



## On Soft slightly $\pi g$ -continuous functions

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### Abstract

In this paper, we introduce and study the concept of soft slightly  $\pi g$ -continuous functions which is weaker than soft  $\pi g$ -continuous functions and obtain its fundamental properties. The relationship between soft slightly  $\pi g$ -continuity and other related functions is also analyzed.

**Keywords:** Soft  $\pi g$ -closed set; soft  $\pi g$ -open set; soft clopen set; soft  $\pi g$ -Continuity; soft slightly continuity; soft slightly  $\pi g$ -continuity.

### 1. Introduction

Molodtsov [8] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Muhammad Shabir and Munazza Naz [10] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. Kharal et al. [5] introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [1] in 2013 studied and discussed the properties of Soft continuous mappings which are defined over an initial universe set with a fixed set of parameters. In this paper, soft slightly  $\pi g$ -continuity is introduced and studied. Moreover, basic properties for soft slightly  $\pi g$ -continuous functions are investigated and relationship between soft slightly  $\pi g$ -continuous functions and its graphs are studied.

### 2. Preliminaries

#### Definition: 2.1[8]

Let  $U$  be the initial universe and  $P(U)$  denote the power set of  $U$ . Let  $E$  denote the set of all parameters. Let  $A$  be a non-empty subset of  $E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ .

#### Definition: 2.2[6]

A subset  $(A, E)$  of a topological space  $X$  is called soft generalized-closed (soft  $g$ -closed), if  $cl(A, E) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft open in  $X$ .

#### Definition: 2.3[2]

A subset  $(A, E)$  of a topological space  $X$  is called soft regular closed, if  $cl(int(A, E)) = (A, E)$ . The complement of soft regular closed set is soft regular open set.

#### Definition: 2.4[2]

The finite union of soft regular open sets is said to be soft  $\pi$ -open. The complement of soft  $\pi$ -open is said to be soft  $\pi$ -closed.

#### Definition: 2.5[2]

A subset  $(A, E)$  of a topological space  $X$  is called soft  $\pi g$ -closed in a soft topological space  $(X, \tau, E)$ , if  $cl(A, E) \tilde{\subset} (U, E)$  whenever  $(A, E) \tilde{\subset} (U, E)$  and  $(U, E)$  is soft  $\pi$ -open in  $X$ .

**Definition: 2.6[1]**

Let  $(F, E)$  be a soft set over  $X$ . The soft set  $(F, E)$  is called soft point, denoted by  $(x_e, E)$ , if for element  $e \in E$ ,  $F(e) = \{x\}$  and  $F(e') = \emptyset$  for all  $e' \in E - \{e\}$ .

**Definition: 2.7[11]**

Let  $(X, \tau, E)$  and  $(Y, \tau', E)$  be two topological spaces. A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be Soft Semi continuous (Soft pre-continuous, Soft  $\alpha$ -continuous, Soft  $\beta$ -continuous), if  $f^{-1}(G, E)$  is soft semi open (soft pre-open, soft  $\alpha$ -open, soft  $\beta$ -open) in  $(X, \tau, E)$  for every soft open set  $(G, E)$  of  $(Y, \tau', E)$ .

**Definition: 2.8[3]**

Let  $(X, \tau, E)$  and  $(Y, \tau', E)$  be two topological spaces. A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be Soft regular continuous (Soft  $\pi$ -continuous, Soft  $g$ -continuous, Soft  $\pi g$ -continuous), if  $f^{-1}(G, E)$  is soft regular open (soft  $\pi$ -open, soft  $g$ -open, soft  $\pi g$ -open) in  $(X, \tau, E)$  for every soft open set  $(G, E)$  of  $(Y, \tau', E)$ .

**Definition: 2.9[3]**

A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft  $\pi g$ -irresolute, if  $f^{-1}(G, E)$  is soft  $\pi g$ -open in  $(X, \tau, E)$  for every soft  $\pi g$ -open set  $(G, E)$  of  $(Y, \tau', E)$ .

**Definition: 2.10[2]**

A space  $(X, \tau, E)$  is called soft  $\pi g$ - $T_{\frac{1}{2}}$  [6], if every soft  $\pi g$ -closed set is soft closed, or equivalently every soft  $\pi g$ -open set is soft open.

**Definition: 2.11[10]**

A soft topological space  $(X, \tau, E)$  is a soft  $-T_0$  space, if for each pair of distinct soft points  $x$  and  $y$  in  $X$ , there exist soft open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$ .

**Definition: 2.12[3]**

A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is called  $\tilde{S}\pi g$ -open, if image of each open set in  $X$  is  $\tilde{S}\pi g$ -open in  $Y$ .

**Definition: 2.13[4]**

A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is called soft contra  $\pi g$ -continuous, if  $f^{-1}(F, E)$  is soft  $\pi g$ -closed in  $X$  for every soft open set  $(F, E)$  of  $Y$ .

**Definition: 2.14[4]**

A space  $(X, \tau, E)$  is said to be soft  $\pi g$ -compact, if every soft  $\pi g$ -open cover of  $X$  has a finite sub cover.

**Definition: 2.15[4]**

Soft countably  $\pi g$ -compact, if every soft  $\pi g$ -open countably cover of  $X$  has a finite subcover.

**Definition: 2.16[4]**

soft  $\pi g$ -Lindelöf, if every soft  $\pi g$ -open cover of  $X$  has a countable subcover.

**Definition: 2.17[4]**

A space  $(X, \tau, E)$  is called soft  $\pi g$ -connected provided that  $X$  cannot be written as the union of two disjoint non-empty soft  $\pi g$ -open sets.

**Definition: 2.18[10]**

A soft topological space  $(X, \tau, E)$  is a soft  $\pi g$ - $T_0$  space, if for each pair of distinct soft points  $x$  and  $y$  in  $X$ , there exist soft open sets  $(F, E)$  and  $(G, E)$  such that  $x \in (F, E)$  and  $y \notin (F, E)$  or  $y \in (G, E)$  and  $x \notin (G, E)$ .

### 3. Slightly $\pi g$ -continuous functions

**Definition: 3.1**

A function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is called soft slightly continuous, if  $f^{-1}(G, E)$  is soft open in  $X$  for each soft clopen subset  $(G, E)$  of  $Y$ .

**Definition: 3.2**

A function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be soft slightly  $\pi g$ -continuous, if  $f^{-1}(G, E)$  is soft  $\pi g$ -open in  $X$  for each soft clopen subset  $(G, E)$  of  $Y$ .

**Theorem: 3.3**

The following statements are equivalent for a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$

1.  $f$  is soft slightly  $\pi g$ -continuous.
2. for every soft clopen subset  $(G, E)$  of  $Y$ ,  $f^{-1}(G, E)$  is soft  $\pi g$ -closed in  $X$ .
3. for every soft clopen subset  $(G, E)$  of  $Y$ ,  $f^{-1}(G, E)$  is soft  $\pi g$ -clopen in  $X$ .

**Proof:**

(1) $\implies$ (2)

Let  $(G, E)$  be soft clopen in  $Y$ . Then  $Y \setminus (G, E)$  is soft clopen in  $Y$ . Since  $f$  is soft slightly  $\pi g$ -continuous,  $f^{-1}(Y \setminus (G, E))$  is soft  $\pi g$ -open in  $X$ .  $f^{-1}(Y \setminus (G, E)) = X - f^{-1}(G, E)$  is soft  $\pi g$ -open in  $X$  implies  $f^{-1}(G, E)$  is soft  $\pi g$ -closed in  $X$ .

(2) $\implies$ (3)

Let  $(G, E)$  be soft clopen in  $Y$ . Then  $Y \setminus (G, E)$  is soft clopen in  $Y$ . By (2)  $f^{-1}(Y \setminus (G, E))$  is soft  $\pi g$ -closed in  $X$ . Hence  $f^{-1}(G, E)$  is soft  $\pi g$ -open in  $X$  implies  $f^{-1}(G, E)$  is soft  $\pi g$ -clopen in  $X$ .

(3) $\implies$ (1): obvious

**Theorem: 3.4**

Every soft slightly continuous function is soft slightly  $\pi g$ -continuous.

**Proof:** Obvious.

**Remark: 3.5**

The converse of the above theorem is not true in general as shown in the following examples.

**Example 3.6**

Let  $X = \{h_1, h_2, h_3, h_4\}$ ,  $Y = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2\}$ . Let  $F_1, F_2, F_3, F_4$  and  $G_1, G_2$  are functions from  $E$  to  $P(X)$  and  $E$  to  $P(Y)$  are defined as follows:

$F_1(e_1) = \{h_3\}$ ,  $F_1(e_2) = \{h_1\}$ ;  $F_2(e_1) = \{h_4\}$ ,  $F_2(e_2) = \{h_2\}$ ;  $F_3(e_1) = \{h_3, h_4\}$ ,  $F_3(e_2) = \{h_1, h_2\}$ ;  $F_4(e_1) = \{h_1, h_4\}$ ,  $F_4(e_2) = \{h_2, h_4\}$   $F_5(e_1) = \{h_2, h_3, h_4\}$ ,  $F_5(e_2) = \{h_1, h_2, h_3\}$ ;  $F_6(e_1) = \{h_1, h_3, h_4\}$ ,  $F_6(e_2) = \{h_1, h_2, h_4\}$  and  $G_1(e_1) = \{h_1\}$ ,  $G_1(e_2) = \{h_1\}$ ;  $G_2(e_1) = \{h_2, h_3\}$ ,  $G_2(e_2) = \{h_2, h_3\}$ . Then  $\tau = \{\tilde{\emptyset}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$  is a soft topological space over  $X$  and  $\tau' = \{\tilde{\emptyset}, \tilde{Y}, (G_1, E), (G_2, E)\}$  is a soft topological space over  $Y$ . If the function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is defined as  $f(h_1) = h_1$ ,  $f(h_3) = h_2$  then  $f$  is soft slightly  $\pi g$ -continuous but not soft slightly continuous.

**Theorem: 3.7**

Let  $(X, \tau, E)$  be a soft  $\pi g$ - $T_{\frac{1}{2}}$  space. Then the function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft slightly  $\pi g$ -continuous if and only if it is soft slightly continuous.

**Proof:**

Let  $(G, E)$  be soft clopen in  $Y$ . Since  $f$  is soft slightly  $\pi g$ -continuous,  $f^{-1}(G, E)$  is soft  $\pi g$ -open in  $X$  implies  $f^{-1}(G, E)$  is soft open in  $X$ . Therefore  $f$  is soft slightly continuous. Conversely every soft slightly continuous is soft slightly  $\pi g$ -continuous.

**Theorem: 3.8**

Suppose  $\tilde{S}\pi GO(X)$  is soft closed under arbitrary unions. Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be a function. Then  $f$  is soft slightly  $\pi g$ -continuous if and only if for each point  $x \in X$  and each soft clopen set  $(V, E)$  containing  $f(x)$ , there exists a soft  $\pi g$ -open set  $(U, E)$  containing  $x$  such that  $f(U, E) \tilde{c} (V, E)$ .

**Proof:**

Let  $x \in X$  and  $(V, E)$  be soft clopen then  $f(x) \in (V, E)$ . Since  $f$  is soft slightly  $\pi g$ -continuous,

$f^{-1}(V, E)$  is soft  $\pi g$ -open in  $X$ . If we put  $(U, E) = f^{-1}(V, E)$  then  $x \in (U, E)$  and  $f(U, E) \subseteq (V, E)$ . Conversely let  $(V, E)$  be a soft clopen subset of  $Y$  and let  $x \in f^{-1}(V, E)$ . Since  $f(x) \in (V, E)$ , there exists a soft  $\pi g$ -open set  $(U_x, E)$  in  $X$  containing  $x$  such that  $(U_x, E) \subseteq f^{-1}(V, E)$ . We obtain  $f^{-1}(V, E) = \cup\{(U_x, E) : x \in f^{-1}(V, E)\}$ . Thus  $f^{-1}(V, E)$  is soft  $\pi g$ -open.

**Definition: 3.9**

A space  $(X, \tau, E)$  is said to be soft locally indiscrete, if every soft open set of  $X$  is soft closed in  $X$ .

**Theorem : 3.10**

If a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft slightly  $\pi g$ -continuous and  $(Y, \tau', E)$  is soft locally indiscrete, then  $f$  is soft  $\pi g$ -continuous.

**Proof:**

Let  $(A, E)$  be a soft open set in  $Y$ . Since  $Y$  is soft locally indiscrete, every soft open set is soft closed. Since  $f$  is soft slightly  $\pi g$ -continuous,  $f^{-1}(A, E)$  is soft  $\pi g$ -open in  $X$ . Hence  $f$  is soft slightly continuous.

**Theorem: 3.11**

If a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft slightly  $\pi g$ -continuous and  $(X, \tau, E)$  is soft  $\pi g-T_{\frac{1}{2}}$  space then  $f$  is soft slightly continuous.

**Proof:**

Let  $(A, E)$  be a soft clopen set in  $Y$ . By hypothesis  $f^{-1}(A, E)$  is soft  $\pi g$ -open in  $X$ . Since  $X$  is soft  $\pi g-T_{\frac{1}{2}}$  space,  $f^{-1}(A, E)$  is soft open in  $X$ . Hence  $f$  is soft slightly continuous.

**Definition: 3.12**

A space  $(X, \tau, E)$  is said to be soft submaximal, if each soft dense subset of  $X$  is soft open.

**Definition: 3.13**

A space  $(X, \tau, E)$  is said to be soft extremally disconnected, if the soft closure of each soft open set of  $X$  is soft open in  $X$ .

**Definition: 3.14**

The graph  $G(f)$  of a function  $f : (X, \tau, E) \rightarrow (Y, \tau', E)$  is said to be soft slightly  $\pi g$ -graph, if for each  $(x, y) \in (X \times Y) \setminus G(f)$ , there exist a soft  $\pi g$ -open set  $(A, E)$  in  $X$  containing  $x$  and a soft clopen set  $(B, E)$  in  $Y$  containing  $y$  such that  $(A \times B, E) \cap G(f) = \emptyset$ .

**Theorem: 3.15**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be soft function and let  $g: (X, \tau, E) \rightarrow (X \times Y, \tau \times \tau', E)$  be the soft graph function of  $f$ , defined by  $g(x) = (x, f(x))$  for every  $x \in X$ . Then  $f$  is soft slightly  $\pi g$ -continuous, if  $g$  is soft slightly  $\pi g$ -continuous.

**Proof:**

Let  $(V, E) \in \tilde{S}CO(Y)$  then  $X \times (V, E) \in \tilde{S}CO(X \times Y)$ . Since  $g$  is soft slightly  $\pi g$ -continuous,  $f^{-1}(V, E) = g^{-1}(X \times (V, E)) \in \tilde{S}\pi GO(X)$ . Thus  $f$  is soft slightly  $\pi g$ -continuous.

**Theorem: 3.16**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be soft slightly  $\pi g$ -continuous function and let  $g: (X, \tau, E) \rightarrow (X \times Y, \tau \times \tau', E)$  be the soft graph function of  $f$ , defined by  $g(x) = (x, f(x))$  for every  $x \in X$ . If for each soft clopen subset  $(W, E)$  of  $(X \times Y, \tau \times \tau', E)$  and for each  $x \in g^{-1}(W, E)$ , the set  $g^{-1}(W, E) \cap f^{-1}(W_x, E)$  where  $(W_x, E)$  is a vertical cut of  $(W, E)$  at  $x$  is soft  $\pi g$ -open relative to  $f^{-1}(W_x, E)$  then  $g$  is soft slightly  $\pi g$ -continuous.

**Proof:**

Let  $(W, E)$  be any soft clopen subset of  $X \times Y$  and let  $x \in g^{-1}(W, E)$ , be an arbitrarily chosen soft point. Then  $(W, E) \cap (\{x\} \times Y)$  is soft clopen in  $\{x\} \times Y$  containing  $g(x)$ . Also  $\{x\} \times Y$  is soft homeomorphic to  $Y$ . Hence, the vertical cut  $W_x = \{y \in Y : (x, y) \in (W, E)\}$  is a soft clopen subset of  $Y$ . Since  $f$  is soft slightly  $\pi g$ -continuous,  $f^{-1}(W_x, E)$  is a soft  $\pi g$ -open subset of  $X$ . By hypothesis  $g^{-1}(W, E) \cap f^{-1}(W_x, E)$  is soft  $\pi g$ -open relative to  $f^{-1}(W_x, E)$ , so  $g^{-1}(W, E) \cap f^{-1}(W_x, E)$  is soft  $\pi g$ -open in  $X$ . Therefore  $g^{-1}(W, E)$  is soft  $\pi g$ -open in  $X$ . Then  $g$  is soft slightly  $\pi g$ -continuous.

**Theorem: 3.17**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  and  $g: (Y, \tau', E) \rightarrow (Z, \tau'', E)$  be functions. Then the following properties hold:

1. if  $f$  is soft  $\pi g$ -irresolute and  $g$  is soft slightly  $\pi g$ -continuous, then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft slightly  $\pi g$ -continuous.
2. if  $f$  is soft  $\pi g$ -irresolute and  $g$  is soft  $\pi g$ -continuous, then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft slightly  $\pi g$ -continuous.
3. if  $f$  is soft  $\pi g$ -irresolute and  $g$  is soft slightly continuous, then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft slightly  $\pi g$ -continuous.

**Theorem: 3.18**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  and  $g: (Y, \tau', E) \rightarrow (Z, \tau'', E)$  be functions. If  $f$  is soft  $M$ -  $\pi g$ -open surjective and  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft slightly  $\pi g$ -continuous then  $g$  is soft slightly  $\pi g$ -continuous.

**Proof:**

Let  $(A, E)$  be any soft clopen in  $Z$ . Since  $g \circ f$  is soft slightly  $\pi g$ -continuous,  $(g \circ f)^{-1}(A, E) = f^{-1}(g^{-1}(A, E))$  is soft soft  $\pi g$ -open. Since  $f$  is soft  $M$ -  $\pi g$ -open, then  $f(f^{-1}(g^{-1}(A, E))) = g^{-1}(A, E)$  is soft soft  $\pi g$ -open in  $Y$ . Hence  $g$  is soft slightly  $\pi g$ -continuous.

**Theorem: 3.19**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be surjective soft  $\pi g$ -irresolute, soft  $M$ - $\pi g$ -open and let  $g: (Y, \tau', E) \rightarrow (Z, \tau'', E)$  be a function. Then  $g \circ f: (X, \tau, E) \rightarrow (Z, \tau'', E)$  is soft slightly  $\pi g$ -continuous if and only if  $g$  is soft slightly  $\pi g$ -continuous.

**4. Soft  $\pi g$ -compact space and Soft  $\pi g$ -connected space**

**Definition: 4.1**

A space  $(X, \tau, E)$  is said to be soft mildly compact, if every soft clopen cover of  $X$  has a finite sub cover.

**Theorem: 4.2**

If a function  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft slightly  $\pi g$ -continuous and  $(A, E)$  is soft  $\pi g$ -compact relative to  $X$ , then  $f(A, E)$  is soft mildly compact relative to  $Y$ .

**Proof:**

Let  $\{(V_i, E) : i \in I\}$  be any soft cover of  $f(A, E)$  by soft clopen sets of the subspace  $f(A, E)$ . For each  $i \in I$  there exists a soft clopen set  $(A_i, E)$  of  $Y$  such that  $(V_i, E) = (A_i, E) \cap f(A, E)$ . For each  $x \in (A, E)$ , there exists  $i(x) \in I$  such that  $f(x) \in (A_{i(x)}, E)$  and there exists  $(F_x, E) \in \mathcal{S}\pi GO(X, x)$  such that  $f(F_x, E) \tilde{c} (A_{i(x)}, E)$ . Since the family  $\{(F_x, E) : x \in (A, E)\}$  is a soft cover of  $(A, E)$  by  $\mathcal{S}\pi g$ - open sets of  $X$ , there exists a finite subset  $(A_0, E)$  of  $(A, E)$  such that  $(A, E) \tilde{c} \cup \{(F_x, E) : x \in (A_0, E)\}$ . Hence, we obtain  $f(A, E) \tilde{c} \cup \{f(F_x, E) : x \in (A_0, E)\}$  which is a subset of  $\cup \{(A_{i(x)}, E) : x \in (A_0, E)\}$ . Thus,  $f(A, E) = \cup \{(V_{i(x)}, E) : x \in (A_0, E)\}$ . Hence  $f(A, E)$  is soft mildly compact relative to  $Y$ .

**Corollary: 4.3**

If  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft slightly  $\pi g$ -continuous surjection and  $X$  is soft  $\pi g$ -compact, then  $Y$  is soft mildly compact.

**Definition: 4.4**

A space  $(X, \tau, E)$  is said to be:

1. Soft mildly countably compact, if every soft clopen countably cover of  $X$  has a finite subcover.
2. Soft mildly Lindelöf, if every soft cover of  $X$  by soft clopen sets has a soft countable subcover.
3. Soft  $\pi g$ -closed compact, if every soft  $\pi g$ -closed cover of  $X$  has a finite subcover.
4. Soft countably  $\pi g$ -closed-compact, if every countable cover of  $X$  by soft  $\pi g$ -closed sets has a finite subcover.

**Theorem: 4.5**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be a soft slightly  $\pi g$ -continuous surjection. Then the following statements hold:

1. If  $X$  is soft  $\pi g$ -Lindelöf, then  $Y$  is soft mildly Lindelöf.

2. If  $X$  is soft countably  $\pi g$ -compact, then  $Y$  is soft mildly countably compact.

**Proof:**

(1) Let  $\{(V_\alpha, E) : \alpha \in I\}$  be a soft clopen cover of  $Y$ . Since  $f$  is soft slightly  $\pi g$ -continuous, then  $\{f^{-1}(V_\alpha, E) : \alpha \in I\}$  is a soft  $\pi g$ -open cover of  $X$ . Since  $X$  is soft  $\pi g$ -Lindelof, there exists a countable subset  $I_0$  of  $I$  such that  $X = \cup\{f^{-1}(V_\alpha, E) : \alpha \in I_0\}$ . Thus  $Y = \cup\{(V_\alpha, E) : \alpha \in I_0\}$  and hence  $Y$  is soft mildly Lindelöf. Proof of (2) Similar to (1).

**Theorem: 4.6**

Let  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  be a soft slightly  $\pi g$ -continuous surjection. Then the following statements hold:

1. If  $X$  is soft  $\pi g$ -closed compact, then  $Y$  is soft mildly compact.
2. If  $X$  is soft  $\pi g$ -closed Lindelöf, then  $Y$  is soft mildly Lindelöf.
3. If  $X$  is soft countably  $\pi g$ -closed compact, then  $Y$  is soft mildly countably compact.

**Theorem: 4.7**

If  $f: (X, \tau, E) \rightarrow (Y, \tau', E)$  is soft slightly  $\pi g$ -continuous surjective function and  $X$  is soft  $\pi g$ -connected, then  $Y$  is soft connected.

**Proof:**

Suppose  $Y$  is not soft connected. Then there exist non-empty disjoint soft clopen subsets  $(U, E)$  and  $(V, E)$  of  $Y$  such that  $Y = (U, E) \cup (V, E)$ . Since  $f$  is soft slightly  $\pi g$ -continuous, we have  $f^{-1}(U, E)$  and  $f^{-1}(V, E)$  are non-empty disjoint soft  $\pi g$ -open sets in  $X$ . Moreover,  $f^{-1}(U, E) \cup f^{-1}(V, E) = X$ . This shows that  $X$  is not soft  $\pi g$ -connected which is a contradiction. Hence  $Y$  is soft connected.

**Theorem: 4.8**

If  $f$  is a soft slightly  $\pi g$ -continuous function from a soft  $\pi g$ -connected space  $X$  onto any space  $Y$ , then  $Y$  is not a soft discrete space.

**Proof:**

Suppose that  $Y$  is soft discrete. Let  $(A, E)$  be a proper nonempty soft open subset of  $Y$ . Then  $f^{-1}(A, E)$  is any proper nonempty soft  $\pi g$ -clopen subset of  $X$ , which is a contradiction to the assumption that  $X$  is soft  $\pi g$ -connected. Therefore  $Y$  is not a soft discrete space.

**Theorem: 4.9**

A space  $X$  is soft  $\pi g$ -connected, if every soft slightly  $\pi g$ -continuous function from a space  $X$  into any soft  $T_0$ -space  $Y$  is constant.

**Proof:**

Suppose that  $X$  is not soft  $\pi g$ -connected. Let every soft slightly  $\pi g$ -continuous function from  $X$  into any soft  $T_0$ -space then  $Y$  is constant. Since  $X$  is not soft  $\pi g$ -connected, there exists a proper nonempty soft  $\pi g$ -clopen subset  $(A, E)$  of  $X$ . Then  $f$  is a non-constant and soft slightly  $\pi g$ -continuous such that  $Y$  is soft  $T_0$ , which is a contradiction. Hence,  $X$  is soft  $\pi g$ -connected.

**Theorem: 4.10**

If a function  $f: X \rightarrow \prod Y_\alpha$  is soft slightly  $\pi g$ -continuous, then  $P_\alpha \circ f : X \rightarrow Y_\alpha$  is soft slightly  $\pi g$ -continuous for each  $\alpha \in \Lambda$ , where  $P_\alpha$  is the projection of  $\prod Y_\alpha$  onto  $Y_\alpha$ .

**Proof:**

Let  $(V_\alpha, E)$  be any soft clopen set of  $Y_\alpha$ . Then  $P_\alpha^{-1}(V_\alpha, E)$  is soft clopen in  $\prod Y_\alpha$ . Hence  $(P_\alpha \circ f)^{-1}(V_\alpha, E) = (f^{-1}(P_\alpha^{-1}(V_\alpha, E)))$  is soft  $\pi g$ -open in  $X$ . Therefore,  $P_\alpha \circ f$  is soft slightly  $\pi g$ -continuous.

**Theorem: 4.11**

If a function  $f: \prod X_\alpha \rightarrow \prod Y_\alpha$  is soft slightly soft  $\pi g$ -continuous, then  $f_\alpha: X_\alpha \rightarrow Y_\alpha$  is soft slightly  $\pi g$ -continuous for each  $\alpha \in \Lambda$ .

**Proof:**

Let  $(V_\alpha, E)$  be any soft clopen set of  $Y_\alpha$ . Then  $P_\alpha^{-1}(V_\alpha, E)$  is soft clopen in  $\prod Y_\alpha$  and  $f^{-1}(P^{-1}(V_\alpha, E)) = f^{-1}(V_\alpha, E) \times \prod \{X_\alpha : \alpha \in \Lambda \setminus \{\alpha\}\}$ . Since  $f$  is soft slightly  $\pi g$ -continuous,  $f^{-1}(P^{-1}(V_\alpha, E))$  is soft  $\pi g$ -open in  $\prod X_\alpha$ . Since the projection  $P_\alpha$  of  $\prod X_\alpha$  onto  $X_\alpha$  is soft open and soft continuous,  $f_\alpha^{-1}(V_\alpha, E)$  is soft  $\pi g$ -open in  $X_\alpha$ . Hence  $f_\alpha$  is soft slightly  $\pi g$ -continuous.

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