



Some New Results Of The integrals in the groups Mod-n

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Abstract.

Let $G = \mathbb{Z}_n$ be abelian group Mod n . we shall define a new integral of the all elements of the $G = \mathbb{Z}_n$ (n is a finite number , $n \in \mathbb{N}$, $n \geq 2$).

We shall study a new – results of the integrals properties in the group \mathbb{Z}_n . We gave the definitions of integrals in the \mathbb{Z}_n , and we shall gave the anew- definitions of the neat (semi) integrals, and the some a new results of this integrals.

Introduction

Let $G = \mathbb{Z}_n$ be abelian group Mod n , $n \geq 2$. We shall starts with the a new – definition of the integrals, by the following :

Definition A : The integrals in the group \mathbb{Z}_n define b

$$\forall n \geq 2, \int_0^k \bar{x} d\mathbb{Z}_n = \bar{x} \mathbb{Z}_n \int_0^{kr} = k \bar{x}$$
$$= \{0\bar{x}, 1\bar{x}, 2\bar{x}, 3\bar{x}, \dots, (n-1)\bar{x}\}$$

With $K = \{0, 1, 2, \dots, n-1\}$

Example 1 :Take $G = \mathbb{Z}_3$, So

$$\langle \bar{0} \rangle \int_0^k \bar{0} dz_3 = \int_0^2 \bar{0} dz_3 = \{0\bar{0}, 1\bar{0}, 2\bar{0}\} = \{\bar{0}\} =$$
$$\int_0^2 \bar{1} dz_3 = \{0\bar{1}, 1\bar{1}, 2\bar{1}\} = \mathbb{Z}_3$$
$$\int_0^2 \bar{2} dz_3 = \{0\bar{2}, 1\bar{2}, 2\bar{2}\} = \{0, \bar{2}, \bar{1}\} = \mathbb{Z}_3$$

$$\text{So } \int_0^2 \bar{1} dz_3 = Z_3 = \int_0^2 \bar{2} dz_3$$

Clearly , $\int_0^k dz_n = Zn \quad \forall n \geq 2$

Example 2 .Take $G = Z_6$ It's easily to show that

$$\int_0^k \bar{0} dz_6 = \int_0^5 dz_6 = \langle \bar{0} \rangle$$

$$\int_0^5 \bar{1} dz_6 = Z_6$$

$$\begin{aligned} \int_0^5 \bar{2} dz_6 &= \{0\bar{2}, 1\bar{2}, 2\bar{2}, 3\bar{2}, 4\bar{2}, 5\bar{2}\} \\ &= \{\bar{0}, \bar{2}, \bar{4}, \bar{0}, \bar{2}, \bar{4}\} = \langle \bar{2} \rangle \\ &= \{\bar{3}\}, \end{aligned}$$

So

$$\int_0^5 \bar{3} dz_6 = \{0\bar{3}, 1\bar{3}, 2\bar{3}, 3\bar{3}, 4\bar{3}, 5\bar{3}\}$$

$$\int_0^5 \bar{5} dz_6 = Z_6$$

Here,

$$1 - \int_0^5 \bar{1} dz_6 = \int_0^5 \bar{5} dz_6 = Z_6$$

$$2 - \int_0^5 (\bar{1} +_6 \bar{2}) dz_6 = \int_0^5 \bar{3} dz_6 = \langle \bar{3} \rangle$$

$$\int_0^5 (\bar{1} +_6 \bar{2}) dz_6$$

Definition B :

The odd integrals, between two different Mod – group , define by

$$\int_0^{k1+k2} (Zn + Zm) dz_{n+m} = \int_0^{k1} Zn dz_{n+m} + \int_0^{k2} Zm dz_{n+m}$$

$$= Zn + Zm$$

Example 3 :

$$\int_0^{k1+k2} (Z_2 + Z_3) dz_{2+3}$$

$$= \int_0^{k1} Z_2 dz_2 + \int_0^{k2} Z_3 dz_3 =$$

$$= \int_0^1 Z_2 dz_2 + \int_0^2 Z_3 dz_3$$

$$= Z_2 + Z_3 = \{(\bar{0}, \bar{0}), (\bar{0}, \bar{1}), (\bar{0}, \bar{2}), (\bar{1}, \bar{0}), (\bar{1}, \bar{1}), (\bar{1}, \bar{2})\}$$

We are ready to give the new-definition to the neat – integrals subgroups –

Definition C : [2]

Let $G = Z_n$ be a group Mod – n and H be a subgroup to G.

Then the integrals of the elements of the H. Is said to be neat – integrals element, for all prime number P , for all $h \neq 0 \in H$

$$\text{If } \int_0^p x dZ_n = Px = h \text{ for some } Z_n \text{ then } \int_0^p ho dH = Pho = h \text{ for some } ho \in H .$$

We shall said a subgroup H of Z_n is a neat – integrals in Z_n if for all $h \in H$, h is neat – integral element , that mean H is neat – integrals in Z_n , For all $h \in H$, for all prime number P , $x \in Z_n$

$$\text{If } \int_0^p x dZ_n = h = \int_0^p ho dh \text{ for some } ho \in H .$$

Example 4 : Take $G = Z_6$ and $H = \langle \bar{3} \rangle = \{ \bar{0}, \bar{3} \}$ then we can show that the all element of H (We have only $\bar{3}$) is a neat – integrals

Take $p = 2$

$$\text{Clearly } \int_0^2 \bar{x} dz_6 = 2\bar{x} = \bar{3}$$

Clearly that, then is no solution in Z_6 , ($\bar{x} \notin z_6$)

Take $p = 3$

$$\text{So } \int_0^3 \bar{x} dz_6 = 3\bar{x} = \bar{3} \Rightarrow 3(\bar{1}) = \bar{3} \text{ in } Z_6$$

To show $\int_0^3 \bar{x} dz_6 = \bar{3}$ it has a solution in H.

$$\int_0^3 \bar{x} dz_6 = \bar{3} = \int_0^3 \bar{3} dh = 3\bar{3} = \bar{3} = \bar{3} \in H$$

Take p = 5

$$\text{Clearly } \int_0^5 \bar{x} dz_6 = 5\bar{x} = \bar{3} \in G$$

$$5(\bar{3}) = \bar{3} \in H.$$

$$\text{So } \int_0^5 \bar{x} dz_6 = \int_0^5 \bar{x} dh$$

Thus, $\forall p, p \geq 2, \forall h \in H$

$$\text{If } \int_0^p \bar{x} dZ_n = h \text{ in } G$$

$$\text{Then } \int_0^p \bar{x} dH = Ph = h \text{ in } H$$

Therefore, H is integral - neat. We shall denote H by P-integral neat subgroup of G

Example 5: Take $G = Z_{12}$ and $H = \langle \bar{2} \rangle$.
and take $P = 2$

$$\int_0^2 \bar{x} dZ_{12} = 2\bar{x} = \bar{2} \text{ in } G$$

$$\text{We have } 2(\bar{1}) = \bar{2} \text{ in } G$$

$$\text{But } \int_0^2 \bar{x} dz_{12} \neq \int_0^2 \bar{x} dH = \bar{2}$$

$$\text{Which means, That } \int_0^2 \bar{x} dz_{12} = \bar{2}$$

It has no solution in H

So H is not P-integral neat in G.

Definition D :

A subgroup H of the G is said to be Po – integral neat in G if , $\forall h \in H \int_0^{po} \bar{x} dZ_n = po \bar{x} = h$

$$\text{Then } \int_0^{po} \bar{x} dH = po \bar{x} = h = po ho \quad \text{for some } ho \in H.$$

Example 6 :Take $G = Z_8$ and $H = \langle \bar{2} \rangle$

If $P = 2$, $h = \bar{2} \in H$

It has solution in $Z_8 \int_0^2 \bar{x} dZ_n = 2 \bar{x} = \bar{2}$

But $\int_0^2 \bar{x} dZ_n = \bar{2}$ it has no solution in H .

So H is not P- integral neat in G .

Now , take $P = 3$, $h = \bar{2} \in H$

its has solution in $Z_8 (3\bar{6} = \bar{2}) \int_0^3 \bar{x} dZ_8 = 3\bar{x} = \bar{2}$

Clearly $\int_0^3 \bar{x} dZ_8 = \bar{2}$ it has solution in H .

$$\int_0^3 \bar{x} dZ_8 = \bar{2} = \int_0^3 \bar{x} dH = 3\bar{6} = \bar{2} \text{ and } ho = \bar{6} \in H$$

Now , test $\bar{4} \in H$

$$\int_0^3 \bar{x} dZ_8 = 3\bar{x} = \bar{4} \text{ in } G$$

$$3\bar{x} = 3(\bar{4}) = \bar{4} \text{ and } \bar{4} \in H$$

$$\text{So } \int_0^3 \bar{x} dZ_8 = \int_0^3 \bar{x} dH \quad \text{in } H$$

Test $\bar{6} \in H$

$$\begin{aligned} \int_0^3 \bar{x} dZ_8 &= 3\bar{x} = \bar{6} && \text{in } G \\ &= 3(\bar{2}) = \bar{6} && \bar{2} \in H \end{aligned}$$

Clearly $\int_0^3 \bar{x} dz_{z_8} = \bar{6} = \int_0^3 \bar{x} dH$ in H and His 3- neat integrals in G .We are ready to show some results of P- neat integrals .

TheoremA : For any P- neat integrals in abelian group G is a Po –neat in G

Theorem B : [1]

Let A and B be two p- neat integrals in G then

i) $A \cap B$ is a P- neat integrals in

ii) $A + B$ is a P – neat integrals in G

Proof :

Let h be any element in $A \cap B$ and for all prime number P

$$\text{Suppose } \int_0^P \bar{x} dG = P\bar{x} = h \quad \text{in } G$$

So there exist an element $g \in G$ such that

$$\int_0^P \bar{x} dG = P\bar{x} = pg = h \in A \cap B$$

Since $h \in A \cap B$ $h \in A$ and $h \in B$

$$\text{Thus, } \int_0^P \bar{x} dG = P\bar{x} = pg = h \in A \quad \text{in } G$$

But A is p- neat integral in G

$$\text{So } \int_0^P \bar{x} dG = \int_0^P \bar{x} dA = P\bar{x} = pa = h \quad \text{for some } a \in A$$

and B is p- neat integral in G

We have

$$\int_0^P \bar{x} dG = \int_0^P \bar{x} dB = P\bar{x} = pb = h \quad \text{in } B$$

Hence, $Pb = h = Pa$

So $P(a - b) = 0$ and thus $a = b \in A \cap B$

$$\text{Therefore } \int_0^P \bar{x} dG = \int_0^P \bar{x} d(A \cap B) = h \in A \cap B$$

We get $A \cap B$ is p- neat integral in G

ii) To prove , $A + B$ be a p- neat integral in G

Let Z be any element in $A + B$ and suppose that $\int_0^P \bar{x} dG = z$ in G

So $P\bar{x} = z$

Since $z \in A + B$, $z = a + b$ for some $a \in A$ $b \in B$

We have $P\bar{x} = a + b \in A + B$

$$\text{and } \int_0^p \overline{x_1} dGn = a \quad \text{and } \int_0^p \overline{x_2} dgn = b$$

But we have A and B are P-neat integrals in G

$$\text{So } \int_0^p \overline{x_1} dG = \int_0^p d_o dA \quad \text{for some } d_o \in A$$

$$\int_0^p \overline{x_2} dG = \int_0^p d_o dB \quad \text{for some } b_o \in b$$

Thus, $Pd_o = a$

$Pd_o = b$ and we get $P(d_o + b_o) = a + b$

$$\text{So, } P\overline{x} = a + b = pa_0 + pb_o = p(d_o + b_o)$$

$$\int_0^p \overline{x} dG = \int_0^p (d_o + b_o) = a + b$$

Which mean that

$$\text{It } \int_0^p \overline{x} dG = z \in A + B \quad \text{in } G$$

$$\text{Then } \int_0^p \overline{x} d(A + B) = z \in A + B \quad \text{in } G$$

Theorem 3: If A is only neat – integral subgroup of A subgroup B of G then

- i) A is a neat- integral of G
- ii) B is p-neat integral in G then B/A is a neat – integral of G/A

$$\text{Proof :/ and for all P it } \int_0^p \overline{x} dG = a \quad \text{in } G \quad \text{and for all P it } \int_0^p \overline{x} dG = a \quad \text{in } G$$

So $P\overline{x} = a \in A$.But A is a neat-integral of B so

$$\int_0^p \overline{x_1} dB = a \in A \subseteq B$$

$$\text{So } \int_0^p \overline{x_1} dA = a = pd_o = pd_o = \text{for some } d_o \in A.$$

$$\text{Hence, } \int_0^p \overline{x} dG = a = \int_0^p d_o dA$$

Therefore A is p-neat in G

ii) Let $b + A \in B/A$ and $\forall p (b \in B)$

$$\int_0^p \overline{x} dG/A = b + A, \quad \overline{x} \in G/A$$

$$\text{So } \int_0^p g dG/A = b + A \quad \text{in } G/A$$

$$P(g + A) = b + A \in G/A$$

$$\text{So } Pg + pA = b + A \Rightarrow Pg + A = b + A$$

$$\text{So } Pg = b \Rightarrow \int_0^p g dG = b \quad \text{in } G$$

But B is P- neat integral in G

$$\text{Thus, } \int_0^p g dG = b = \int_0^p b dB$$

$$P b_0 = b \quad \text{some } b_0 \in B$$

Since we have

$$\int_0^p g + A d G/A = P(g + A) = b + A$$

$$P(g + A) = p b_0 + A = p(b_0 + A) = b + A$$

$$\text{Thus, } \int_0^p g + A d G/A = \int_0^p b_0 + A d B/A$$

We get B/A is p- neat integral

References

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