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# On Generalized ( $\theta$, $\theta$)-3-Derivations in Prime Near-Rings

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## Abstract

Let N be a near-ring and $\theta$ is a mapping on N. In this paper we introduce the notion of generalized ($\theta,\theta$)-3-derivation in near-ring N. Also we investigate the commutativity of addition of near-rings satisfying certain identities involving generalized ($\theta,\theta$)-3-derivation on prime near-rings.

**Keywords:** Near-ring; prime near-ring; ($\theta,\theta$)-3-derivation; generalized ($\theta,\theta$)-3-derivation.

## 1. Introduction

Let N be a near-ring and $\theta$ is a mapping on N. This paper consists of two sections. In section one ,we recall some basic definitions and other concepts, which be used in our paper, we explain these concepts by examples and remarks. In section two, we define the concepts of generalized ($\theta,\theta$)-3-derivation in near-ring N and we explore the commutativity of addition and ring behavior of prime near-rings satisfying certain conditions involving generalized ($\theta,\theta$)-3-derivations .

## 2.BASIC CONCEPTS

**Definition 2.1:[1]** A right near-ring (resp. a left near-ring ) is a nonempty set N equipped with two binary operations + and . such that

(i) (N, +) is a group ( not necessarily abelian )

(ii) (N, .) is a semigroup .

(iii) For all x,y,z $\in$N , we have

$$(x+y)z = xz + yz \text{ ( resp. } z(x+y) = zx + zy \text{ )}$$

**Example 2.2:[1]** Let G be a group ( not necessarily abelian ) then all mapping of G into itself form a right near-ring M(G) with regard to point wise addition and multiplication by composite .

**Lemma 2.3:[1]** Let N be left ( resp. right ) near-ring , then

(i)$x0 = 0$ ( resp. $0x = 0$ ) for all $x \in N$ .

(ii) $-(xy) = x(-y)$ ( resp. $-(xy) = (-x)y$ ) for all $x,y \in N$ .

**Definition 2.4:[2]** A right near-ring (resp. left near-ring ) is called zero symmetric right near-ring ( resp. zero symmetric left near-ring ) if $x0 = 0$ ( resp. $0x = 0$ ) , for all $x \in N$ .

**Definition 2.5:[2]** Let $\{N_i\}$ be a family of near-rings ($i \in I$ , I is an indexing set ) . $N = N_1 \times N_2 \times \ldots \times N_n$ with regard to component wise addition and multiplication , N is called the direct product of near-rings $N_i$ .

**Definition 2.6:[2]** A near-ring N is called a prime near-ring if $aNb = \{0\}$, where $a,b \in N$, implies that either $a = 0$ or $b = 0$ .

**Definition 2.7:[3]** Let N be a near-ring . The symbol Z will denote the multiplicative center of N , that is $Z = \{x \in N / xy = yx \text{ for all } y \in N\}$ .

**Definition 2.8:[3]** Let N be a near-ring . For any $x,y \in N$ the symbol $(x,y)$ will denote the additive commutator $x + y - x - y$ .

**Definition 2.9:[3]** Let N be a near-ring . For any $x,y \in N$ the symbol $[x,y] = xy - yx$ stands for multiplicative commutator of x and y .

**Properties 2.10:[3]** Let R be a ring , then for all $x,y,z \in R$, we have :

1-$[x,yz] = y[x,z] + [x,y]z$

2-$[xy,z] = x[y,z] + [x,z]y$

3-$[x+y,z] = [x,z] + [y,z]$

4-$[x,y+z] = [x,y] + [x,z]$

**Definition 2.11:[4]** Let N be a near-ring .An 3-additive mapping $d: N \times N \times N \rightarrow N$ is said to be 3-derivation if the relations

$$d(x_1x_1',x_2,x_3) = d(x_1,x_2,x_3)\, x_1' + x_1\, d(x_1',x_2,x_3)$$

$$d(x_1,x_2x_2',x_3) = d(x_1,x_2,x_3)\, x_2' + x_2\, d(x_1,x_2',x_3)$$

$$d(x_1,x_2,x_3x_3') = d(x_1,x_2,x_3)\, x_3' + x_3\, d(x_1,x_2,x_3')$$

hold for all $x_1, x_1', x_2, x_2', x_3, x_3' \in N$ .

**Definition 2.12:[4]** Let N be a near-ring and n be a fixed positive integer . An n- additive mapping d : N x N x….x N $\rightarrow$ N is called $(\sigma,\tau)$- n-derivation of N if there exist functions $\sigma,\tau$ : N$\rightarrow$ N such that the relations

$d(x_1x_1^{/},x_2,\ldots,x_n) = d(x_1,x_2,\ldots,x_n)\,\sigma(x_1^{/}) + \tau(x_1)\,d(x_1^{/},x_2,\ldots,x_n)$

$d(x_1,x_2x_2^{/},\ldots,x_n) = d(x_1,x_2,\ldots,x_n)\,\sigma(x_2^{/}) + \tau(x_2)\,d(x_1,x_2^{/},\ldots,x_n)$

…..

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$d(x_1,x_2,\ldots,x_nx_n^{/}) = d(x_1,x_2,\ldots,x_n)\,\sigma(x_n^{/}) + \tau(x_n)\,d(x_1,x_2,\ldots,x_n^{/})$

hold for all $x_1, x_1^{/}, x_2, x_2^{/}, \ldots, x_n, x_n^{/} \in N$ .

**Definition 2.13:[4]** Let N be a near-ring and n be a fixed positive integer . Let d : N x N x….x N $\rightarrow$ N be a $(\sigma,\tau)$- n-derivation of N . An n- additive mapping f : N x N x….x N $\rightarrow$ N is called a right generalized $(\sigma,\tau)$- n-derivation associated with $(\sigma,\tau)$- n-derivation d if the relations

$f(x_1x_1^{/},x_2,\ldots,x_n) = f(x_1,x_2,\ldots,x_n)\,\sigma(x_1^{/}) + \tau(x_1)\,d(x_1^{/},x_2,\ldots,x_n)$

$f(x_1,x_2x_2^{/},\ldots,x_n) = f(x_1,x_2,\ldots,x_n)\,\sigma(x_2^{/}) + \tau(x_2)\,d(x_1,x_2^{/},\ldots,x_n)$

…..

…..

$f(x_1,x_2,\ldots,x_nx_n^{/}) = f(x_1,x_2,\ldots,x_n)\,\sigma(x_n^{/}) + \tau(x_n)\,d(x_1,x_2,\ldots,x_n^{/})$

hold for all $x_1, x_1^{/}, x_2, x_2^{/}, \ldots, x_n, x_n^{/} \in N$ .

An n- additive mapping f : N x N x….x N $\rightarrow$ N is called a left generalized $(\sigma,\tau)$- n-derivation associated with $(\sigma,\tau)$- n-derivation d if the relations

$f(x_1x_1^{/},x_2,\ldots,x_n) = d(x_1,x_2,\ldots,x_n)\,\sigma(x_1^{/}) + \tau(x_1)\,f(x_1^{/},x_2,\ldots,x_n)$

$f(x_1,x_2x_2^{/},\ldots,x_n) = d(x_1,x_2,\ldots,x_n)\,\sigma(x_2^{/}) + \tau(x_2)\,f(x_1,x_2^{/},\ldots,x_n)$

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$f(x_1,x_2,\ldots,x_nx_n^{/}) = d(x_1,x_2,\ldots,x_n)\,\sigma(x_n^{/}) + \tau(x_n)\,f(x_1,x_2,\ldots,x_n^{/})$

hold for all $x_1, x_1^{/}, x_2, x_2^{/}, \ldots, x_n, x_n^{/} \in N$ .

Lastly an n- additive mapping f : N x N x….x N → N is called a generalized $(\sigma,\tau)$- n-derivation associated with $(\sigma,\tau)$- n-derivation d if it is both right

generalized $(\sigma,\tau)$- n-derivation as well as left generalized $(\sigma,\tau)$- n-derivation of N associated with $(\sigma,\tau)$- n-derivation d .

## 3.GENERALIZED $(\theta,\theta)$-3-DERIVATIONS

First we introduce the basic definition in this paper

**Definition 3.1:** Let N be a near-ring and $\theta$ is a mapping on N. Let d : N x N x N → N be a $(\theta,\theta)$-3-derivation of N . An 3 - additive mapping f : N x N x N → N is called a right generalized $(\theta,\theta)$-3-derivation associated with $(\theta,\theta)$-3-derivation d if the relations

$f(x_1x_1^{/},x_2,x_3) = f(x_1,x_2,x_3)\,\theta(x_1^{/}) + \theta(x_1)\,d(x_1^{/},x_2,x_3)$

$f(x_1,x_2x_2^{/},x_3) = f(x_1,x_2,x_3)\,\theta(x_2^{/}) + \theta(x_2)\,d(x_1,x_2^{/},x_3)$

$f(x_1,x_2,x_3x_3^{/}) = f(x_1,x_2,x_3)\,\theta(x_3^{/}) + \theta(x_3)\,d(x_1,x_2,x_3^{/})$

hold for all $x_1, x_1^{/}, x_2, x_2^{/}, x_3, x_3^{/} \in N$ .

An 3- additive mapping f : N x N x N → N is called a left generalized $(\theta,\theta)$-3- derivation associated with $(\theta,\theta)$-3-derivation d if the relations

$f(x_1x_1^{/},x_2,x_3) = d(x_1,x_2,x_3)\,\theta(x_1^{/}) + \theta(x_1)\,f(x_1^{/},x_2,x_3)$

$f(x_1,x_2x_2^{/},x_3) = d(x_1,x_2,x_3)\,\theta(x_2^{/}) + \theta(x_2)\,f(x_1,x_2^{/},x_3)$

$f(x_1,x_2,x_3x_3^{/}) = d(x_1,x_2,x_3)\,\theta(x_3^{/}) + \theta(x_3)\,f(x_1,x_2,x_3^{/})$

hold for all $x_1, x_1^{/}, x_2, x_2^{/}, x_3, x_3^{/} \in N$ .

Lastly an 3-additive mapping f : N x N x N → N is called a generalized $(\theta,\theta)$-3- derivation associated with $(\theta,\theta)$- 3-derivation d if it is both right generalized $(\theta,\theta)$- 3-derivation as well as left generalized $(\theta,\theta)$- 3-derivation of N associated with $(\theta,\theta)$- 3-derivation d .

We now explain this definition by the following example

**Example 3.2 :** Let R be a commutative ring and S be zero symmetric left near-ring which is not a ring such that (S,+) is abelian , it can be easily verified that the set M = R x S is a zero symmetric left near-ring with respect to component wise addition and multiplication . Now suppose that

$$N = \left\{ \begin{pmatrix} (0,0) & (x,x^{/}) & (y,y^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z,z^{/}) \end{pmatrix}, \; (x,x^{/}) \;,\; (y,y^{/}), (z,z^{/}), (0,0) \in M \right\}.$$

It can be easily seen that N is a non commutative zero symmetric left near-ring with respect to matrix addition and matrix multiplication .

Define d , f : N x N x N $\rightarrow$ N and $\theta$ : N $\rightarrow$ N such that

$$d\left( \begin{pmatrix} (0,0) & (x_1,x_1{}^{/}) & (y_1,y_1{}^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z_1,z_1{}^{/}) \end{pmatrix}, \begin{pmatrix} (0,0) & (x_2,x_2{}^{/}) & (y_2,y_2{}^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z_2,z_2{}^{/}) \end{pmatrix}, \begin{pmatrix} (0,0) & (x_3,x_3{}^{/}) & (y_3,y_3{}^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z_3,z_3{}^{/}) \end{pmatrix} \right)$$
$$= \begin{pmatrix} (0,0) & (x_1 x_2 x_3, 0) & (0,0) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) \end{pmatrix}$$

$$f\left( \begin{pmatrix} (0,0) & (x_1,x_1{}^{/}) & (y_1,y_1{}^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z_1,z_1{}^{/}) \end{pmatrix}, \begin{pmatrix} (0,0) & (x_2,x_2{}^{/}) & (y_2,y_2{}^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z_2,z_2{}^{/}) \end{pmatrix}, \begin{pmatrix} (0,0) & (x_3,x_3{}^{/}) & (y_3,y_3{}^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z_3,z_3{}^{/}) \end{pmatrix} \right) =$$
$$\begin{pmatrix} (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) \end{pmatrix}$$

And

$$\theta \begin{pmatrix} (0,0) & (x,x^{/}) & (y,y^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z,z^{/}) \end{pmatrix} = \begin{pmatrix} (0,0) & (x,x^{/}) & (y,y^{/}) \\ (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (z,z^{/}) \end{pmatrix}$$

It can be easily seen that d is $(\theta, \theta)$- 3-derivation of N and f is a nonzero generalized $(\theta, \theta)$- 3-derivation associated with d ,where $\theta$ is an automorphism on N .

The following lemmas help us to prove the main theorems :

**Lemma 3.3 :** Let N be a prime near-ring , d a nonzero $(\theta, \theta)$- 3-derivation of N and x$\in N$ , where $\theta$ is an automorphism on N .

(i)If d(N,N,N) x = {0} , then x = 0 .

(ii)If x d(N,N,N) = {0} , then x = 0 .

**Proof :** (i) By our hypothesis d(N,N,N) x = {0}

i.e.: $d(x_1,x_2,x_3)\, x = 0$ , for all $x_1 , x_2 , x_3 \in N$ (3.1)

Putting $x_1x_1{}^{/}$ in place of $x_1$ , where $x_1{}^{/} \in N$, in (3.1) and using [2 ,Lemma 4 ] we get

$d(x_1,x_2,x_3)\,\theta(x_1^{/})\,x + \theta(x_1)\,d(x_1^{/},x_2,x_3)\,x = 0$ , for all $x_1, x_1^{/}, x_2, x_3 \in N$ .

Using (3.1) in previous equation , we get

$d(x_1,x_2,x_3)\,\theta(x_1^{/})\,x = 0$ , for all $x_1, x_1^{/}, x_2, x_3 \in N$ .

Since $\theta$ is an automorphism , then we have

$d(x_1,x_2,x_3)\,N\,x = \{0\}$ , for all $x_1, x_2, x_3 \in N$ . Since $d \neq 0$ , primeness of N implies that $x = 0$ .

(ii) It can be proved in a similar way .

**Lemma 3.4 :** Let N be a near-ring admitting a generalized $(\theta,\theta)$-3- derivation f associated with $(\theta,\theta)$- 3-derivation d of N , where $\theta$ is an automorphism on N, then

$(d(x_1,x_2,x_3)\,\theta(x_1^{/}) + \theta(x_1)\,f(x_1^{/},x_2,x_3))\,y = d(x_1,x_2,x_3)\,\theta(x_1^{/})y + \theta(x_1)\,f(x_1^{/},x_2,x_3)y$

$(d(x_1,x_2,x_3)\,\theta(x_2^{/}) + \theta(x_2)\,f(x_1,x_2^{/},x_3))y = d(x_1,x_2,x_3)\,\theta(x_2^{/})\,y + \theta(x_2)\,f(x_1,x_2^{/},x_3)y$

$(d(x_1,x_2,x_3)\,\theta(x_3^{/}) + \theta(x_3)\,f(x_1,x_2,x_3^{/}))y = d(x_1,x_2,x_3)\,\theta(x_3^{/})y + \theta(x_3)\,f(x_1,x_2,x_3^{/})y$

hold for all $x_1, x_1^{/}, x_2, x_2^{/}, x_3, x_3^{/} \in N$ .

**Proof :** For all $x_1, x_1^{/}, x_1^{//}, x_2, x_3 \in N$ , we have

$f((x_1x_1^{/})x_1^{//},x_2,x_3) = f(x_1x_1^{/},x_2,x_3)\,\theta(x_1^{//}) + \theta(x_1x_1^{/})\,d(x_1^{//},x_2,x_3)$

Therefore

$$f((x_1x_1^{/})x_1^{//},x_2,x_3) = (d(x_1,x_2,x_3)\,\theta(x_1^{/}) + \theta(x_1)\,f(x_1^{/},x_2,x_3))\,\theta(x_1^{//}) + \theta(x_1)\,\theta(x_1^{/})\,d(x_1^{//},x_2,x_3) \tag{3.2}$$

Also

$f(x_1(x_1^{/}x_1^{//}),x_2,x_3) = d(x_1,x_2,x_3)\,\theta(x_1^{/}x_1^{//}) + \theta(x_1)\,f(x_1^{/}x_1^{//},x_2,x_3)$

Thus , we get

$$f(x_1(x_1^{/}x_1^{//}),x_2,x_3) = d(x_1,x_2,x_3)\,\theta(x_1^{/})\,\theta(x_1^{//}) + \theta(x_1)\,f(x_1^{/},x_2,x_3)\,\theta(x_1^{//}) + \theta(x_1)\,\theta(x_1^{/})\,d(x_1^{//},x_2,x_3) \tag{3.3}$$

Combining relations (3.2) and (3.3) , we get

$(d(x_1,x_2,x_3)\,\theta(x_1^{/}) + \theta(x_1)\,f(x_1^{/},x_2,x_3))\,\theta(x_1^{//}) = d(x_1,x_2,x_3)\,\theta(x_1^{/})\,\theta(x_1^{//}) + \theta(x_1)\,f(x_1^{/},x_2,x_3)\,\theta(x_1^{//})$ , for all $x_1, x_1^{/}, x_1^{//}, x_2, x_3 \in N$ .

Since $\theta$ is an automorphism , putting y in place of $\theta(x_1^{//})$ , we find that

$(d(x_1,x_2,x_3)\,\theta(x_1^{/}) + \theta(x_1)\, f(x_1^{/},x_2,x_3)\,)\, y = d(x_1,x_2,x_3)\,\theta(x_1^{/})\, y +$

$\theta(x_1)\, f(x_1^{/},x_2,x_3)\, y$ , for all $x_1 , x_1^{/} , x_2 , x_3 , y \in N$ .

Similarly other ( n -1 ) relations can be proved .

**Lemma 3.5 :** Let N be a prime near-ring admitting a generalized $(\theta,\theta)$-3- derivation f associated with a nonzero $(\theta , \theta)$- 3-derivation d of N and $x \in N$ , where $\theta$ is an automorphism on N .

(i)If f( N , N , N ) x = {0} , then x = 0 .

(ii)If x f( N , N , N ) = {0} , then x = 0 .

**Proof :** (i) By our hypothesis we have

$$f(x_1,x_2,x_3)\, x = 0 \text{ , for all } x_1 , x_2 , x_3 \in N \qquad (3.4)$$

Putting $x_1x_1^{/}$ in place of $x_1$ , where $x_1^{/} \in N$, in (3.4) and using lemma 3.4 we get

$d(x_1,x_2,x_3)\,\theta(x_1^{/})\, x + \theta(x_1)\, f(x_1^{/},x_2,x_3)\, x = 0$, for all $x_1 , x_1^{/} , x_2 , x_3 \in N$ .

Using (3.4) in previous equation , we get

$d(x_1,x_2,x_3)\,\theta(x_1^{/})\, x = 0$ , for all $x_1 , x_1^{/} , x_2 , x_3 \in N$ .

Since $\theta$ is an automorphism, then we have

$d(x_1,x_2,x_3)\, N\, x = \{0\}$ , for all $x_1 , x_2 , x_3 \in N$ . Since $d \neq 0$ , primeness of N implies that x = 0 .

(ii) It can be proved in a similar way.

Now , we will prove the main theorems :

**Theorem 3.6 :** Let N be a prime near-ring and $f_1$ , $f_2$ be any two generalized $(\theta,\theta)$-3-derivations of N with associated nonzero $(\theta , \theta)$- 3-derivations $d_1$ and $d_2$ respectively , where $\theta$ is an automorphism on N . If $[ f_1 (N , N , N) , f_2 (N , N , N)] = \{0\}$ , then (N , +) is abelian.

**Proof :** Assume that $[ f_1 (N , N , N) , f_2 (N , N , N)] = \{0\}$ . If both z and z + z commute element wise with $f_2 (N , N , N)$ , then for all $x_1 , x_2 , x_3 \in N$ we have

$$z\, f_2(x_1,x_2,x_3) = f_2(x_1,x_2,x_3)\, z \qquad (3.5)$$

And

$$(z + z)\, f_2(x_1,x_2,x_3) = f_2(x_1,x_2,x_3)\,(z + z) \qquad (3.6)$$

Substituting $x_1 + x_1^/$ instead of $x_1$ in (3.6) we get

$(z + z)\, f_2(x_1+ x_1^/, x_2, x_3) = f_2(x_1+ x_1^/, x_2, x_3)\,(z + z)$ for all $x_1, x_1^/, x_2, x_3 \in N$ .

From (3.5) and (3.6) the previous equation can be reduced to

$z\, f_2(x_1+ x_1^/ - x_1 - x_1^/, x_2, x_3) = 0$ for all $x_1, x_1^/, x_2, x_3 \in N$.

i.e.; $z\, f_2( (x_1, x_1^/), x_2, x_3) = 0$ for all $x_1, x_1^/, x_2, x_3 \in N$ .putting $z = f_1(y_1, y_2, y_3)$, we get $f_1(y_1, y_2, y_3)\, f_2( (x_1, x_1^/), x_2, x_3) = 0$ for all $x_1, x_1^/, x_2, x_3, y_1, y_2, y_3 \in N$ . By Lemma 3.5 (i) we conclude that $f_2( (x_1, x_1^/), x_2, x_3) = 0$

for all $x_1, x_1^/, x_2, x_3 \in N$. (3.7)

Since we know that for each $w \in N$ , $w(x_1, x_1^/) = w (x_1+ x_1^/ - x_1 - x_1^/) = w x_1 + w x_1^/ - w x_1 - w x_1^/ = ( w x_1 , w x_1^/ )$ which is again an additive commutator of a near-ring N , putting $w(x_1, x_1^/)$ in place of additive commutator $(x_1, x_1^/)$ in (3.7) we get

$f_2(w (x_1, x_1^/), x_2, x_3) = 0$ for all $x_1, x_1^/, x_2, x_3, w \in N$.

i.e.; $d_2 (w, x_2, x_3)\, \theta(x_1, x_1^/) + \theta(w)\, f_2( (x_1, x_1^/), x_2, x_3) = 0$ for all $x_1, x_1^/, x_2, x_3, w \in N$. Using (3.7) in previous equation yields $d_2 (w, x_2, x_3)\, \theta(x_1, x_1^/) = 0$ for all $x_1, x_1^/, x_2, x_3, w \in N$ . Since $\theta$ is an automorphism , using Lemma 3.3 we conclude that $(x_1, x_1^/) = 0$ . Hence $(N ,+)$ is abelian .

**Theorem 3.7 :** Let N be a prime near-ring and $f_1$ , $f_2$ be any two generalized $(\theta,\theta)$-3-derivations of N with associated nonzero $(\theta, \theta)$- 3-derivations $d_1$ and $d_2$ respectively , where $\theta$ is an automorphism on N . If $f_1(x_1, x_2, x_3)\, f_2 (y_1, y_2, y_3) + f_2(x_1, x_2, x_3)\, f_1(y_1, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_2, y_3 \in N$ , then $( N , + )$ is abelian.

**Proof :** By hypothesis we have ,

$f_1(x_1, x_2, x_3)\, f_2 (y_1, y_2, y_3) + f_2(x_1, x_2, x_3)\, f_1(y_1, y_2, y_3) = 0$
for all $x_1, x_2, x_3, y_1, y_2, y_3 \in N$ . (3.8)

Substituting $y_1 + y_1^/$ instead of $y_1$ in (3.8) we get

$f_1(x_1, x_2, x_3)\, f_2 (y_1+ y_1^/, y_2, y_3) + f_2(x_1, x_2, x_3)\, f_1(y_1+ y_1^/, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^/, y_2, y_3 \in N$ .

So we get

$f_1(x_1, x_2, x_3)\, f_2 (y_1, y_2, y_3) + f_1(x_1, x_2, x_3)\, f_2 ( y_1^/, y_2, y_3) + f_2(x_1, x_2, x_3)\, f_1(y_1, y_2, y_3) +$
$f_2(x_1, x_2, x_3)\, f_1( y_1^/, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^/, y_2, y_3 \in N$ .

Using (3.8) again in last equation we get

$f_1(x_1, x_2, x_3) f_2(y_1, y_2, y_3) + f_1(x_1, x_2, x_3) f_2(y_1^{/}, y_2, y_3) + f_1(x_1, x_2, x_3) f_2(-y_1, y_2, y_3) + f_1(x_1, x_2, x_3) f_2(-y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$ .

Thus , we get

$f_1(x_1, x_2, x_3) f_2((y_1, y_1^{/}), y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$ .

Now using Lemma 3.5 (i) we conclude that $f_2((y_1, y_1^{/}), y_2, y_3) = 0$ for all $y_1, y_1^{/}, y_2, y_3 \in N$ .

Putting $w(y_1, y_1^{/})$ in place of $(y_1, y_1^{/})$ , where $w \in N$ , in the previous equation and using it again we get $d_2(w, y_2, y_3)\theta(y_1, y_1^{/}) = 0$ for all $y_1, y_1^{/}, y_2, y_3, w \in N$ . Since $\theta$ is an automorphism , using Lemma 3.3 we conclude that $(y_1, y_1^{/}) = 0$ . Hence $(N,+)$ is abelian.

**Theorem 3.8 :** Let N be a prime near-ring , $f_1$ and $f_2$ be any two generalized $(\theta,\theta)$-3-derivations of N with associated nonzero $(\theta,\theta)$- 3-derivations $d_1$ and $d_2$ respectively , where $\theta$ is an automorphism on N, such that

$f_1(x_1, x_2, x_3)\theta f_2(y_1, y_2, y_3) + \theta f_2(x_1, x_2, x_3) f_1(y_1, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_2, y_3 \in N$ , then $(N,+)$ is abelian .

**Proof :** By hypothesis we have ,

$$f_1(x_1, x_2, x_3)\theta f_2(y_1, y_2, y_3) + \theta f_2(x_1, x_2, x_3) f_1(y_1, y_2, y_3) = 0$$
for all $x_1, x_2, x_3, y_1, y_2, y_3 \in N$ . (3.9)

Substituting $y_1 + y_1^{/}$ , where $y_1^{/} \in N$ , for $y_1$ in (3.9) we get

$f_1(x_1, x_2, x_3)\theta f_2(y_1 + y_1^{/}, y_2, y_3) + \theta f_2(x_1, x_2, x_3) f_1(y_1 + y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$ .

Thus , we get

$f_1(x_1, x_2, x_3)\theta f_2(y_1, y_2, y_3) + f_1(x_1, x_2, x_3)\theta f_2(y_1^{/}, y_2, y_3) + \theta f_2(x_1, x_2, x_3)$

$f_1(y_1, y_2, y_3) + \theta f_2(x_1, x_2, x_3) f_1(y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$ .

Using (3.9) in previous equation implies

$f_1(x_1, x_2, x_3)\theta f_2(y_1, y_2, y_3) + f_1(x_1, x_2, x_3)\theta f_2(y_1^{/}, y_2, y_3) + f_1(x_1, x_2, x_3)\theta f_2(-y_1, y_2, y_3) + f_1(x_1, x_2, x_3)\theta f_2(-y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$ .

Therefore

$f_1(x_1, x_2, x_3)\theta f_2((y_1, y_1^{/}), y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$ .

Now using Lemma 3.5 , in previous equation , we conclude that

$\theta f_2((y_1, y_1^{/}), y_2, y_3) = 0$ for all $y_1, y_1^{/}, y_2, y_3 \in N$ .

Since $\theta$ is an automorphism of N , we conclude that $f_2((y_1, y_1^{/}), y_2, y_3) = 0$

For all $y_1, y_1^{/}, y_2, y_3 \in N$. Putting $w(y_1, y_1^{/})$ in place of $(y_1, y_1^{/})$ , where $w \in N$, in the previous equation and using it again we get $d_2(w, y_2, y_3)\, \theta(y_1, y_1^{/}) = 0$ for all $y_1, y_1^{/}, y_2, y_3$ , $w \in N$ . Since $\theta$ is an automorphism, using Lemma 3.3 we conclude that $(y_1, y_1^{/}) = 0$. Hence (N ,+) is abelian.

## Conclusion

In present paper we define the notion of generalized $(\theta, \theta)$-3- derivations in near-rings . Also we study and discuss the commutativity of addition of prime near-ring with generalized $(\theta, \theta)$-3- derivations .

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