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On The Karush – Kuhn – Tucker Reformulation of Bi – Level Geometric Programming Problem with an Interval Coefficients as Multiple Parameters

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ABSTRACT

This paper presents a new approach to solve a special class of bi – level nonlinear programming (NLP) problems with an interval coefficients as multiple parameters. Geometric programming (GP) is a powerful technique developed for solving nonlinear programming (NLP) problems and it is useful in the study of a variety of optimization problems. Many applications of GP in various fields of science and engineering are used to solve certain complex decision making problems. In this paper a new mathematical formulations for a new class of nonlinear optimization models called bi – level geometric programming (BLGP) problem is presented. These problems are not necessarily convex and thus not solvable by standard nonlinear programming techniques. This paper proposed a method to solve BLGP problem where coefficient of objective function as well as coefficient of constraints are multiple parameters. Especially the multiple parameters are considered in an interval which are the Arithmetic mean (A.M), Geometric mean (G.M) and Harmonic mean (H. M) of the end points of the interval. In this paper, the values of objective function in interval range of parameters for A. M., G. M. and H. M. are preserved the same relationship. Also, BLGP problem can be converted to a single objective by using the classical karush – kuhn – Tucker (KKT) reformulation and the ability of calculating the bounds of objective value in KKT is basically presented in this paper that may help researchers in constructing more realistic model in optimization field. Finally, numerical example is given to illustrate the efficiency of the method.

Keywords: Bi – Level optimization problem; Geometric programming; KKT reformulation; Arithmetic mean; Geometric mean; Harmonic mean.

1. Introduction

Bi – Level programming (BLP) is a special situation of multi-level programming in which there are only two levels of optimization, that is, the levels of decision (optimization). The hierarchical optimization structure appears naturally in critical resource management and policy making, including tourism resource planning, water resource management, financial planning, land-use planning, production planning, transportation planning and power market pricing [1]. BLP is closely related to the economics problem addressed by Stackelberg (1952) through its development of strategic game theory. The original formulation for BLP appeared in 1973, in a paper authored by Bracken and McGill (1973), although it was Candler and Norton (1977) who first used the designation “bi-level” programming. However, it was not until the early 1980s that these problems started to receive the attention they deserved. Bi – level and multi-level programming techniques are developed for solving decentralized decision- making problems with decision entities (also called decision makers) in a hierarchical system [2, 3, 4]. The decision maker at the upper level is termed the leader, and at the lower level is termed the follower (Bard 1998).Each decision maker tries to optimize their own objectives by considering the objective of the other level only partially or not at all, but the decision of each level affects the strategy of other level decision makers.

Optimality condition for nonlinear bi – level programming (NLBLP) was discussed by Bard [5] as based upon those of Fiacco and McCormick [6] and kolstad and lasdon, who utilized descent algorithm techniques [7] and a variable metric algorithm [8].

Geometric programming (GP) is an optimization technique developed to solve a special type of nonlinear programming (NLP) that was originally developed by Duffin et al [9] for application to engineering design problems requiring a compromise between many alternatives. Geometric programming has been studied by others such as Dembo [10] and kyparisis [11], but none have considered its application to multilevel programming. This paper extends posynomial geometric programming (PGP) to bi – level programming [12,13] where the coefficient of objective function and the constraints are considered as multiple parameters. Because GP need not be convex, standard NLP techniques for its solution cannot necessarily be used. KKT conditions can be used to solve BLGP to convert it into sing le objective for searching an optimal solution [14].

When the objective function and the constraint coefficient are interval parameters, the derived objective value should lie in an interval as well. Liu [15] develops a solution method to calculate the bounds of the objective value in GP with interval parameters. The ability of calculating the bounds of objective value is basically developed in this paper that may help researchers in constructing more realistic model in optimization field. Also,we have shown the values of objective function at the multiple parameters such as A.M., G.M. and H. M. in KKT reformulation preserves the same relation.

2- Geometric Programming

A geometric programming (GP) problem in primal form was formulated by Duffin et al. [9] as follows:

Mathematical Formulation 1:

$$\underset{i\in R^m}{Minimize}\ g_0\ (t,c) \tag{1a}$$

s.t.

$$g_k(t,c)\ \leq\ 1 \qquad \text{for } k=1,2,\dots,p, \tag{1b}$$

$$t_j > 0 \qquad \text{for } j=1,2,\dots,m. \tag{1c}$$

where

$$g_k\ (t,c)=\sum_{i\in J_k} c_i t_1^{a_{i1}} t_2^{a_{i2}} \dots . t_m^{a_{im}}$$

$$\text{for } k = 0, 1, 2, \dots p, \tag{1d}$$

$$J_k=\{\ m_k, m_k+1\ ,\ m_k+2,\dots\dots,n_k\}$$

$$\text{for } k = 0, 1, 2, \dots, p. \tag{1e}$$

$$m_0=1,\ m_1=n_0+1,\ m_2=n_1+1,\dots.,m_p=n_{p-1}+1, \tag{1f}$$

$$n_p=n \tag{1g}$$

$$c=(c_1,c_2,\dots,c_n) \text{ and } c_i>0 \text{ for } i=1,2,..n \tag{1h}$$

3- Bi – level Geometric Programming (BLGP) Problem

A bi – level programming (BLP) problem is formulated for a problem in which two decision maker make decisions successively [16].

Mathematical Formulation2:

The variable t in equations (1a) – (1d) of primal form for GP can be partitioned into two nonempty disjoint classes x and y, where the leader problem has control decision variable x and the follower problem has control decision variable y. It is assumed that player 1, who controls the leader problem, has the first choice and selects x, followed by player 2, who controls the follower problem and selects y.

Define $x=(x_1,x_2,\ldots,x_r^u)$ and $y=(y_1,y_2,\ldots,y_r^l)$, where r^u and r^l are the dimensions of x and y, respectively, $r^u + r^l = m$, and $t \subset R^m$. Each BLGP problem consists of a leader and a follower problem as described as:

The upper – level or leader problem is given by:

$$\underset{x}{Minimize}\; G_0(x,y,c) \tag{2a}$$

s.t.

$$G_{k^u}(x,y,c) \le 1 \quad \text{for} \quad k^u = 1,2,\ldots,p^u, \tag{2b}$$

$$x_{j^u} > 0 \quad \text{for} \quad j^u = 1,2,\ldots,r^u < m. \tag{2c}$$

and where y as a function of x is implicitly defined by a lower – level or follower's problem as given by:

$$\underset{y}{Minimize}\; g_0(x,y,c) \tag{2d}$$

s.t.

$$g_{k^l}(x,y,c) \le 1 \quad \text{for} \quad k^l = 1,2,\ldots,p^l, \tag{2e}$$

$$y_{j^l} > 0 \quad \text{for} \quad j^l = 1,2,\ldots,r^l < m. \tag{2f}$$

Where in the above,

$$r^u + r^l = m, \tag{3}$$

$$G_{k^u}(x,y,c) = \sum_{i\in J_{k^u}} c_i^u \prod_{j=1}^{r^u} x_j^{a_{ij}^u} + \sum_{i\in J_{k^u}} c_i^{*u} \prod_{j=r^u+1}^{m} y_j^{a_{ij}^u},$$

$$\text{for } k^u = 0,1,2,\ldots,p^u, \tag{4}$$

$$g_{k^l}(x,y,c) = \sum_{i\in J_{k^l}} c_i^l \prod_{j=1}^{r^l} y_j^{a_{ij}^l} + \sum_{i\in J_{k^l}} c_i^{*l} \prod_{j=r^l+1}^{m} x_j^{a_{ij}^l},$$

$$\text{for } k^l = 0,1,2,\ldots,p^l. \tag{5}$$

The notation used for the limits of summations in equations (4) and (5) is defined as

$$J_{k^u} = \left\{ r_{k^u}^u, r_{k^u}^u + 1, \ldots, n_{k^u}^u \right\} \qquad \text{for } k^u = 0,1,2,\ldots,p^u, \tag{6}$$

$$J_{k^l} = \left\{ r_{k^l}^l, r_{k^l}^l + 1, \ldots, n_{k^l}^l \right\} \qquad \text{for } k^l = 0,1,2,\ldots,p^l, \tag{7}$$

where

$$r_0^u = 1,\ r_1^u = n_0^u + 1,\ r_2^u = n_1^u + 1, \ldots, r_{p^u}^u = n_{p^{u-1}}^u + 1,\ n_{p^u}^u = n^u, \tag{8}$$

$$r_0^l = 1,\ r_1^l = n_0^l + 1,\ r_2^l = n_1^l + 1, \ldots, r_{p^l}^l = n_{p^{l-1}}^l + 1,\ n_{p^l}^l = n^l. \tag{9}$$

In equation (6), $n_{k^u}^u$ is the row dimension of a_{ij}^u in the G_{k^u} equation for $k^u = 0,1,2,\ldots,p^u$. Similarly, in equation (7), $n_{k^l}^l$ is the row dimension of a_{ij}^l in the g_{k^l} equation for $k^l = 0,1,2,\ldots,p^l$.

Considering the coefficient of variables in objective functions and constraints in both of upper and lower level as the multiple parameters the problem defined in (2) – (5) can be reformulated as:

The upper level is given by:

$$\underset{x}{Minimize}\ \ G_0(x, y, \bar{c}) \tag{10a}$$

s.t.

$$G_{k^u}(x, y, \bar{c}) \le 1 \qquad \text{for } k^u = 1,2,\ldots,p^u, \tag{10b}$$

$$x_{j^u} > 0 \qquad \text{for } j^u = 1,2,\ldots,r^u < m \tag{10c}$$

The lower level is given by:

$$\underset{y}{Minimize}\ \ g_0(x, y, \bar{c}) \tag{10d}$$

s.t.

$$g_{k^l}(x, y, \bar{c}) \le 1 \qquad \text{for } k^l = 1,2,\ldots,p^l, \tag{10e}$$

$$y_{j^l} > 0 \qquad \text{for } j^l = 1,2,\ldots,r^l < m \tag{10f}$$

Where in the above,

$$r^u + r^l = m, \tag{11}$$

$$G_{k^u}(x, y, \bar{c}) = \sum_{i \in J_{k^u}} \bar{c}_i^u \prod_{j=1}^{r^u} x_j^{a_{ij}^u} + \sum_{i \in J_{k^u}} \bar{c}_i^{*u} \prod_{j=r^u+1}^{m} y_j^{a_{ij}^u},$$

$$\text{for } k^u = 0,1,2,\ldots,p^u, \tag{12}$$

$$g_{k^l}(x, y, \bar{c}) = \sum_{i \in J_{k^l}} \bar{c}_i^l \prod_{j=1}^{r^l} y_j^{a_{ij}^l} + \sum_{i \in J_{k^l}} \bar{c}_i^{*l} \prod_{j=r^l+1}^{m} x_j^{a_{ij}^l},$$

$$\text{for } k^{l} = 0,1,2,\ldots,p^{l}, \tag{13}$$

$$\bar{c}^{u} = \left\{ c^{L^{u}}, c^{A.M.^{u}}, c^{G.M.^{u}}, c^{H.M.^{u}}, c^{U^{u}} \right\}, \tag{14}$$

$$\bar{c}^{l} = \left\{ c^{L^{l}}, c^{A.M.^{l}}, c^{G.M.^{l}}, c^{H.M.^{l}}, c^{U^{l}} \right\}. \tag{15}$$

The notation used for the limits of summations in equations (12) and (13) as the same notation used for the limits of summation in equation (4) and (5), and L : lower bound, A.M. : Arithmetic mean, G.M. : Geometric mean, H.M. : Harmonic mean and U : upper bound of parameter interval.

Using the multiple parameter as defined above [17], we can define the BLGP problem in lower bound as:

The upper level is given by:

$$F^{L} = \underset{x}{Minimize}\; G_{0}\left(x, y, c^{L^{u}} \right) \tag{16a}$$

s.t.

$$G_{k^{u}}\left(x, y, c^{L^{u}} \right) \le 1 \qquad \text{for } k^{u} = 1,2,\ldots,p^{u}, \tag{16b}$$

$$x_{j^{u}} > 0 \qquad \text{for } j^{u} = 1,2,\ldots,r^{u} < m. \tag{16c}$$

The lower level is given by:

$$f^{L} = \underset{y}{Minimize}\; g_{0}\left(x, y, c^{L^{l}} \right) \tag{16d}$$

s.t.

$$g_{k^{l}}\left(x, y, c^{L^{l}} \right) \le 1 \qquad \text{for } k^{l} = 1,2,\ldots,p^{l}, \tag{16e}$$

$$y_{j^{l}} > 0 \qquad \text{for } j^{l} = 1,2,\ldots,r^{l} < m. \tag{16f}$$

Similarly, BLGP problem in upper bound of interval parameter can be defined as:

The upper level is given by:

$$F^{U} = \underset{x}{Minimize}\; G_{0}\left(x, y, c^{U^{u}} \right) \tag{17a}$$

s.t.

$$G_{k^{u}}\left(x, y, c^{U^{u}} \right) \le 1 \qquad \text{for } k^{u} = 1,2,\ldots,p^{u}, \tag{17b}$$

$$x_{j^{u}} > 0 \qquad \text{for } j^{u} = 1,2,\ldots,r^{u} < m. \tag{17c}$$

The lower level is given by:

$$f^{U} = \underset{y}{Minimize}\; g_{0}\left(x, y, c^{U^{l}} \right) \tag{17d}$$

s.t.

$$g_{k^l}\left(x,y,c^{U^l}\right)\le 1 \qquad \text{for } k^l=1,2,\ldots,p^l, \tag{17e}$$

$$y_{j^l}>0 \qquad \text{for } j^l=1,2,\ldots,r^l<m. \tag{17f}$$

Also, BLGP problem in A.M. of interval parameter is defined as:

The upper level is given by:

$$F^{A.M.}=\underset{x}{Minimize}\; G_0\left(x,y,c^{A.M.^u}\right) \tag{18a}$$

s.t.

$$G_{k^u}\left(x,y,c^{A.M.^u}\right)\le 1 \qquad \text{for } k^u=1,2,\ldots,p^u, \tag{18b}$$

$$x_{j^u}>0 \qquad \text{for } j^u=1,2,\ldots,r^u<m. \tag{18c}$$

The lower level is given by:

$$f^{A.M.}=\underset{y}{Minimize}\; g_0\left(x,y,c^{A.M.^l}\right) \tag{18d}$$

s.t.

$$g_{k^l}\left(x,y,c^{A.M.^l}\right)\le 1 \qquad \text{for } k^l=1,2,\ldots,p^l, \tag{18e}$$

$$y_{j^l}>0 \qquad \text{for } j^l=1,2,\ldots,r^l<m. \tag{18f}$$

Similarly, BLGP problem in G.M. of interval parameter is defined as:

The upper level is given by:

$$F^{G.M.}=\underset{x}{Minimize}\; G_0\left(x,y,c^{G.M.^u}\right) \tag{19a}$$

s.t.

$$G_{k^u}\left(x,y,c^{G.M.^u}\right)\le 1 \qquad \text{for } k^u=1,2,\ldots,p^u, \tag{19b}$$

$$x_{j^u}>0 \qquad \text{for } j^u=1,2,\ldots,r^u<m. \tag{19c}$$

The lower level is given by:

$$f^{G.M.}=\underset{y}{Minimize}\; g_0\left(x,y,c^{G.M.^l}\right) \tag{19d}$$

s.t.

$$g_{k^l}\left(x,y,c^{G.M.^l}\right)\le 1 \qquad \text{for } k^l=1,2,\ldots,p^l, \tag{19e}$$

$$y_{j^l} > 0 \qquad \text{for } \; j^l = 1,2,\ldots,u^l < m. \tag{19f}$$

and BLG problem in H.M. of interval parameter can be defined as:

the upper level is given by:

$$F^{H.M.} = \underset{x}{Minimize}\; G_0\left(x, y, c^{H.M.^u}\right) \tag{20a}$$

s.t.

$$G_{k^u}\left(x, u, c^{H.M.^u}\right) \leq 1 \qquad \text{for } \; k^u = 1,2,\ldots,p^u, \tag{20b}$$

$$x_{j^u} > 0 \qquad \text{for } \; j^u = 1,2,\ldots,r^u < m. \tag{20c}$$

The lower level is given by:

$$f^{H.M.} = \underset{y}{Minimize}\; g_0\left(x, y, c^{H.M.^l}\right) \tag{20d}$$

s.t.

$$g_{k^l}\left(x, u, c^{H.M.^l}\right) \leq 1 \qquad \text{for } \; k^l = 1,2,\ldots,p^l, \tag{20e}$$

$$y_{j^l} > 0 \qquad \text{for } \; j^l = 1,2,\ldots,r^l < m. \tag{20f}$$

4 Optimality Condition for BLGP

The most popular approach to solve the nested bi – level optimization problems is using the KKT conditions and transforms the original problem to its first level auxiliary problem [18]. A necessary condition, for (x^*, y^*) to solve BLGP problem (10a) – (10f) where coefficient of objective function and coefficient of constraints are multiple parameters, is that there exists $\lambda^* \in R^{p^u}, \tilde{\lambda}^*_{k^l} \in R^{p^l}$ *and* $\tilde{\lambda}^*_0 \in R^{p^l}$ such that $\left(\tilde{x}^*, \tilde{y}^*, \tilde{\lambda}^*_{k^u}, \tilde{\lambda}^*_{k^l}, \tilde{\lambda}^*_0\right)$ is feasible for the following:

$$\underset{\tilde{x},\tilde{y}}{Minimize}\;\; g_0(\tilde{x}, \tilde{y}, \tilde{c}) \tag{21a}$$

s.t.

$$\nabla_{x_i} G_0\,(\tilde{x}, \tilde{y}, \tilde{c}) + \lambda_{i,k^u} \nabla_{x_i} G_{k^u}\,(\tilde{x}, \tilde{y}, \tilde{c}) + \lambda_{j,k^l} \nabla_{x_i} g_{k^l}\,(\tilde{x}, \tilde{y}, \tilde{c}) = 0$$

$$\text{for } i \in J_{k^u}, \tag{21b}$$

$$\begin{aligned} &\nabla_{y_i} G_0\,(\tilde{x}, \tilde{y}, \tilde{c}) + \lambda_{i,k^u} \nabla_{y_j} G_{k^u}\,(\tilde{x}, \tilde{y}, \tilde{c}) + \lambda_{j,k^l} \nabla_{y_i} g_{k^l}\,(\tilde{x}, \tilde{y}, \tilde{c}) \\ &+ \lambda_{j,0} \nabla_{y_i} g_0(\tilde{x}, \tilde{y}, \tilde{c}) = 0 \qquad \text{for } i \in j_{k^l} \end{aligned} \tag{21c}$$

$$G_{k^u}(\tilde{x},\tilde{y},\tilde{c}) \geq 0 \qquad \text{for } k^u = 1,2,\ldots,p^u, \tag{21d}$$

$$g_{k^l}(\tilde{x},\tilde{y},\tilde{c}) \geq 0 \qquad \text{for } k^l = 1,2,\ldots,p^l, \tag{21e}$$

$$\lambda_{i,k^u} G_{k^u}(\tilde{x},\tilde{y},\tilde{c}) = 0 \qquad \text{for } k^u = 1,2,\ldots,p^u \text{ and } i \in J_{k^u}, \tag{21f}$$

$$\lambda_{j,k^l} g_{k^l}(\tilde{x},\tilde{y},\tilde{c}) = 0 \qquad \text{for } k^l = 1,2,\ldots,p^l \text{ and } j \in J_{k^l}, \tag{21g}$$

$$\lambda_{i,k^u} \geq 0,\ \lambda_{j,k^l} \geq 0 \text{ and } \lambda_{j,0} \geq 0 \text{ for all } i \in J_{k^u} \text{ and all } j \in J_{k^l} \tag{21h}$$

where G_0, G_{k^u}, g_0 and g_{k^l} are assumed to be continuously differentiable functions, the dimensions of $\tilde{x}$ and $\tilde{y}$ are r^u and r^l respectively, and for the Kuhn –Tucker multipliers: $\tilde{\lambda}_{k^u} \in R^{p^u}, \tilde{\lambda}_{k^l} \in R^{p^l}$ *and* $\tilde{\lambda}_0 \in R^{p^l}$.

The following example illustrate the methodology proposed in this paper for solving BLGP problem with multiple parameters of objective function and constraints coefficient.

5 Illustrative Example

To demonstrate the solution method for BLGP, let us consider the following illustrative example.

Upper level:

$$\min_{x_1} G_0 = (1, 1.5, 1.4, 0.75, 2)\ x_1^{-1}x_2^{-1} + (15, 17.5, 17.3, 0.6, 20)\, x_2\, x_3$$
$$+ (10, 11, 10.95, 0.09, 12)\ x_3^{-1}$$

where x_2 and x_3 solve the lower level:

$$\min_{x_2, x_3} g_0 = (15, 17.5, 17.3, 0.6, 20)\ x_1^{-1}x_2^{-1}x_3^{-1} + (5, 7.5, 7.1, 0.15, 10)$$
$$x_1^{-2}\, x_3 + (2, 3, 2.8, 0.38, 4)\ x_2^{-2}$$

$s.t.$

$$(6,7,6.9,0.15,8)\ x_1 x_2 + (1,1.5,1.4,0.75,2)\ x_2^{-1}x_3^2 \leq 1,$$
$$x_1, x_2, x_3 > 0$$

According to model given in (21a) – (21h), the above problem can be transformed to its corresponding KKT reformulations as:

First; $f^L = \min_{x_1,x_2,x_3} \left(15 x_1^{-1}x_2^{-1}x_3^{-1} + 5 x_1^{-2} x_3 + 2x_2^{-2}\right)$

$s.t.$

$$-x_1^{-2}x_2^{-1}+\lambda_1(6x_2)=0$$
$$\left(-x_1^{-1}x_2^{-2}+15x_3\right)+\lambda_1\left(6x_1-x_2^{-2}x_3^{2}\right)+\lambda_2\left(-15x_1^{-1}x_2^{-2}x_3^{-1}-4x_2^{-3}\right)=0$$
$$\left(15x_2-10x_3^{-2}\right)+\lambda_1\left(2x_2^{-1}x_3\right)+\lambda_2\left(-15x_1^{-1}x_2^{-1}x_3^{-2}+5x_1^{-2}\right)=0$$
$$6x_1x_2+x_2^{-1}x_3^{2}\le 1$$
$$\lambda_1\left(6x_1x_2+x_2^{-1}x_3^{2}-1\right)=0$$
$$x_1,x_2,x_3>0 \quad , \quad \lambda_1\ge 0 \text{ and } \lambda_2\ge 0$$

Optimal value of above KKT problem is $f^{L}=423.2721$ for

$x_1=0.1348523$, $x_2=0.7583583$ and $x_3=0.5413233.$
$\lambda_1=15.93613$ *and* $\lambda_2=0$

Second; $f^{U}=\min\limits_{x_1,x_2,x_3}\left(20x_1^{-1}x_2^{-1}x_3^{-1}+10x_1^{-2}x_3+4x_2^{-2}\right)$

s.t.

$$-2x_1^{-2}x_2^{-1}+\lambda(8x_2)=0$$
$$\left(-2x_1^{-1}x_2^{-2}+30x_3\right)+\lambda_1\left(8x_1-2x_2^{-2}x_3^{2}\right)+\lambda_2\left(-20x_1^{-1}x_2^{-2}x_3^{-1}-8x_2^{-3}\right)=0$$
$$\left(20x_2-12x_3^{-2}\right)+\lambda_1\left(4x_2^{-1}x_3\right)+\lambda_2\left(-20x_1^{-1}x_2^{-1}x_3^{-2}+10x_1^{-2}\right)=0$$
$$8x_1x_2+2x_2^{-1}x_3^{2}\le 1$$
$$\lambda_1\left(8x_1x_2+2x_2^{-1}x_3^{2}-1\right)=0$$
$$x_1,x_2,x_3>0 \quad , \quad \lambda_1\ge 0 \text{ and } \lambda_2\ge 0$$

Optimal value of above KKT problem is $f^{U}=1275.421$ for

$x_1=0.07572633, x_2=1.159941$ *and* $x_3=0.4152380.$

Also $\lambda_1=32.4022$ *and* $\lambda_2=0$

Third; $f^{A.M.}=\min\limits_{x_1,x_2,x_3}\left(17.5x_1^{-1}x_2^{-1}x_3^{-1}+7.5x_1^{-2}x_3+3x_2^{-2}\right)$

s.t.

$$-1.5x_1^{-2}x_2^{-1}+\lambda_1(7x_2)=0$$
$$\left(-1.5x_1^{-1}x_2^{-2}+17.5x_3\right)+\lambda_1\left(7x_1-1.5x_2^{-2}x_3^{2}\right)+\lambda_2\left(-17.5x_1^{-1}x_2^{-2}x_3^{-1}-6x_2^{-3}\right)=0$$
$$\left(17.5x_2-11x_3^{-2}\right)+\lambda_1\left(3x_2^{-1}x_3\right)+\lambda_2\left(-17.5x_1^{-1}x_2^{-1}x_3^{-2}+7.5x_1^{-2}\right)=0$$
$$7x_1x_2+1.5x_2^{-1}x_3^{2}\le 1$$
$$\lambda_1\left(7x_1x_2+1.5x_2^{-1}x_3^{2}-1\right)=0$$
$$x_1,x_2,x_3>0 \quad , \quad \lambda_1\ge 0 \text{ and } \lambda_2\ge 0$$

Optimal value of above KKT problem is $f^{A.M.} = 761.1063$ for

$x_1 = 0.09808134$, $x_2 = 0.9705042$ and $x_3 = 0.4646420.$

Also, $\lambda_1 = 23.64956$ and $\lambda_2 = 0$

Fourth; $f^{G.M.} = \min_{x_1,x_2,x_3} \left(17.3x_1^{-1}x_2^{-1}x_3^{-1} + 7.1x_1^{-2}x_3 + 2.8x_2^{-2}\right)$

s.t.

$$-1.4x_1^{-2}x_2^{-1} + \lambda_1(6.9x_2) = 0$$
$$\left(-1.4x_1^{-1}x_2^{-2} + 17.3x_3\right) + \lambda_1\left(6.9x_1 - 1.4x_2^{-2}x_3^2\right) + \lambda_2\left(-17.3x_1^{-1}x_2^{-2}x_3^{-1} - 5.6x_2^{-3}\right) = 0$$
$$\left(17.3x_2 - 10.95x_3^{-2}\right) + \lambda_1\left(2.8x_2^{-1}x_3\right) + \lambda_2\left(-17.3x_1^{-1}x_2^{-1}x_3^{-2} + 7.1x_1^{-2}\right) = 0$$
$$6.9x_1x_2 + 1.4x_2^{-1}x_3^2 \le 1$$
$$\lambda_1\left(6.9x_1x_2 + 1.4x_2^{-1}x_3^2 - 1\right) = 0$$
$$x_1, x_2, x_3 > 0 \quad , \quad \lambda_1 \ge 0 \textit{ and } \lambda_2 \ge 0$$

Optimal value of above KKT problem is $f^{G.M.} = 705.6133$ for

$x_1 = 0.1026093$, $x_2 = 0.9285441$ and $x_3 = 0.4766750.$

Also, $\lambda_1 = 22.35115$ and $\lambda_2 = 0$

Fifth; $f^{H.M.} = \min_{x_1,x_2,x_3} \left(0.6x_1^{-1}x_2^{-1}x_3^{-1} + 0.15x_1^{-2}x_3 + 0.38x_2^{-2}\right)$

s.t.

$$-0.75x_1^{-2}x_2^{-1} + \lambda_1(0.15x_1) = 0$$
$$\left(-0.75x_1^{-1}x_2^{-2} + 0.6x_3\right) + \lambda_1\left(0.15x_1 - 0.75x_2^{-2}x_3^2\right) + \lambda_2\left(-0.6x_1^{-1}x_2^{-2}x_3^{-1} - 0.76x_2^{-2}\right) = 0$$
$$\left(0.6x_2 - 0.09x_3^{-2}\right) + \lambda_1\left(1.5x_2^{-1}x_3\right) + \lambda_2\left(-0.6x_1^{-1}x_2^{-1}x_3^{-2} + 0.15x_1^{-2}\right) = 0$$
$$0.15x_1x_2 + 0.75x_2^{-1}x_3^2 \le 1$$
$$\lambda_1\left(0.15x_1x_2 + 0.75x_2^{-1}x_3^2 - 1\right) = 0$$
$$x_1, x_2, x_3 > 0 \quad , \quad \lambda_1 \ge 0 \text{ and } \lambda_2 \ge 0$$

Similarly optimal value of above problem is $f^{H.M.} = 0.2849358$ for

$x_1 = 1.732673, x_2 = 3.730051$ and $x_3 = 0.3898307.$

Also, $\lambda_1 = 0.1197036$ and $\lambda_2 = 2.967498$

From above discussion we observe that the value of the objective function using KKT approach in interval of parameters for A.M., G.M. and H.M. preserve the same relationship. That is the values so obtained for the objective functions are in the form A.M.> G.M.> H.M. in between the values of the function in lower bound and upper bound of the interval.

6 Conclusion

In many real world geometric programming (GP) problem, the parameters may not be known precisely which leads to the formulation of mathematical programming problem with multiple parameters.

This paper presents a mathematical formulation for the bi – level geometric programming (BLGP) problem that is applicable for the needs of engineering design problems with an interval coefficients as multiple parameters.

In this paper the interval of objective function as well as constraints coefficient is considered such as A.M., G. M. and H.M. of the end points of certain interval in BLGP as multiple parameters for finding the optimal solution of objective functions. The idea is to find the upper bound and lower bound of the objective function and constraints within the interval of parameters as well as to compute the optimal values of the objectives at the indicated points such as A.M., G.M., H.M. of the interval. In this paper the technique of karush – kuhn – Tucker (KKT) is extended for solving BLGP problems and convert it into single objective for searching an optimal solution. KKT approach will be able to calculate the bounds of objective value for the problems where the objective function and the constraint coefficient are interval parameters. Also the optimal objective value should lie in an interval and preserves the same relation. At the end we acquired the derived result in range and they are in order A.M. > G.M. > H.M. Finally, it is hoped that the approach presented will open up many new vistas of research on BLGP for its actual implementation to the real world decision problem and also it can be applied in multi-level programming.

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