Sigma_{j=1}^2 b'_j h_j) + (\Sigma_{i=1}^2 b_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0,$$
$$(\Sigma_{i=1}^2 a_i g_i)(\Sigma_{j=1}^2 c'_j h_j) + (\Sigma_{i=1}^2 b_i g_i)(\Sigma_{j=1}^2 d'_j h_j) + (\Sigma_{i=1}^2 c_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0,$$

$$(\Sigma_{i=1}^2 a_i g_i)(\Sigma_{j=1}^2 d'_j h_j) + (\Sigma_{i=1}^2 d_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0\cdot$$

Since R is linear M-Armendariz, we have $a_i a'_j = 0$ for all $1 \leq i, j \leq 2$. Thus $a'_j a_i = 0$ for all $1 \leq i, j \leq 2$, since R is reduced. Hence $(\Sigma_{j=1}^2 a'_j h_j)(\Sigma_{i=1}^2 a_i g_i) = 0$. If we multiply the second equation on the left side by $(\Sigma_{j=1}^2 a'_j h_j)$, then

$$(\Sigma_{j=1}^2 a'_j h_j)(\Sigma_{i=1}^2 b_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0\cdot$$

Thus, $((\Sigma_{i=1}^2 b_i g_i)(\Sigma_{j=1}^2 a'_j h_j))^2 = 0$. Since $R[M]$ is reduced by Lemma 2.1, it follows that $(\Sigma_{i=1}^2 b_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0$. Since R is linear M-Armendariz, we have $b_i a'_j = 0$ for all $1 \leq i, j \leq 2$. Hence $(\Sigma_{i=1}^2 b_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0$ and so $(\Sigma_{i=1}^2 a_i g_i)(\Sigma_{j=1}^2 b'_j h_j) = 0$. Thus $a_i b'_j = 0$ for all i, j, since R is linear M-Armendariz. Also, if we multiply the fourth equation on the left side by $\Sigma_{j=1}^2 a'_j h_j$, then

$$\Sigma_{j=1}^2 a'_j h_j(\Sigma_{i=1}^2 d_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0\cdot$$

By Lemma 2.1, we have $(\Sigma_{i=1}^2 d_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0$. Thus $d_i a'_j = 0$ for all i, j, since R is linear M-Armendariz. Similarly, we have $(\Sigma_{i=1}^2 a_i g_i)(\Sigma_{j=1}^2 d'_j h_j) = 0$. Thus $a_i d'_j = 0$ for all i, j. Now if we multiply the third equation on the left side by $\Sigma_{j=1}^2 a'_j h_j$ then $(\Sigma_{j=1}^2 a'_j h_j)(\Sigma_{i=1}^2 c_i g_i)(\Sigma_{j=1}^2 a'_j h_j) = 0\cdot$ Similar argument shows that $c_i a'_j = 0$ for all i, j. Hence the third equation becomes

$$(\Sigma_{i=1}^2 a_i g_i)(\Sigma_{j=1}^2 c'_j h_j) + (\Sigma_{i=1}^2 b_i g_i)(\Sigma_{j=1}^2 d'_j h_j) = 0. \qquad (*)$$

Since $d'_j a_i = 0$ for all i, j, we have $(\Sigma_{j=1}^2 d'_j h_j)(\Sigma_{i=1}^2 a_i g_i) = 0\cdot$ Now if we multiply equation $(*)$ on the ride side by $(\Sigma_{i=1}^2 a_i g_i)$ then

$$(\Sigma_{i=1}^2 a_i g_i)(\Sigma_{j=1}^2 c'_j h_j)(\Sigma_{i=1}^2 a_i g_i) = 0\cdot$$

A similar argument shows that $a_i c'_j = 0$ for all i, j. Thus from equation $(*)$ it follows that $b_i d'_j = 0$ for all i, j. Now it easy to see that $A_iB_j = 0$ for each $1 \leq i, j \leq 2$.

(2)⇒(1) Suppose that $T(R)$ is linear M-Armendariz. Note that R is isomorphic to the subring

$$\left\{\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} \mid a \in R\right\}$$

of $T(R)$. Thus R is linear M-Armendariz, since each subring of linear M-Armendariz ring is also linear M-Armendariz.By analogy with proof of [15, Theorem 2.3], we can show that R is reduced. □

Remark 2.4. *Let R be a ring and let*

$$T_n(R) = \left\{\begin{pmatrix} a & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a & \cdots & a_{3n} \\ 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a \end{pmatrix} \mid a, a_{ij} \in R\right\}.$$

Where n is a positive integer. Then Proposition 2.3 suggests that $T_n(R)$ may be also linear M-Armendariz for $n \geq 4$ if R is linear M-Armendariz and reduced. But the following example eliminate the possibility.

Example 2.5. *Let M be a monoid with $\mid M \mid \geq 2$ and R a ring. Take $e \neq g \in M$. Let $\alpha = e_{12}e + (e_{12} - e_{13})g$ and $\beta = e_{34}e + (e_{24} + e_{34})g$ be in $T_n(R)[M]$ where $e_{ij}s$ are the matrix units in $T_n(R)(n \geq 4)$. Then $\alpha\beta = 0$, but $(e_{12} - e_{13})e_{34} \neq 0$. Thus $T_n(R)$ is not linear M-Armendariz ring $(n \geq 4)$.*

Corollary 2.6. *Let M be a strictly totaly ordered monoid and R is linear M-Armendariz ring. If R is reduced, then the trivial extension $T(R, R)$ is linear M-Armendariz.*

Proof. Note that $T(R, R)$ is isomorphic to the ring

$$\left\{\begin{pmatrix} a & b & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} \mid a, b \in R\right\}$$

Now the result follows from Proposition 2.3, and by [16, Lemma 2.6], every subring of linear M-Armendariz ring is also linear M-Armendariz. □

Proposition 2.7. *For an abelian ring R, the following conditions are equivalent:*

(1) *R is linear M-Armendariz;*
(2) *eR and $(1 - e)R$ are linear M-Armendariz for all idempotent e of R;*
(3) *eR and $(1 - e)R$ are linear M-Armendariz for some idempotent e of R.*

Proof.
(1)⇒(2) is obvious since eR and $(1 - e)R$ are subrings of R.
(2)⇒(3) is obvious.
(3)⇒(1) Let $\alpha = \Sigma_{i=1}^{2} a_i g_i, \beta = \Sigma_{j=1}^{2} b_j h_j \in R[M]$ and assume $\alpha\beta = 0$. Next for some $e = e^2 \in R$ let $\alpha_1 = e\alpha, \alpha_2 = (1 - e)\alpha, \beta_1 = e\beta$ and $\beta_2 = (1 - e)\beta$. Then $0 = \alpha\beta = \alpha_1\beta_1 + \alpha_2\beta_2$ and so $\alpha_1\beta_1 = e\alpha_1\beta_1 = 0, \alpha_2\beta_2 = (1 - e)\alpha_2\beta_2 = 0$. By the conditions we have that $a_i b_j e = 0$ and $a_i b_j (1 - e) = 0$ for all $1 \leq i, j \leq 2$ and hence $a_i b_j = 0$ for all $1 \leq i, j \leq 2$. Thus R is a linear M-Armendariz ring. □

Proposition 2.8. *Let R be a ring an Δ be a multiplicative monoid in R consisting of central regular elements. Then R is linear M-Armendariz if and only if $\Delta^{-1}R$ is linear M-Armendariz.*

Proof. Let R be a linear M-Armendariz ring and $S = \Delta^{-1}R$. Put $\alpha\beta = 0$, where $\alpha = \Sigma_{i=1}^{2} a_i g_i, \beta = \Sigma_{j=1}^{2} b_j h_j \in S[M]$. We assume that $a_i = \varepsilon_i u^{-1}, b_j = \eta_j v^{-1}$ with $\varepsilon_i, \eta_j \in R$ for all $1 \leq i, j \leq 2$ and $u, v \in \Delta$. Then we have $0 = \alpha\beta = \Sigma_{i=1}^{2}\Sigma_{j=1}^{2} a_i b_j g_i h_j = \Sigma_{i=1}^{2}\Sigma_{j=1}^{2} \varepsilon_i \eta_j u^{-1} v^{-1} g_i h_j = (\Sigma_{i=1}^{2}\Sigma_{j=1}^{2} \varepsilon_i \eta_j g_i h_j)((uv)^{-1})$. Hence $\Sigma_{i=1}^{2}\Sigma_{j=1}^{2} \varepsilon_i \eta_j g_i h_j = 0$ in $R[M]$. Since R is linear M-Armendariz rings, $\varepsilon_i \eta_j = 0$, for all $1 \leq i, j \leq 2$ and so $a_i b_j = \varepsilon_i u^{-1} \eta_j v^{-1} = \varepsilon_i \eta_j u^{-1} v^{-1} = 0$ for all $1 \leq i, j \leq 2$. Thus $S = \Delta^{-1}R$ is linear M-Armendariz rings. The converse is obtained by [16, Lemma 2.6]. □

Corollary 2.9. *Let M be a monoid and $R_i, i \in I$, be rings. Then the following statements are equivalent:*

(1) $\bigoplus_{i \in I} R_i$ *is linear M-Armendariz;*
(2) R_i *is linear M-Armendariz for each $i \in I$.*

3. PP-RINGS AND BAER-RINGS

Lemma 3.1. *Let M be a monoid with $|M| \geq 2$ and R is linear M-Armendariz. If $a, b, c \in R$ are such that $ab = 0$ and $c^2 = 0$, then $acb = 0$.*

Proof. Take $g \in M$ with $g \neq e$. then $(ae - acg)(be + cbg) = 0$. Thus $acb = 0$ since, R is linear M-Armendariz. □

Proposition 3.2. *Let M be a monoid with $|M| \geq 2$. Then every linear M-Armendariz ring is abelian.*

Proof. Using Lemma 3.1 by analogy with the proof of Huh et.al. [1, Corollary 8], we can complete the proof.

□

Lemma 3.3. *Let M be a monoid and R is linear M-Armendariz ring. If ϕ is an idempotent of the monoid ring $R[M]$ and ϕ_0 the coefficient of e, in ϕ, then ϕ_0 is an idempotent of R and $\phi R[M] = \phi_0 R[M]$.*

Proof. Let $\phi = \phi_0 e + \phi_1 g_1 + \cdots + \phi_n g_n \in R[M]$. Then $0 = (1-f)f = ((1-\phi_0)e + \phi_1 g_1 + \cdots + \phi_n g_n)(\phi_0 e + \phi_1 g_1 + \cdots + \phi_n g_n)$. Since R is linear M-Armendariz, it follows that $(1-\phi_0)\phi_0 = 0, \phi_i \phi_0 = 0$ and $(1-\phi_0)\phi_i = 0$ for all i. Thus ϕ_0 is an idempotent of R, $\phi_0 \phi = \phi$ and $\phi\phi_0 = \phi_0$. Now clearly we have $\phi R[M] = \phi_0 R[M]$. □

By Kaplansky [10], or Birkenmeier et al. [7], a ring R is called Baer if the right annihilator of every nonempty subset of R is generated by an idempotent. A ring R is called a right PP-ring if each principal right ideal of R is projective, or equivalently if the right annihilator of each element of R is generated by an idempotent. A ring R is called a PP-ring if it is both a right and a left PP-ring. Baer rings are clearly right PP-rings. In Fraser and Nicholson [9], it was shown that if R is a reduced ring, then R is a PP-ring if and only if $R[x]$ is a PP-ring. It was shown in Kim and Lee [14], that if R is an Armendariz ring, then R is a PP-ring (a Baer ring) if and only if $[x]$ is a PP-ring (a Baer ring). For monoid rings, it was shown by Groenewald [13], Theorems 1 and 2, that for a $u.p.$-monoid M, R is a reduced PP-ring (reduced Baer ring) if and only if $R[M]$ is a reduced PP-ring (reduced Baer ring, respectively). For monoid rings, and R be an M-Armendariz, it was shown by Liu [11], Theorems 3.4 and 3.5, that R is PP-ring (Baer ring) if and only if $R[M]$ is PP-ring (Baer ring, respectively). Here we have the following result.

Theorem 3.4. *Let M be a monoid with $|M| \geq 2$ and R is linear M-Armendariz ring. Then R is a PP-ring if and only if $R[M]$ is a PP-ring.*

Proof. Assume that R is a PP-ring. Let $\alpha = a_1g_1 + a_2g_2 + \cdots + a_ng_n \in R[M]$. Then there exists $\phi_i^2 = \phi_i \in R$ such that $r_R(a_i) = \phi_i R$, for $i = 1, 2, \ldots, n$. Let $\phi = \phi_1\phi_2\cdots\phi_n$. Then by Proposition 3.2, $\phi^2 = \phi \in R$ and $\phi R = \cap_{i=1}^n (r_R(a_i))$. Thus $\alpha(\phi e) = a_1\phi g_1 + a_2\phi g_2 + \cdots + a_n\phi g_n = 0$. Hence $(\phi e)R[M] \subseteq r_{R[M]}(\alpha)$. Let $\beta = b_1h_1 + b_2h_2 + \cdots + b_mh_m r_{R[M]}(\alpha)$. Then $\alpha\beta = 0$. Since R is linear M-Armendariz, we have $a_ib_j = 0$ for all i, j. Then $b_j \in \phi_1\phi_2\cdots\phi_nR = \phi R$ for all j. Hence $\beta = b_1h_1 + b_2h_2 + \cdots + b_mh_m = (\phi e)(b_1h_1) + (\phi e)(b_2h_2) + \cdots + (\phi e)(b_mh_m) \in (\phi e)R[M]$. Consequently $r_{R[M]}(\alpha) = (\phi e)R[M]$. Thus $R[M]$ is a PP-ring.

Conversely, assume that $R[M]$ is a PP-ring. Let $a \in R$. By Lemma 3.3 there exists an idempotent ϕ of R such that $r_{R[M]}(a) = \phi R[M]$. Now it is easy to see that $r_R(a) = \phi R$, and therefore R is a PP-ring. □

Theorem 3.5. *Let M be a monoid with $|M| \geq 2$ and R is linear M-Armendariz ring. Then R is a Baer ring if and only if $R[M]$ is a Baer ring.*

Proof. Assume that R is Baer. Let W be a nonempty subset of $R[M]$ and let W^* be the set of all coefficients of elements of W. Then W^* is a nonempty subset of R and so $r_R(W^*) = \phi R$ for some idempotent $\phi \in R$. Clearly $\phi e \in r_{R[M]}(W)$. Thus $(\phi e)R[M] \subseteq r_{R[M]}(W)$. Conversely, let $\beta = b_1h_1 + b_2h_2 + \cdots + b_mh_m \in r_{R[M]}(W)$. Then $W\beta = 0$ and hence $\alpha\beta = 0$ for any $\alpha = a_1g_1 + a_2g_2 + \cdots + a_ng_n \in W$. Since R is linear M-Armendariz, we have $a_ib_j = 0$ for all i, j. Then $b_j \in r_R(W^*) = \phi R$ for all $1 \leq j \leq m$. Hence $\beta = b_1h_1 + b_2h_2 + \cdots + b_mh_m = (\phi e)(b_1h_1) + (\phi e)(b_2h_2) + \cdots + (\phi e)(b_mh_m) \in (\phi e)R[M]$. Therefore $R[M]$ is Baer.

Conversely, assume that $R[M]$ is Baer and μ a nonempty subset of R. Then $r_{R[M]}(\mu e) = \phi R[M]$ for some idempotent $\phi \in R$ by Lemma 3.3. Hence $r_R(\mu) = \phi R$ and therefore R is a Baer ring. □

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