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I (D,B)-supra pre maps via supra topological ordered spaces

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Abstract

Sayed [19] defined a supra pre-open set and studied its basic properties. In this work, we introduce various forms of supra continuous (supra open, supra closed, supra homeomorphism) maps in supra topological ordered spaces by using the notions of supra pre-open sets and increasing (decreasing, balancing) sets. We illustrate the relationships among these maps with the help of examples. Moreover, we investigate under what conditions these maps preserve some separation axioms between supra topological ordered spaces.

Keywords: $I(D,B)$-supra pre-continuous map; $I(D,B)$-supra pre-open map $I(D,B)$-supra pre-homeomorphism map, Ordered supra pre-separation axioms.

2010 Mathematics Subject Classification: 54F05, 54F15

1. Introduction

A set$X$ that is both a topological space and a partially ordered set is variously called a topological ordered space and is denoted by the triple (X, τ, $\preccurlyeq$), where τ is a topology and $\preccurlyeq$ is a partial order relation on X. With regard to this definition, the topology and the partial order relation operate independently of one another. The study of topological ordered spaces was initiated by Nachbin [18] in 1965.

In 1968, McCartan [17] introduced and studied ordered separation axioms by utilizing a monotone neighborhood. Mashhour et al. [16] introduced a notion of supra topological spaces and generalized some properties of topological spaces to supra topological spaces such as continuity and some separation axioms. In 1991, Arya and Gupta [7] utilized semi open sets [15] to introduce semi separation axioms in topological ordered spaces. In 2002, Leela and Balasubramanian [14] utilized β-open sets [1] to introduce βT_1-ordered spaces and βT_2-ordered spaces. Also, in 2002, Kumar [13] introduced and studied the concepts of continuity, openness, closeness and homeomorphism between topological ordered spaces. In 2004, Das [8] introduced ordered separation axioms in some ordered spaces by generalized results obtained by McCartan [17]. Sayed [20] introduced and studied supra pre-open sets and supra pre-continuous maps. In 2012, Rao and Chudamani [19] generalized the concepts and results obtained in [13] by using supra pre-open sets. In 2016, Abo-Elhamayel and Al-shami [2] introduced the concepts of x-supra continuous (x-supra open, x-supra closed, x-supra homeomorphism) maps in supra topological ordered spaces, for $x = \{I, D, B\}$ and studied their properties. El-Shafei et al. [10] introduced a notion of supra R-open sets and defined new types of supra separation axioms. Also, they initiated some ordered maps via supra topological ordered spaces in [11] and investigated strong separation axioms in supra topological ordered spaces in [12].Al-shami[6] studied ordered maps by using some generalized supra open sets and investigated their main properties.

The aim of the present paper is to establish some types of x-supra pre-continuous (x-supra pre -open, x-supra pre-closed, x-supra pre-homeomorphism) maps in supra topological spaces, for $x =$ {increasing, decreasing, balancing}(briefly, for $x = \{I, D, B\}$). Also, we investigate necessary and sufficient conditions for these maps to preserve some separation axioms. Many of the findings that raised at are generalizations of those findings in supra topological ordered spaces which introduced in [2].

Throughout this paper, $(X, \tau, \preccurlyeq_1)$ and $(Y, \tau, \preccurlyeq_2)$ stand for topological ordered spaces and $(X, \mu, \preccurlyeq_1)$ and $(Y, \mu, \preccurlyeq_2)$ stand for supra topological ordered spaces. A diagonal relation is denoted by Δ.

In the following, we recall some concepts and notions that will be needed in the sequels.

Definition 1.1. [18] Let $(X, \preccurlyeq)$ be a partially ordered set. Then:

i. $i(b) = \{a \in X: b \preccurlyeq a\}$and $d(b) = \{a \in X: a \preccurlyeq b\}$.
ii. $i(B) = \cup\{i(b): b \in B\}$and $d(B) = \cup\{d(b): b \in B\}$.
iii. A set B is called increasing (resp. decreasing), If $B = i(B)$ $(resp. B = d(B))$.

Definition 1.2. [20] A subset E of a supra topological space (X, μ) is called supra pre-open if $E \subseteq int(cl(E))$ and its complement is called supra pre-closed.

Definition 1.3. [2] A map $g: (X, \tau) \to (Y, \theta)$ is said to be supra open (resp. supra closed) if the image of any open (resp. closed) subset of X is a supra open (resp. supra closed) subset of Y.

Definition 1.4.[16]

i. A map $g: (X, \tau) \to (Y, \theta)$ is said to be supra continuous if the inverse image of each open subset of Y is a supra open subset of X.
ii. Let (X, τ) be a topological space and μ be a supra topology on X. We say that μ is associated supra topology with τ if $\tau \subseteq \mu$.

Definition 1.5. [20] A map $g: (X, \tau) \to (Y, \theta)$ is said to be:

i. Supra pre-continuous if the inverse image of each open subset of Y is a supra pre-open subset of X.
ii. Supra pre-open (resp. supra pre -closed) if the image of each open (resp. closed) subset of X is a supra pre -open (resp. supra pre -closed) subset of Y.

In what follows, we give the definition of supra pre-homeomorphism maps.

Definition 1.6. A map $g: (X, \tau) \to (Y, \theta)$ is said to be supra pre-homeomorphism if it is bijective, supra pre -continuous and supra pre -open.

Definition 1.7. A map $g: (X, \preccurlyeq_1) \to (Y, \preccurlyeq_2)$ is called

i. Order preserving (or increasing) if $a \preccurlyeq_1 b$, then $f(a) \preccurlyeq_2 f(b)$, for each $a, b \in X$.
ii. Order embedding if $a \preccurlyeq_1 b$ if and only if $f(a) \preccurlyeq_2 f(b)$, for each $a, b \in X$.

Theorem 1.8.

i. If $g: (X, \preccurlyeq_1) \to (Y, \preccurlyeq_2)$ is an increasing map, then the inverse image of each increasing (resp. decreasing) is increasing (resp. decreasing).
ii. If $g: (X, \preccurlyeq_1) \to (Y, \preccurlyeq_2)$ is a decreasing map, then the inverse image of each increasing (resp. decreasing) is decreasing (resp. increasing).

Definition 1.9. ([16], [20]) Let E be a subset of a supra topological space (X, μ). Then:

i. Supra interior of E, denoted by $sint(E)$, is the union of all supra open sets contained in E.
ii. Supra closure of E, denoted by $scl(E)$, is the intersection of all supra closed sets containing E.
iii. Supra pre-interior of E, denoted by $spint(E)$, is the union of all supra pre-open sets contained in E.
iv. Supra pre-closure of E, denoted by $spcl(E)$, is the intersection of all supra pre-closed sets containing E.

Definition 1.10. [17] A topological ordered space $(X, \tau, \preccurlyeq)$ is called:

i. Lower (Upper) strong T_1-ordered if for each $a, b \in X$ such that $a \npreceq b$, there exists an increasing (a decreasing) open set G containing $a(b)$ such that $b(a)$ belongs to G^c.
ii. Strong T_1-ordered if it is both strong lower T_1-ordered and strong upper T_1-ordered.

iii. Strong T_0-ordered if it is strong lower T_1-ordered or strong upper T_1-ordered.
iv. Strong T_2-ordered if for every $a, b \in X$ such that $a \not\preccurlyeq b$, there exist disjoint open setss W_1 and W_2 of a and b, respectively, such that W_1 is increasing and W_2 is decreasing.

Definition 1.11. [12] A supra topological ordered space $(X, \mu, \preccurlyeq)$ is called:

i. Lower (Upper) SST_1-ordered if for each $a, b \in X$ such that $a \not\preccurlyeq b$, there exists an increasing (a decreasing) supra open set G containing $a(b)$ such that $b(a)$ belongs to G^c.
ii. SST_1-ordered space if it is both lower and upper SST_1-ordered space.
iii. SST_0-ordered space if it is lower SST_1-ordered space or upper SST_1-ordered space.
iv. SST_2-ordered if for every $a, b \in X$ such that $a \not\preccurlyeq b$, there exist disjoint supra open sets W_1 and W_2 of a and b, respectively, such that W_1 is increasing and W_2 is decreasing.

2. Supra pre-continuous Maps in Supra Topological Ordered Spaces

The concepts of I-supra pre-continuous, D-supra pre-continuous and B-supra pre-continuous maps in supra topological ordered spaces are presented and their main properties are investigated. The relationships among them are illustrated with the help of examples. Under what conditions these three types of supra pre-continuous maps preserve some of ordered supra pre-separation axioms are studied as well.

Definition 2.1. A subset E of $(X, \mu, \preccurlyeq)$ is said to be:

i. I-supra (resp. D-supra, B-supra) pre-open if it is supra pre-open and increasing (resp. decreasing, balancing).
ii. I-supra (resp. D-supra, B-supra) pre-closed if it is supra pre-closed and increasing (resp. decreasing, balancing).

Definition 2.2. A map $f: (X, \mu, \preccurlyeq_1) \to (Y, \tau, \preccurlyeq_2)$ is called I-supra (resp. D-supra, B-supra) pre-continuous at $p \in X$if for each open set H containing $f(p)$, there exists an I-supra (resp. a D-supra, a B-supra) pre-open set G containing p such that $f(G) \subseteq H$.
Also, the map is called I-supra (resp. D-supra, B-supra) pre-continuous if it is continuous at each point $p \in X$.

Theorem 2.3. A map $f: (X, \mu, \preccurlyeq_1) \to (Y, \tau, \preccurlyeq_2)$ is I-supra (resp. D-supra, B-supra) pre-continuous if and only if the inverse image of each open subset of Y is an I-supra (resp. a D-supra, a B-supra) pre-open subset of X.
Proof. We only prove the theorem in case of f is an I-supra pre-continuous map.
To prove the necessary part, let G be an open subset of Y, Then we have the following two cases:
$f^{-1}(G) = \emptyset$which is an I-supra pre-open subset of X.
$f^{-1}(G) \neq \emptyset$. By choosing $p \in X$ such that $p \in f^{-1}(G)$, we obtaine that $f(p) \in G$. So there exists an I-supra pre-open set H_p containing p such that $f(H_p) \subseteq G$. Since p is chosen arbitrary, then $f^{-1}(G) = \cup_{p \in f^{-1}(G)} H_p$. Thus $f^{-1}(G)$ is an I-supra pre-open subset of X.
To prove the sufficient part, let G be an open subset of Ycontaining $f(p)$. Then $p \in f^{-1}(G)$. By hypothesis,$f^{-1}(G)$ is an I-supra pre-open set. Since $f(f^{-1}(G)) \subseteq G$, then f is an I-supra pre-continuous at $p \in X$ and since p is chosen arbitrary, then f is an I-supra pre-continuous.

Remark 1.

i. Every I-supra (D-supra, B-supra) pre-continuous map is supra pre-continuous.
ii. Every B-supra pre-continuous map is I-supra pre-continuous and D-supra pre-continuous.

The following two examples illustrate that a supra pre-continuous map (resp. I-supra pre-continuous) need not be I-supra pre-continuous or D-supra pre-continuous or B-supra pre-continuous (resp. B-supra pre-continuous).

Example 2.4. Let the supra topology $\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and the topology $\tau = \{\emptyset, Y, \{x\}\}$ on $X = \{a, b, c, d\}$and $Y = \{x, y, z\}$, respectively. Let the partial ordered relation $\preccurlyeq_1 = \Delta \{(a, b), (b, d), (a,$

$d)\}$ on X and let a map $f: X \rightarrow Y$ is defined as follows $f(a) = f(c) = f(d) = x$and $f(b) = y$. Obviously, f is supra pre-continuous. Now, $\{x\}$ is an open subset of Y, whereas $f^{-1}(\{x\}) = \{a,\ c,\ d\}$ is neither a decreasing nor an increasing supra pre-open subset of X. Then f is not I-supra (D-supra, B-supra) pre-continuous.

Example 2.5. We replace only the partial order relation in Example 2.4, by $\preccurlyeq_1 == \Delta\ \{(b,\ d)$. Then the map f is I-supra pre-continuous, but not B-supra pre-continuous.

The relationships among types of supra continuous maps are illustrated in the following figure.

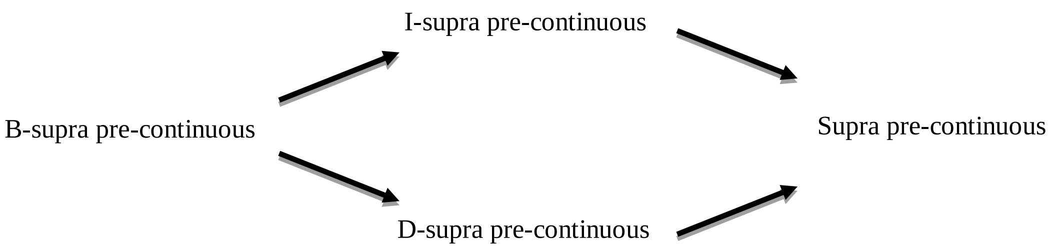

Fig 1: The relationships among types of supra continuous maps

Definition 2.6. Let E be a subset of $(X,\ \mu,\ \preccurlyeq_1)$. Then:

i. $E^{ispo} = \cup\{G: G$ is an I-supra pre-open set contained in $E\}$.
ii. $E^{dspo} = \cup\{G: G$ is a D-supra pre-open set contained in $E\}$.
iii. $E^{\mathrm{b}spo} = \cup\{G: G$ is a B-supra pre-open set contained in $E\}$.
iv. $E^{ispcl} = \cup\{G: G$ is an I-supra pre-closed set containing $E\}$.
v. $E^{dspo} = \cup\{G: G$ is a D-supra pre-closed set containing $E\}$.
vi. $E^{\mathrm{b}spo} = \cup\{G: G$ is a B-supra pre-closed set containing $E\}$.

Lemma 2.7. Let E be a subset of $(X,\ \mu,\ \preccurlyeq_1)$. Then:

i. $(E^{dspo})^c = (E^c)^{ispcl}$.
ii. $(E^{ispo})^c = (E^c)^{dspcl}$.
iii. $(E^{\mathrm{b}spo})^c = (E^c)^{\mathrm{b}spcl}$.

Proof.
$(E^{dspo})^c = \{\cup F: F$is a D-supra pre-open set contained in $E\}^c$
$=\cap \{F^c: F^c$is an I-supra pre-closed set containing $E^c\}$
$= (E^c)^{ispo}$.
The proof of ii and iii is similar to that of i.

Theorem 2.8. Let $g: (X,\ \mu,\ \preccurlyeq_1) \rightarrow (Y, \tau,\ \preccurlyeq_2)$ be a map. Then the following five statements are equivalent:

i. g is I-supra pre-continuous;
ii. The inverse image of each closed subset of Y is a D-supra pre-closed subset of X;
iii. $(g^{-1}(H))^{dspcl} \subseteq g^{-1}(cl(H))$, for every $H \subseteq Y$;
iv. $g\left(A^{dspcl}\right) \subseteq cl\left(g(A)\right)$, for every $A \subseteq X$;
v. $g^{-1}\left(int(H)\right) \subseteq (g^{-1}(H))^{ispo}$.

Proof.
i→ii: Consider H is a closed subset of Y. Then H^c is open. Therefore $g^{-1}(H^c) = (g^{-1}(H))^c$ is an I-supra pre-open subset of X. So $g^{-1}(H)$ is D-supra pre-closed.
ii→iii: For any subset H of Y, we have that $cl(H)$ is closed. Since $g^{-1}\left(cl(H)\right)$ is a D-supra pre-closed subset of X, then $(g^{-1}(H))^{dspcl} \subseteq (g^{-1}(cl(H)))^{dspcl} = g^{-1}(cl(H))$.

iii→iv: Consider A is a subset of X. Then $A^{dspcl} \subseteq (g^{-1}(g(A)))^{dspcl} \subseteq g^{-1}(cl(g(A)))$. Therefore $g(A^{dspcl}) \subseteq g(g^{-1}(cl(g(A)))) \subseteq cl(g(A))$.
iv→v: Consider H is a subset of Y. By Lemma 2.7, we obtain that $g(X-(g^{-1}(H))^{ispo}) = g(((g^{-1}(H))^c)^{dspcl})$. By iv, $g(((g^{-1}(H))^c)^{dspcl}) \subseteq cl(g(g^{-1}(H))^c) = cl(g(g^{-1}(H^c))) \subseteq cl(Y-H) = Y - int(H$. So$X-(g^{-1}(H))^{ispo} \subseteq g^{-1}(Y-int(H)) = X - g^{-1}(int(H))$. Thus $g^{-1}(int(H)) \subseteq (g^{-1}(H))^{ispcl}$.
v→i: Consider H is an open subset of Y. Then $g^{-1}(H) = g^{-1}(int(H)) \subseteq (g^{-1}(H))^{ispo}$. Since $(g^{-1}(H))^{ispo} \subseteq g^{-1}(H)$, then $g^{-1}(H)$ is an I-supra pre-open subset of X. So $g$is I-supra pre-continuous.

Theorem 2.9. Let $g: (X,\ \mu,\ \preccurlyeq_1) \to (Y, \tau,\ \preccurlyeq_2)$ be a map. Then the following five statements are equivalent:
i. g is D-supra pre-continuous;
ii. The inverse image of each closed subset of Y is a I-supra pre-closed subset of X;
iii. $(g^{-1}(H))^{ispcl} \subseteq g^{-1}(cl(H))$, for every $H \subseteq Y$;
iv. $g(A^{ispcl}) \subseteq cl(g(A))$, for every $A \subseteq X$;
v. $g^{-1}(int(H)) \subseteq (g^{-1}(H))^{dspo}$.

Proof. The proof is similar to that of Theorem 2.8.

Theorem 2.10. Let $g: (X,\ \mu,\ \preccurlyeq_1) \to (Y, \tau,\ \preccurlyeq_2)$ be a map. Then the following five statements are equivalent:
i. g is B-supra pre-continuous;
ii. The inverse image of each closed subset of Y is a B-supra pre-closed subset of X;
iii. $(g^{-1}(H))^{bspcl} \subseteq g^{-1}(cl(H))$, for every $H \subseteq Y$;
iv. $g(A^{bspcl}) \subseteq cl(g(A))$, for every $A \subseteq X$;
v. $g^{-1}(int(H)) \subseteq (g^{-1}(H))^{bspo}$.

Proof. The proof is similar to that of Theorem 2.8.

Definition 2.11. A supra topological ordered space $(X,\ \mu,\ \preccurlyeq)$is called:
i. Lower (Upper) strong supra p-T_1-ordered (briefly, lower (upper) SSp-T_1-ordered) if for each $a, b \in X$ such that $a \not\preccurlyeq b$, there exists an increasing (a decreasing) supra pre-open set G containing $a(b)$ such that $b(a)$ belongs to G^c.
ii. SSp -T_0-ordered space if it is lower SSp -T_1-ordered or upper SSp -T_1-ordered.
iii. SSp -T_1-ordered space if it is both lower SSp -T_1-ordered and upper SSp -T_1-ordered.
iv. SSp -T_2-ordered if for every $a, b \in X$ such that $a \not\preccurlyeq b$, there exist disjoint supra p-open sets $W_1$and W_2 containing a and b, respectively, such that W_1 is increasing and W_2 is decreasing.
v. Lower (resp. upper) supra p-regularly ordered if for every decreasing (resp. increasing) supra pre-closed set $F$and for each $a \notin F$, there exist disjoint supra pre-open sets $W_1$and W_2 containing F and a, respectively, such that W_1 is decreasing (resp. increasing) and W_2 is increasing (resp. decreasing).
vi. Supra p-normally ordered if for every disjoint supra pre-closed sets $F_1$and F_2 such that F_1 is decreasing and F_2 is increasing, there exist disjoint supra pre-open sets $W_1$and W_2 containing $F_1$and F_2, respectively, such that W_1 is decreasing and W_1 is increasing.

Theorem 2.12. Let a bijective map$f: (X,\ \mu,\ \preccurlyeq_1) \to (Y, \tau,\ \preccurlyeq_2)$ be I-supra pre-continuous and f^{-1} be order preserving. If $(Y, \tau,\ \preccurlyeq_2)$ is lower T_1-ordered, then $(X,\ \mu,\ \preccurlyeq_1)$ is lower SSp-T_1-ordered.
Proof. Let $a, b \in X$ such that $a \not\preccurlyeq_1 b$. Then there exist $x, y \in Y$ such that $x = f(a), y = f(b)$. Since f^{-1} is order preserving, then $x \not\preccurlyeq_2 y$. Since $(Y, \tau,\ \preccurlyeq_2)$ is lower T_1-ordered, then there exists an increasing neighborhood W of $x$in Y such that $x \in W$and $y \notin W$. Therefore there exists an open set G such that $x \in G \subseteq W$. Since f is bijective I-supra pre-continuous, then $a \in f^{-1}(G)$ which is I-supra pre-open and $b \notin f^{-1}(G)$. Thus $(X,\ \mu,\ \preccurlyeq_1)$ is lower SSp-T_1-ordered.

Theorem 2.13. Let a bijective map $f: (X,\ \mu,\ \preccurlyeq_1) \to (Y, \tau,\ \preccurlyeq_2)$ be I-supra pre-continuous and f^{-1} be order preserving. If $(Y, \tau,\ \preccurlyeq_2)$ is upperT_1-ordered, then $(X,\ \mu,\ \preccurlyeq_1)$ is upper SSp -T_1-ordered.
Proof. The proof is similar to that of Theorem 2.12.

Theorem 2.14. Let a bijective map $f: (X, \mu, \preccurlyeq_1) \to (Y, \tau, \preccurlyeq_2)$ be I-supra pre-continuous and f^{-1} be order preserving. If $(Y, \tau, \preccurlyeq_2)$ is T_i-ordered, then $(X, \mu, \preccurlyeq_1)$ is SSp-T_i-ordered, for $i = 0, 1, 2$.
Proof. We prove the theorem in case of $i = 2$.

Let $a, b \in X$ such that $a \not\preccurlyeq_1 b$. Then there exist $x, y \in Y$ such that $x = f(a), y = f(b)$. Since f^{-1} is order preserving, then $x \not\preccurlyeq_2 y$. Since $(Y, \tau, \preccurlyeq_2)$ is T_2-ordered, then there exists disjoint neighborhoods W_1 and W_2 of x and y, respectively, such that W_1 is increasing and W_2 is decreasing. Therefore there exist disjoint open sets G and H containing x and y, respectively.. Since f is bijective B-supra pre-continuous, then $a \in f^{-1}(G)$ which is an increasing supra pre -open subset of X, $b \in f^{-1}(H)$ which is a decreasing supra pre-open subset of X and $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Thus $(X, \mu, \preccurlyeq_1)$ is SSp-T_i-ordered.

In a similar way, we can prove the theorem in case of $i = 0, 1$.

Theorem 2.15. Consider $f: (X, \mu, \preccurlyeq_1) \to (Y, \tau, \preccurlyeq_2)$ is a bijective supra pre-continuous such that $f$is ordered embedding. If $(Y, \tau, \preccurlyeq_2)$ is strong T_i-ordered, then $(X, \mu, \preccurlyeq_1)$ is SSp-T_i-ordered, for $i = 0, 1, 2$.
Proof. We prove the theorem in case of $i = 2$. Let $a, b \in X$ such that $a \not\preccurlyeq_1 b$. Then there exist $x, y \in Y$ such that $x = f(a), y = f(b)$. Since f is ordered embedding, then $x \not\preccurlyeq_2 y$. Since $(Y, \tau, \preccurlyeq_2)$ is strong T_2-ordered, then there exist disjoint supra open sets W_1 and W_2 containing x and y, respectively, such that W_1 is increasing and W_2 is decreasing. Since f is bijective supra pre-ccontinuous and order preserving, then $f^{-1}(W_1)$ is an increasing supra pre-open set containing a, $f^{-1}(W_2)$ is a decreasing supra pre-open set containing b and $f^{-1}(W_1) \cap f^{-1}(W_2) = \emptyset$. Thus $(X, \mu, \preccurlyeq_1)$ is SSp-T_i-ordered.

Similarly, one can prove the theorem in case of $i = 0, 1$.

Theorem 2.16. Consider $f: (X, \mu, \preccurlyeq_1) \to (Y, \tau, \preccurlyeq_2)$ is an injective B-supra pre-continuous map. If $(Y, \tau, \preccurlyeq_2)$ is a T_i-space, then $(X, \mu, \preccurlyeq_1)$ is an SSp-T_i-ordered space, for $i = 1, 2$.
Proof. We prove the theorem in case of $i = 2$ and the other case is similar. Let $a, b \in X$ such that $a \not\preccurlyeq_1 b$. Then there exist $x, y \in Y$ such that $x = f(a), y = f(b)$. Since f is injective, then $x \neq y$ and since $(Y, \tau, \preccurlyeq_2)$ is a T_2-space, then there exist disjoint open sets G and H such that $x \in G$and $y \in H$. Therefore $a \in f{-}1(G)$ which is an increasing supra pre-open subset of X, $b \in f{-}1(H)$ which is a decreasing supra pre-open subset of X and $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Thus $(X, \mu, \preccurlyeq_1)$ is an SSp-T_2-ordered space.

3. Supra pre-open (Supra pre-closed) Maps in Supra Topological Ordered Spaces

In this section, we introduce the concepts of I-supra pre-open (I-supra pre-closed), D-supra pre-open (D-supra pre-closed) and B-supra pre-open (B-supra pre-closed) maps in supra topological ordered spaces. We demonstrate their main properties and illustrate relationships among them with the help of examples. Finally, we present some results concerning the image and per image of some separation axioms under these maps.

Definition 3.1. A map $g: (X, \tau, \preccurlyeq_1) \to (Y, \mu, \preccurlyeq_2)$ is said to be:

i. I-supra (resp. D-supra, B-supra) pre-open if the image of any open subset of X is an I-supra (resp. D-supra, B-supra) pre-open subset of Y.
ii. I-supra (resp. D-supra, B-supra) pre-closed if the image of any closed subset of X is an I-supra (resp. D-supra, B-supra) pre-closed subset of Y.

Remark 1.

i. Every I-supra (D-supra, B-supra) pre-open map is supra pre-open.
ii. Every I-supra (D-supra, B-supra) pre-closed map is supra pre-closed.
iii. Every B-supra pre-open (resp. B-supra pre-closed) map is I-supra pre-open and D-supra pre-open (resp. I-supra pre-closed and D-supra pre-closed).

The following two examples illustrate that a supra pre-open (resp. D-supra pre-open) map need not be I-supra pre-open or D-supra pre-open or B-supra pre-open (resp. B-supra pre-open).

Example 3.2. Let $\tau = \{\emptyset, X, \{1, 2\}\}$ be a topology and $\preccurlyeq_2 = \Delta \{(1, 3), (3, 2), (1, 2)\}$ be a partial order relation on $X = \{1, 2, 3\}$. Let the supra topology associated with τ be $\{\emptyset, X, \{1\}, \{1, 2\}, \{1, 3\}\}$. The identity map $f: X \to X$ is supra pre-open. Now, $\{1, 2\}$ is an open subset of X. Since $f(\{1, 2\}) = \{1, 2\}$ is neither an increasing nor a decreasing supra pre-open subset of Y. Then f is not x-supra pre-open map for $x = \{I,\ D,$ B$\}$.

Example 3.3. We replace only the partial order relation in Example 3.2 by $\preccurlyeq_2 = \Delta\ \{(1,\ 3), (1,\ 2)\}$. Then the map f is D-supra pre-open, but is not B-supra pre-open.

The following two examples illustrate that a supra pre-closed (resp. an I-supra pre-closed) map need not be I-supra pre-closed or D-supra pre-closed or B-supra pre-closed (resp. B-supra pre-closed).

Example 3.4. Let the topology $\tau = \{\emptyset,\ X,\ \{a,\ b\}\}$on $X = \{a,\ b,\ c\}$, the supra topology associated with τ be $\{\emptyset,\ X, \{c\},\ \{a,\ b\}\}$ and the partial order relation $\preccurlyeq_2 = \Delta\ \{(a,\ c), (c,\ b), (a, b)\}$ on X. The map $f\colon X \to X$ is defined as follows $f(a) = f(c) = c$and $f(b) = b$. Obviously, f is supra pre-closed. Now, $\{c\}$ is a closed subset of X, but $f(\{c\}) = \{c\}$ is neither a decreasing nor an increasing supra pre-closed subset of Y. Then the map f is not $I(\ D,$ B)-supra pre-closed.

Example 3.5. We replace only the partial order relation in Example 3.4 by $\preccurlyeq_2 = \Delta\ \{(b,\ c)\}$. Then the map f is I-supra pre-closed, but is not B-supra pre-closed.

The relationships among the presented types of supra open (supra closed) maps are illustrated in the following figure.

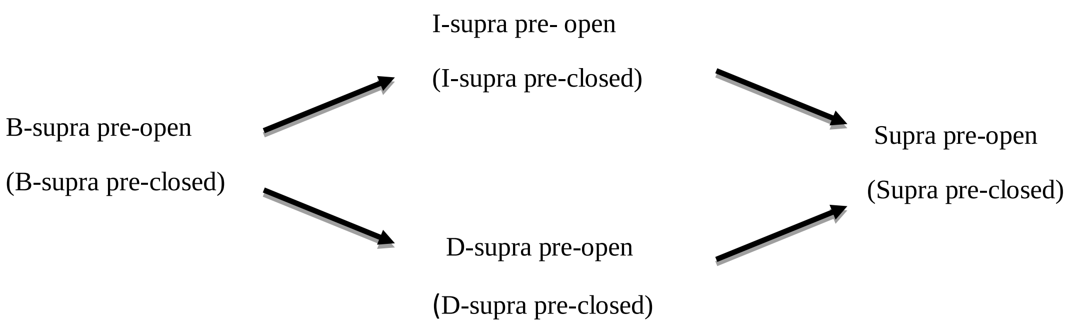

Fig 2: The relationships among types of supra open (supra closed) maps

Theorem 3.6. The following statements are equivalent, for a map $f\colon (X,\ \tau,\ \preccurlyeq_1) \to (Y,\ \mu,\ \preccurlyeq_2)$:

i. f is I-supra pre-open;
ii. $intf^{-1}(H) \subseteq f^{-1}(H^{ispo})$, for every $H \subseteq Y$;
iii. $f\big(int(G)\big) \subseteq (f(G))^{ispo}$, for every $G \subseteq X$.

Proof.i→ii: Since $int(f^{-1}(H))$ is an open subset of X, then $f(int\big(f^{-1}(H)\big))$ is an I-supra pre-open subset of Y. Obviously, $f\left(int\big(f^{-1}(H)\big)\right) \subseteq f\big(f^{-1}(H)\big) \subseteq H$. So $int\big(f^{-1}(H)\big) \subseteq f^{-1}(H^{ispo})$.

ii→iii: By replacing H by $f(G)$ in ii, we obtain that $int(f^{-1}\big(f(G)\big)) \subseteq f^{-1}(\big(f(G)\big)^{ispo})$. Since $int(G) \subseteq int(f^{-1}(f(G))) \subseteq f^{-1}(\big(f(G)\big)^{ispo})$, then $f\big(int(G)\big) \subseteq (f(G))^{ispreo}$.

iii→i: Let G be an open subset of X. Then $f\big(int(G)\big) = f(G) \subseteq (f(G))^{ispo}$. So $f(G)$ is an I-supra pre-open set. Thus f is an I-supra pre-open map.

In a similar way one can prove the following two theorems.

Theorem 3.7. The following statements are equivalent, for a map $f\colon (X,\ \tau,\ \preccurlyeq_1) \to (Y,\ \mu,\ \preccurlyeq_2)$:

i. f is D-supra pre-open;
ii. $intf^{-1}(H) \subseteq f^{-1}(H^{dspo})$, for every $H \subseteq Y$;

iii. $f\left(int(G)\right) \subseteq (f(G))^{dspo}$, for every $G \subseteq X$.

Theorem 3.8. The following statements are equivalent, for a map $f: (X, \tau, \preccurlyeq_1) \to (Y, \mu, \preccurlyeq_2)$:

i. f is B-supra pre-open;
ii. $intf^{-1}(H) \subseteq f^{-1}(H^{bspo})$, for every $H \subseteq Y$;
iii. $f\left(int(G)\right) \subseteq (f(G))^{bspo}$, for every $G \subseteq X$.

Theorem 3.9. Let $f: (X, \tau, \preccurlyeq_1) \to (Y, \mu, \preccurlyeq_2)$ be a map. Then we have the following results.

i. f is I-supra pre-closed if and only if $(f(G))^{ispcl} \subseteq f(cl(G))$, for any $G \subseteq X$.
ii. f is D-supra pre-closed if and only if $(f(G))^{dspcl} \subseteq f(cl(G))$, for any $G \subseteq X$.
iii. f is B-supra pre-closed if and only if $(f(G))^{bspcl} \subseteq f(cl(G))$, for any $G \subseteq X$.

Proof. (i) Necessity: Consider f is an I-supra pre-closed map. Then for any $G \subseteq X$, we have that $f(cl(G))$ is an I-supra pre-closed subset of Y. Since $f(G) \subseteq f(cl(G))$, then $(f(G))^{ispcl} \subseteq f(cl(G))$.
Sufficiency: Consider B is a closed subset of X. Then $f(B) \subseteq (f(B))^{ispcl} \subseteq f\left(cl(B)\right) = f(B)$. Therefore $f(B) = f(B)^{ispcl}$. Thus $f(B)$ is an I-supra pre-closed set. Hence f is an I-supra pre-closed map.
The proof of ii and iii is similar to that of (i).

Theorem 3.10. Let $f: (X, \tau, \preccurlyeq_1) \to (Y, \mu, \preccurlyeq_2)$ be a bijective map. Then we have the following results.

i. f is I-supra pre-open if and only if f is D-supra pre-closed.
ii. f is D-supra pre-open if and only if f is I-supra pre-closed.
iii. f is B-supra pre-open if and only if f is B-supra pre-closed.

Proof. (i) Necessity: Let f be an I-supra pre-open map and let G be a closed subset of X. Then G^c is open. Since f is bijective, then $f(G^c) = (f(G))^c$ is I-supra pre-open. Therefore $f(G)$ is a D-supra pre-closed subset of Y. Thus the map f is D-supra pre-closed.
Sufficiency: Let f be a D-supra pre-closed map and let B be an open subset of X. Then B^c is closed. Since f is bijective, then $f(B^c) = (f(B))^c$ is D-supra pre-closed. Therefore $f(B)$ is I-supra pre-open. Thus f the map is I-supra pre-closed.
The proof of ii and iii is similar to that of (i).

Theorem 3.11. The following two statements hold.

i. If the maps $f: (X, \tau, \preccurlyeq_1) \to (Y, \theta, \preccurlyeq_2)$ is open and $g: (Y, \theta, \preccurlyeq_2) \to (Z, \nu, \preccurlyeq_3)$ is I-supra (resp. D-supra, B-supra) pre-open, then a map gof is I-supra (resp. D-supra, B-supra) pre-open.
ii. If the maps $f: (X, \tau, \preccurlyeq_1) \to (Y, \theta, \preccurlyeq_2)$ is closed and $g: (Y, \theta, \preccurlyeq_2) \to (Z, \nu, \preccurlyeq_3)$ is I-supra (resp. D-supra, B-supra) pre-closed, then a map gof is I-supra (resp. D-supra, B-supra) pre-closed.

Proof. It is clear.

Theorem 3.12. If the maps gof is I-supra (resp. D-supra, B-supra) pre-open and $f: (X, \tau, \preccurlyeq_1) \to (Y, \theta, \preccurlyeq_2)$ is surjective continuous, then a map $g: (Y, \theta, \preccurlyeq_2) \to (Z, \nu, \preccurlyeq_3)$ is I-supra (resp. D-supra, B-supra) pre-open.
Proof. Consider f is continuous and let G be an open subset of Y. Then $f^{-1}(G)$ is an open subset of X. Since gof is I-supra pre-open and f is surjective, then $gof\left(f^{-1}(G)\right) = g(G)$ is an I-supra pre-open subset of Z. Therefore g is I-supra pre-open.
A similar proof can be given for the cases between parentheses.

Theorem 3.13. If the maps $gof: (X, \tau, \preccurlyeq_1) \to (Z, \nu, \preccurlyeq_3)$ is closed and $g: (Y, \theta, \preccurlyeq_2) \to (Z, \nu, \preccurlyeq_3)$ is injective I-supra (resp. D-supra, B-supra) pre-continuous, then a map $f: (X, \tau, \preccurlyeq_1) \to (Y, \theta, \preccurlyeq_2)$ is D-supra (resp. I-supra, B-supra) pre-closed.
Proof. Consider g is I-supra pre-continuous and let G be a closed subset of X. Then $gof(G)$ is a closed subset of Z. Since g is injective I-supra pre-continuous, then $g^{-1}\left(gof((G))\right) = f(G)$ is a D-supra pre-closed subset of Y. Therefore f is D-supra pre-closed.

A similar proof can be given for the cases between parentheses.

Theorem 3.14. We have the following results for a bijective map $f\colon (X,\ \tau,\ \preccurlyeq_1) \to (Y,\ \theta,\ \preccurlyeq_2)$:

i. f is I-supra (resp. D-supra, B-supra) pre-open if and only if f^{-1} is I-supra (resp. D-supra, B-supra) pre-continuous.

ii. f is D-supra (resp. I-supra, B-supra) pre-closed if and only if f^{-1} is I-supra (resp. D-supra, B-supra) pre-continuous.

Proof. We prove item i when f is B-supra pre-open, and the other cases follows similar lines.
" $\Rightarrow$ " Let f be a B-supra pre-open map and let G be an open subset of X. Then $(f^{-1})^{-1}(G) = f(G)$ is a balancing supra pre-open subset of Y. Therefore f^{-1} is B-supra pre-continuous.
" $\Leftarrow$ " Let G be an open subset of X and let f^{-1} be a B-supra pre-continuous map. Then $f(G) = (f-1)-1G$ is a balancing supra pre-open subset of Y. Therefore f is B-supra pre-open.
Similarly, one can prove item ii.

Theorem 3.15. Let a bijective map $f\colon (X,\ \tau,\ \preccurlyeq_1) \to (Y,\ \mu,\ \preccurlyeq_2)$ be I-supra pre-open (D-supra pre-closed) and order preserving. If $(X,\ \tau,\ \preccurlyeq_1)$ is lower T_1-ordered, then $(Y,\ \mu,\ \preccurlyeq_2)$ is lower SSp-T_1-ordered.
Proof. We prove the theorem when a map f be I-supra pre-open.
Let $x, y \in Y$ such that $x \not\preccurlyeq_2 y$. Since f is bijective, then there exist $a, b \in X$ such that $a = f^{-1}(x)$, $b = f^{-1}(y)$ and since f is an ordered preserving map, then $a \not\preccurlyeq_1 b$. By hypothesis $(X,\ \tau,\ \preccurlyeq_1)$ is lower T_1-ordered, then there exists an increasing neighborhood W in X such that $a \in W$ and $b \notin W$. Therefore there exists an open set G such that $a \in G \subseteq W$. Thus $x \in f(G)$ which is an I-supra pre-open and $y \notin f(G)$. Hence $(Y,\ \mu,\ \preccurlyeq_2)$ is lower SSpre-T_1-ordered.
The proof for a D-supra pre-closed map is achieved similarly.

Theorem 3.16. Let a bijective map $f\colon (X,\ \tau,\ \preccurlyeq_1) \to (Y,\ \mu,\ \preccurlyeq_2)$ be D-supra pre-open (I-supra pre-closed) and order preserving. If $(X,\ \tau,\ \preccurlyeq_1)$ is upper T_1-ordered, then $(Y,\ \mu,\ \preccurlyeq_2)$ is upper SSp-T_1-ordered.
Proof. The proof is similar to that of Theorem 3.15.

Theorem 3.17. Let a bijective map $f\colon (X,\ \tau,\ \preccurlyeq_1) \to (Y,\ \mu,\ \preccurlyeq_2)$ be B-supra pre-open (B-supra pre-closed) and order preserving. If $(X,\ \tau,\ \preccurlyeq_1)$ is T_i-ordered, then $(Y,\ \mu,\ \preccurlyeq_2)$ is SSpre-T_i-ordered, for $i = 0,1,2$.
Proof. We prove the theorem when a map f is B-supra pre-open and $i = 2$.
For all $x, y \in Y$ such that $x \not\preccurlyeq_2 y$, there exist $a, b \in X$ such that $a = f^{-1}(x)$, $b = f^{-1}(y)$ and since f is an ordered preserving map, then $a \not\preccurlyeq_1 b$. Since $(X,\ \tau,\ \preccurlyeq_1)$ is T_2-ordered, then there exist disjoint neighborhoods W_1 and W_2 of a and b, respectively, such that W_1 is increasing and W_2 is decreasing. Therefore there are disjoint open sets G and H such that $a \in G \subseteq W_1$ and $b \in H \subseteq W_2$. Thus $x \in f(G)$ which is a B-supra pre-open, $y \in f(H)$ which is a B-supra pre-open and $f(G) \cap f(H) = \emptyset$. Thus $(Y,\ \mu,\ \preccurlyeq_2)$ is SSp-T_2-ordered.
In a similar way, we can prove the theorem in case of $i = 0, 1$.
The proof for a B-supra pre-closed map is achieved similarly.

Theorem 3.18. Consider a bijective map $f\colon (X,\ \tau,\ \preccurlyeq_1) \to (Y,\ \mu,\ \preccurlyeq_2)$ be supra pre-open such that f and f^{-1} are order preserving. If $(X,\ \tau,\ \preccurlyeq_1)$ is strong T_i-ordered, then $(Y,\ \mu,\ \preccurlyeq_2)$ is SSp-T_i-ordered, for $i = 0,1,2$.
Proof. We prove the theorem in case of $i = 2$. Let $x, y \in Y$ such that $x \not\preccurlyeq_2 y$. Then there exist $a, b \in X$ such that $a = f^{-1}(x)$, $b = f^{-1}(y)$. Since f is ordered preserving, then $a \not\preccurlyeq_1 b$. Since $(X,\ \tau,\ \preccurlyeq_1)$ is strong T_2-ordered, then there exist disjoint an I-supra open set W_1 and containing a and a D-supra open set W_2 containing b. Since f is a bijective supra pre-open and f^{-1} is an order preserving, then $f(W_1)$ is an I-supra open set containing x, $f(W_2)$ is a D-supra open set containing y and $f(W_1) \cap f(W_2) = \emptyset$. Therefore $(Y,\ \mu,\ \preccurlyeq_2)$ is SSp-T_2-ordered.
Similarly, one can prove the theorem in case of $i = 0,1$

Theorem 3.19. Let $f\colon (X,\ \tau,\ \preccurlyeq_1) \to (Y,\ \mu,\ \preccurlyeq_2)$ be a bijective supra open map such that f and f^{-1} are order preserving. If $(X,\ \tau,\ \preccurlyeq_1)$ is strong T_i-ordered, then $(Y,\ \mu,\ \preccurlyeq_2)$ is SSp-T_i-ordered, for $i = 0,1,2$.

Proof. The proof is similar to that of Theorem 3.18.

4. Supra pre-homeomorphism Maps in Supra Topological Ordered Spaces

The concepts of I-supra pre-homeomorphism, D-supra pre-homeomorphism and B-supra pre-homeomorphism maps are introduced and many of their properties are established. Some illustrative examples are provided.

Definition 4.1. Let τ^* and θ^* be associated supra topologies with $\tau$and θ, respectively. A bijective map $g: (X,\ \tau,\ \preccurlyeq_1) \rightarrow (Y,\ \theta,\ \preccurlyeq_2)$ is called I-supra (resp. D-supra, B-supra) pre-homeomorphism if it is I-supra pre-continuous and I-supra pre-open (resp. D-supra pre-continuous and D-supra pre-open, B-supra pre-continuous and B-supra pre-open).

Remark 1.

i. Every I-supra (resp. D-supra, B-supra) pre -homeomorphism map is supra pre-homeomorphism.
ii. Every B-supra pre-homeomorphism map is I-supra pre-homeomorphism and D-supra pre-homeomorphism.

The following two examples illustrate that a supra pre-homeomorphism (resp. D-supra pre-homeomorphism) map need not be I-supra pre-homeomorphism or D-supra pre-homeomorphism or B-supra pre-homeomorphism (resp. B-supra pre-homeomorphism).

Example 4.2. Let the topology $\tau = \{\emptyset,\ X,\ \{a,\ c\}\}$on $X = \{a,\ b,\ c\}$, the supra topology associated with τ be $\left\{\emptyset,\ X, \{a\},\ \{a,\ c\}\right\}$ and the partial order relation $\preccurlyeq_1 = \triangle \{(c,\ a), (c,\ b)\}$ on X. Let the topology $\theta = \{\emptyset,\ Y,\ \{y,\ z\}\}$on $Y = \{x,\ y,\ z\}$, the supra topology associated with θ be $\left\{\emptyset,\ Y,\ \{y\},\ \{y,\ z\}\right\}$ and the partial order relation $\preccurlyeq_2 = \triangle \{(y,\ z)\}$ on Y. The map $f: (X,\ \tau,\ \preccurlyeq_1) \rightarrow (Y,\ \theta,\ \preccurlyeq_2)$ is defined as follows $f(a) = y,\ f(b) = z$and $f(c) = x$. Now, the map f is supra pre-homeomorphism, but is not x-supra pre-homeomorphism, for $x = \{I,\ D,\ B\}$.

Example 4.3. We replace only the partial order relation $\preccurlyeq_1$ in Example 4.2 by $\preccurlyeq = \triangle \{(a,\ c)\}$. Then the map f is D-supra pre-homeomorphism, but not B-supra pre-homeomorphism.

The relationships among the presented types of supra homeomorphism maps are illustrated in the following figure.

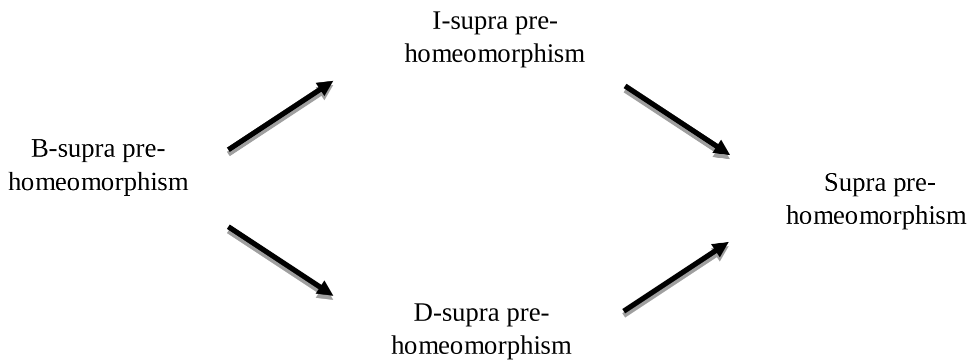

Fig 3: The relationships among types of supra homeomorphism maps

Theorem 4.4. Let a map $f: X \rightarrow Y$ be bijective I-supra pre-continuous. Then the following statements are equivalent:

i. f is I-supra pre-homeomorphism;
ii. f^{-1} is I-supra pre-continuous;
iii. $f$is$D$-supra pre-closed.

Proof.i→ ii: Let G be an open subset of X. By hypothesis, f is bijective, then $(f^{-1})^{-1}(G) = f(G)$. Since f is I-supra pre-continuous, then $(f^{-1})^{-1}(G)$ an I-supra pre-open set in Y. Therefore f^{-1} is I-supra pre-continuous.
ii→ iii: Let G be a closed subset of X. Then G^c is an open subset of X and $(f^{-1})^{-1}(G^c) = f(G^c) = f(G)^c$ is an I-supra pre-open set in Y. Therefore $f(G)$ is a D-supra pre-closed subset of Y. Thus f is D-supra pre-closed.
iii→ i: Let G be an open subset of X. Then G^c is a closed set and $f(G^c) = f(G)^c$ is D-supra pre-closed. Therefore $f(G)$ is an I-supra pre-open subset of Y. Thus f is an I-supra pre-open map. This completes the proof.

In a similar way one can prove the following two theorems.

Theorem 4.5. Let a map $f: X \to Y$ be bijective D-supra pre-continuous. Then the following statements are equivalent:

i. f is D-supra pre-homeomorphism;
ii. f^{-1} is D-supra pre-continuous;
iii. $f$is$I$-supra pre-closed.

Theorem 4.6. Let a map $f: X \to Y$ be bijective$B$-supra pre-continuous. Then the following statements are equivalent:

i. f is B-supra pre-homeomorphism;
ii. f^{-1} is B-supra pre-continuous;
iii. $f$is$B$-supra pre-closed.

Theorem 4.7. Consider $(X,\ \tau,\ \preccurlyeq_1)$, $(Y,\ \theta,\ \preccurlyeq_2)$are two topological ordered spaces, τ^* and θ^* be associated supra topologies with $\tau$and θ, respectively. Let $f: X \to Y$ be a supra pre-homeomorphism map such that f and f^{-1} are order preserving. If X(resp. Y) is strong T_i-ordered, then Y(resp. X) is SSp-T_i-ordered, for $i = 0, 1, 2$.
Proof. Let $(X,\ \tau,\ \preccurlyeq_1)$ be strong T_i-ordered, then by Theorem 3.18, $(Y,\ \theta,\ \preccurlyeq_2)$ is SSp-T_i-ordered, for $i = 0, 1, 2$.
Let $(Y,\ \theta,\ \preccurlyeq_2)$ be strong T_i-ordered, then by Theorem 2.15, $(X,\ \tau,\ \preccurlyeq_1)$ is SSp-T_i-ordered, for $i = 0, 1, 2$.

5. Conclusion

In the present paper, the concepts of increasing supra pre-continuous (supra pre-open, supra pre-closed, supra pre-homeomorphism) maps, decreasing supra pre-continuous (supra pre-open, supra pre-closed, supra pre-homeomorphism) maps and balancing supra pre-continuous (supra pre-open, supra pre-closed, supra pre-homeomorphism) maps are given and studied. The sufficient conditions for maps to preserve some separation axioms (which introduced in [8], [12] and [17]) are determined. In particular, we investigate the equivalent conditions for each concept and present their characterizations. Apart from that, we point out the relationships among them with the help of illustrative examples. We plan to use a notion of somewhere dense [5] to define various kinds of maps in topological ordered spaces. In the end, the presented concepts in this paper are fundamental background for studying several topics in supra topological ordered spaces.

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