Journal of Progressive Research in Mathematics(JPRM)
ISSN: 2395-0218

SCITECH
RESEARCH ORGANISATION

Volume 12, Issue 1
Published online: May 22, 2017|

Journal of Progressive Research in Mathematics
www.scitecresearch.com/journals

# A combination forecasting model based on IOWA operators

Yan Qian, Lei Liu

(University of economics and management, Nanjing University of Aeronautics and Astronautics, Nanjing, 211100)

**Abstract.** With respect to the combination forecasting problems with forecast value and actual value which are actual numbers. The relative distance measure is used as the induced value of the IOWA operator. A new fuzzy combination forecasting model is presented to minimize the error sum of squares. Finally, an example is illustrated to show this model improved the combination forecasting accuracy efficiently.

**Key words:** The relative distance measure; combination forecasting; IOWA operator.

## The introduction

Since Bates and Granger put forward the combination forecast, it has long been one of the hot problems at home and abroad. Combination forecast, that is, considering the characteristics of each single prediction methods and combining different prediction methods. Combination forecasting can rally the merits of each single forecasting methods and overcome its shortcomings. Therefore, it has been widely used in many fields such as economy, statistics, society and so on [2-5]. Social environment is becoming more and more complicated, it is often difficult to obtain satisfactory results in the real life by simple combination of traditional real number combination forecasting. *Yager* has put forward the ordered weighted averaging (*OWA*) operator, it can effectively gather data. *Yager* has put forward the induced ordered weighted (*IOWA*) operator and probes into how to solve *IOWA* weight coefficient of combination forecast model. Then combination forecasting based on *IOWA* operator has attracted more and more attention from scholars both at home and abroad because of its excellent prediction effect. This paper [6] presents a combination forecasting method which is based on the prediction accuracy and the error sum squares. Based on the paper [6], paper [7] is based on the combination forecasting effectiveness. In paper [8], the interval-valued fuzzy combination forecasting method is proposed, which is based on the maximum and minimum closeness degree. The correlation coefficient is used as the guidance value in paper [9] and paper [10], and the least square distance is used as the standard in paper [11].

In this paper, based on the synthetic analysis of existed research results, the concept of relative distance measure is proposed and as the induced value of *IOWA* operator. And the combination forecasting model with the least square error sum is constructed. The example in this paper shows that the model can improve the prediction accuracy

## Preliminary data

**Definition 1[12].** Set $OWA_w : R^n \to R$ as n-variate function, $W=(w_1,w_2,...,w_n)^T$ is a weighted vector related to

$OWA_W$ , and it meet the requirements $\sum_{i=1}^{n} w_i = 1, w_i \geq 0, i = 1, 2, ..., n$ , if

$$OWA_W(a_1, a_2, \cdots, a_n) = \sum_{i=1}^{n} w_i b_i$$

The function $OWA_W$ is '$n$' dimension ordered weighted averaging ($OWA$) operator, $b_i$ is the first $i$ largest number among $a_1, a_2, \cdots, a_n$ according the descending order.

**Definition 2**[12-14]**.** Set $(\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle)$ are 2d number array, make

$$IOWA_W(\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle) = \sum_{i=1}^{n} w_i a_{v-index(i)}$$

$W = (w_1, w_2, ..., w_n)^T$ is the weighted vector of, correspond $\sum_{i=1}^{n} w_i = 1, w_i \geq 0, i = 1, 2, ..., n$, $v - index(i)$ is the coordinate under of the number among $v_1, v_2, ..., v_n$ according descending order, then the function $IOWA_W$ is '$n$' induction dimension ordered weighted average operator produce from $v_1, v_2, ..., v_n$ , for short $IOWA$ operator, $v_i$ is induction of $a_i$ .

In order to strengthen understanding, the paper gives an example: Set $\langle 3, 4 \rangle, \langle 1, 2 \rangle, \langle 5, 1 \rangle, \langle 7, 0 \rangle$ are 2d number array, $w = (0.3, 0.4, 0.2, 0.1)^T$ is the weighted vector of $IOWA_W$ , then

$$\begin{aligned} &IOWA_W(\langle 3, 4 \rangle, \langle 1, 2 \rangle, \langle 5, 1 \rangle, \langle 7, 0 \rangle) \\ &= 0 \times 0.3 + 1 \times 0.4 + 4 \times 0.2 + 2 \times 0.1 = 1.4 \end{aligned}$$

The $IOWA$ operator has the following properties according to definition 2:

**Lemma 1 ( Monotonictiy):** Set $\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle$ and $\langle v_1, a_1' \rangle, \langle v_2, a_2' \rangle, ..., \langle v_n, a_n' \rangle$ are any two data vectors, and $a_i' \geq a_i$ , then

$$IOWA_W(\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle) \geq IOWA_W(\langle v_1, a_1' \rangle, \langle v_2, a_2' \rangle, ..., \langle v_n, a_n' \rangle)$$

**Lemma 2 (Commutativity):** Set $\langle v_1, a_1' \rangle, \langle v_2, a_2' \rangle, ..., \langle v_n, a_n' \rangle$ is arbitrary permutation data vector of $\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle$ , then

$$IOWA_W(\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle) = IOWA_W(\langle v_1, a_1' \rangle, \langle v_2, a_2' \rangle, ..., \langle v_n, a_n' \rangle)$$

**Lemma 3 (Idempotency):** Set $\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle$ is any one data vectors, if $a_i = a, \forall i$ , then

$$IOWA_W(\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle) = a$$

**Lemma 4 ( Intermediate value property):** Set $\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle$ is any one data vectors, then

$$\min_i \{a_i\} \leq IOWA_W(\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle) \leq \max_i \{a_i\}$$

**Lemma 5.** Set $\langle v_1, a_1 \rangle, \langle v_2, a_2 \rangle, ..., \langle v_n, a_n \rangle$ is any one data vectors, if $v_i = a_i, \forall i$ , then the $IOWA$ operator is degenerated into an $OWA$ operator.

## Combined forecasting model based on relative distance and IOWA operator

Set the actual value of the predicted object is represented by a sequence of $\{x_t\}_{t=1}^{n}$, there are m kinds of fuzzy single prediction method to predict it, make $x_{it}$ to express predicted value of the $i$ prediction method in the $t$ moment, and $i=1,2,...,m, t=1,2,...,n$, make $L=(l_1,l_2,...,l_m)$ is the weighted vector of $IOWA$, correspond $\sum_{i=1}^{m} l_i = 1$ and $l_i \geq 0, i=1,2,...,m$.

**Definition 3.** Set

$$a_{it} = 1 - d(x_t, \hat{x}_{it}) \Big/ \sum_{i=1}^{m} d(x_t, \hat{x}_{it}), i=1,2,...,m; t=1,2,...,n, \qquad (1)$$

the $a_{it}$ is relative distance accuracy of the $i$ prediction method in the $t$ moment.

As can be seen from the definition 5: the greater the difference between the predicted value and the actual value, the lower the relative distance accuracy.

According to the formula (1), $a_{it} \in [0,1], i=1,2,...,m, t=1,2,...,n$, the relative prediction accuracy $a_{it}$ is used as the guidance value of $\hat{x}_{it}$, m fuzzy single prediction method and relative prediction accuracy based on difference metric function constitutes m 2d number array, namely, $\langle a_{1t}, \hat{x}_{1t} \rangle, \langle a_{2t}, \hat{x}_{2t} \rangle, ..., \langle a_{mt}, \hat{x}_{mt} \rangle$, according to definition 2 the following concepts can be obtained:

**Definition 4**[13-14]**.** Set

$$IOWA_L(\langle a_{1t}, \hat{x}_{1t} \rangle, \langle a_{2t}, \hat{x}_{2t} \rangle, ..., \langle a_{mt}, \hat{x}_{mt} \rangle) = \sum_{i=1}^{m} l_i \hat{x}_{a-index(it)} \qquad (2)$$

$a-index(it)$ is the coordinate under of the number among $a_{1t}, a_{2t}, ..., a_{mt}$ according descending order, the formula (2) called prediction value of triangular fuzzy number combination of IOWA operator induced by relative prediction precision sequence at $t$ moment

As can be seen from the definition 4:after the IOWA operator is induced, the combined forecasting weight coefficients are independent of each fuzzy prediction method, but determined by the relative forecast accuracy of each fuzzy single item forecast, that is, the smaller the difference between the predicted value and the actual value of the first $i$ fuzzy single prediction method at $t$ moment, the corresponding weight is larger.

**Definition 5**[9]Set the actual sequence of the predicted object is $\{x_t\}_{t=1}^{n}$, prediction sequence is $\{\hat{x}_t\}_{t=1}^{n}$, make $SSE = \sum_{t=1}^{n} (\hat{x}_t - x_t)^2$ as the sum of squared error of prediction sequence and actual prediction.

Make $\delta_{a-index(t)} = x_t - \hat{x}_{a-index(it)}$, according to definition3 and definition 4, the sum of squared error of the predicted value and the actual value can be obtained at $n$ points, namely,

$$SSE_1 = \sum_{t=1}^{n} \left[ x_t - \sum_{i=1}^{m} l_i \hat{x}_{a-index(it)} \right]^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} l_i l_j \left[ \sum_{t=1}^{n} \delta_{a-index(it)} \delta_{a-index(jt)} \right]$$

Based on possible point and IWOA operator of fuzzy combination for forecasting model, it can be expressed as:

$$\min \quad SSE_1 = \sum_{i=1}^{m}\sum_{j=1}^{m} l_i l_j \left[ \sum_{t=1}^{n} \delta_{a-index(it)} \delta_{a-index(jt)} \right]$$

$$s.t. \begin{cases} \sum_{i=1}^{m} l_i = 1 \\ l_i \geq 0, i = 1, 2, .., m \end{cases} \qquad (3)$$

Then steps based on possible point and IWOA operator of fuzzy combination for forecasting model is given below:

1) Input the actual sequence, use of each single prediction methods for forecasting sequence as $\{\hat{x}_{it}\}_{t=1}^{n}, i = 1, 2, ..., m$;

2) Formula (1) and Formula (2)is used to calculate relative distance precision $a_{it}$ of each single prediction methods at all time points, $i = 1, 2, ..., m; t = 1, 2, ..., n$;

3) Regard relative distance precision of each single prediction methods at all time points as the induced value to build a binary array $(\langle a_{1t}, \hat{x}_{1t} \rangle, \langle a_{2t}, \hat{x}_{2t} \rangle, ..., \langle a_{mt}, \hat{x}_{mt} \rangle)$, formula (2) is used to calculate combination forecast based on possible point and IOWA operator;

4) Formula (3) is used to build combination forecast model and calculate the weight coefficient;

5) Use the formula $\hat{x}_i = \sum_{i=1}^{m} l_i \hat{x}_{a-index(it)}$ to calculate combination forecast.

## The example analysis

In this paper, we use the data in the literature [11] to explain the computational process of the proposed method and verify the effectiveness of this method, the prediction effect index system is error variance (SSE)、mean absolute error (AE)、average percentage error (APE) and the mean square error percentage (SPE),

$$SSE = \sum_{t=1}^{n} (x_t - \hat{x}_t)^2$$

$$AE = \sum_{t=1}^{n} |x_t - \hat{x}_t|$$

$$APE = \sum_{t=1}^{n} |(x_t - \hat{x}_t)/x_t|$$

$$SPE = \sqrt{\sum_{t=1}^{n} [(x_t - \hat{x}_t)/x_t]^2}$$

The steps of this method are as follows:

Step number one: the actual value sequence and the data of each single prediction methods are shown in table 1.

Table 1. Prediction Results of Three Kinds of Fuzzy Single Prediction Models

| Time t | Actual value | Single method 1 | Single method 2 | Single method 3 |
|---|---|---|---|---|
| 1 | 72.5 | 75.7532 | 72.3747 | 72.5 |
| 2 | 74.3 | 72.6894 | 76.211 | 74.3 |
| 3 | 78.5 | 78.6441 | 80.0473 | 77.4076 |
| 4 | 83.8 | 81.9153 | 83,8836 | 81.0646 |
| 5 | 87.6 | 89.2122 | 87.7199 | 86.2824 |
| 6 | 93.1 | 91.5578 | 91.5562 | 90.7392 |
| 7 | 95.5 | 95.7235 | 95.3925 | 95.9793 |
| 8 | 102.7 | 104.171 | 99.2288 | 99.4672 |
| 9 | 104.3 | 105.6347 | 103.0651 | 105.5219 |
| 10 | 109.2 | 105.2278 | 106.9014 | 108.9521 |
| 11 | 109.4 | 111.8216 | 110.7377 | 113.0623 |
| 12 | 113.3 | 112.4537 | 114.574 | 114.7418 |
| 13 | 115.9 | 115.6816 | 118.4103 | 117.6775 |

Step number two: calculate relative distance precision of each single prediction methods at all time points, the results are shown in table 2:

Table 2. Relative Prediction Accuracy of Three Kinds of Single Prediction

| Time t | 1 | 2 | 3 | 4 | … | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|
| Method 1 | 0.0015 | 0.5847 | 0.9942 | 0.6783 | … | 0.7216 | 0.8379 | 0.9950 |
| Method 2 | 0.9985 | 0.4153 | 0.3365 | 0.9994 | … | 0.9151 | 0.6326 | 0.3373 |
| Method 3 | 1.0000 | 1.0000 | 0.6693 | 0.3223 | … | 0.3633 | 0.5295 | 0.6677 |

Step number three: according to the relative prediction accuracy of each single prediction method at all time points to calculate combination forecast:

$$IOWA_L\left(\langle 0.0015, 75.7532\rangle, \langle 0.9985, 72.2747\rangle, \langle 1, 72.5\rangle\right)$$
$$= l_1 72.5 + l_2 72.2747 + l_3 75.7532$$

$$IOWA_L\left(\langle 0.5847, 72.6894\rangle, \langle 0.4135, 72.211\rangle, \langle 1, 74.3\rangle\right)$$
$$= l_1 74.3 + l_2 72.6894 + l_3 72.211$$

… … …

$$IOWA_L\left(\langle 0.9950, 115.6816\rangle, \langle 0.3373, 118.4103\rangle, \langle 0.6677, 117.6775\rangle\right)$$
$$= l_1 115.6816 + l_2 117.6775 + l_3 118.4103$$

Step number four: formula (3) is used to build combination forecast model and calculate the weight coefficient,

$$l_1 = 0.8768, \quad l_2 = 0.1232, \quad l_3 = 0$$

Step number five: use the forma $\hat{x}_i = \sum_{i=1}^{m} l_i \hat{x}_{a-index(it)}$ to calculate combination forecast.

In order to verify the effectiveness of the combined forecasting method proposed in this paper, table 3 shows the predicted values of the proposed method, table 4 gives the method proposed in this paper and the prediction results of each single prediction method.

Table 3. Predictive Value of This Paper

| Actual value | Single method1 | Single method 2 | Single method 3 | Single method in this paper |
|---|---|---|---|---|
| 72.5 | 75.7532 | 72.3747 | 72.5 | 72.4846 |
| 74.3 | 72.6894 | 76.211 | 74.3 | 74.1016 |
| 78.5 | 78.6441 | 80.0473 | 77.4076 | 78.4918 |
| 83.8 | 81.9153 | 83,8836 | 81.0646 | 83.0858 |
| 87.6 | 89.2122 | 87.7199 | 86.2824 | 87.5428 |
| 93.1 | 91.5578 | 91.5562 | 90.7392 | 91.5576 |
| 95.5 | 95.7235 | 95.3925 | 95.9793 | 95.4333 |
| 102.7 | 104.171 | 99.2288 | 99.4672 | 103.5915 |
| 104.3 | 105.6347 | 103.0651 | 105.5219 | 105.2192 |
| 109.2 | 105.2278 | 106.9014 | 108.9521 | 108.6995 |
| 109.4 | 111.8216 | 110.7377 | 113.0623 | 110.8712 |
| 113.3 | 112.4537 | 114.574 | 114.7418 | 112.7149 |
| 115.9 | 115.6816 | 118.4103 | 117.6775 | 115.9275 |

Table 4. The Predicted Value of Each Method

| | SSE | AE | APE | MSPE |
|---|---|---|---|---|
| Method 1 | 48.1294 | 20.5347 | 0.2232 | 0.59 |
| Method 2 | 37.0499 | 17.5651 | 0.1791 | 0.38 |
| Method 3 | 46.8712 | 19.5697 | 0.1990 | 0.48 |
| Method in this paper | 7.3344 | 6.9977 | 0.0704 | 0.01 |

As can be seen from table 3, in this paper, the combination method based on relative distance precision and IOWA operator is much better than the single prediction method, so the method in this paper is effective and the method proposed in this paper is easier to compute. The weight coefficient $l_3 = 0$ does not mean that the third methods are invalid, because the IOWA operator is the combination of the predicted value of the sort of determined value and independent of the position of the single prediction value.

## Conclusion

In this paper, the concept of relative distance precision is proposed and as the induced value of IOWA operator, establishing a combined forecasting model which takes the sum of error square sum as the minimum. The effectiveness of the model is verified by a numerical example in the paper. By predicting coherence, we can predict relative prediction accuracy of each individual fuzzy prediction in the next n+T years, which is consistent with the average relative prediction accuracy of the past T years. Meanwhile, the idea of this paper can be extended to other combinatorial forecasting models.

## References:

[1] J.M.Bates,C.W.Granger,The combination of forecasts, Operational Research Quarterly, 1969(20):451-468.

[2] Zhi Xiao. Ke Gong. Yan Zou. A combined forecasting approach based on fuzzy soft sets. Journal of Computational and Applied Mathematics, 2009(1):326-333.

[3] Chonghui Wei, FuluJin, Yaqun He. Prediction method of air materiel consumption based on rough set and neural network[J]. Journal of Southeast University (NATURAL SCIENCE EDITION).2004(S1):68-70.

[4] H.W. Song, R.F. Zhang, Y.L. Zhang, F. Xia, Q.Z. Miao, Energy consumption combination forecast of Hebei Province based on the IOWA operator, Energy Procedia, 5(2011): 2224-2229.

[5] Yucheng Wan, Yaqun He, Zhaohan Sheng. Research on generalized weighted function based on the combination of grey system and neural network[J].Systems engineering theory and Practice, 2003(7):80-87.

[6] Huayou Chen, Chunlin Liu. Combination forecasting method based on IOWA operator[J]. Forecast. 2003(6):61-65.

[7] Qiming Chen, Huayou Chen. Optimal combination forecasting model based on IOWA operator and its application under two kinds of criteria[J].Mathematical statistics and management. 2013(5):847-853.

[8] Feifei Jin, Huayou Chen, Ligang Zhou. The best combination forecasting model of IOWA operator based on the maximum-minimum closeness degree[J]. Practice and understanding of Mathematics. 2013(7):110-116.

[9] Xiao Wang, Xi Liu, Huayou Chen, Lihui Jiang. Interval combination forecasting method based on IOWA operator[J]. Journal of Wuhan University of Technology (information and Management Engineering). 2010(2):221-225.

[10] Huayou Chen, Xiang Li, Lei Yu, Mengjie Yao. Interval combination forecasting method based on correlation coefficient and IOWA operator[J].Statistics and Decision. 2012(6):83-86.

[11] HongjieQiu, YinshanGu, Cui Li. Combined forecasting of fuzzy prediction[J]. Statistics and Decision. 2009(17):155-157.

[12] Zeshui Xu. Uncertain multiple attribute decision making method and its application[M]. Tsinghua University Press. 2004.

[13] Yulan Wang, Huayou Chen. Method and application of forecasting and decision making based on integrated operator[M]. Anhui University Press. 2014.

[14] Huayou Chen. Theory and application of combination forecasting method[M]. Science Press. 2008.

[15] R.R. Yager,Generalized OWA aggregation operators, Fuzzy Optimization and Decision Making. 2004(2):93-107.

[16] R.R. Yager, Induced aggregation operators, Fuzzy Sets and Systems,137(2003):59-69.