SCITECH
RESEARCH ORGANISATION|

Volume 6, Issue 2
November 29, 2017

Journal of Information Sciences and Computing Technologies
www.scitecresearch.com/journals

Pilot Aided Transmissions Technique to Achieve Optimal Effective Capacity Over Imperfect Channel Estimation in Cognitive Radio Networks

Mohamed Elalem
Elmergib University

Abstract—In cognitive radio networks, a secondary user (SU) can share the same frequency band with the primary user (PU) as long as the interference introduced to the later is below a predefined threshold. In this paper, the transmission performance in cognitive radio networks is studied assuming imperfect channel estimation, and taking quality of service (*QoS*) constraints into consideration. It is assumed that the cognitive transmitter can perform channel estimation and send the data at two different rates and power levels depending on the activity of the primary users. The existence of the primary user can be detected by channel sensing. A two-state Markov chain process is used to model the existence of the primary users. The cognitive transmission is also configured as a state transition model depending on whether the rates are higher or lower than the instantaneous rates values. This paper studies the maximum capacity of the cognitive user under the delay constraint. We use the new metric concept of effective capacity of the channel and introduce an optimization problem for rate and power allocation under interference power constraints. An numerical example illustrates the average effective capacity optimization and the impact of other system parameters.

I. INTRODUCTION

Recent researches in spectrum-sharing techniques have enabled different wireless communications technologies to coexist and cooperate towards achieving a better gain from the limited spectrum resources. This started when spectrum utilization measurements showed that most of the allocated spectrum experiences low utilization [1]. Certain administrative authorities, as Federal Communications Commission (FCC) and National Telecommunications and Information Administration (NTIA) for radio spectrum regulation divide the radio spectrum into many frequency bands, and licenses for the often exclusive usage of these bands are provided to operators, typically for a long time such as one or two decades. Depending on the type of radio service that is then provided by the licensees, frequency bands are often idle in many areas, and inefficiently used. The concept of spectrum sharing (the coexisting of different radio systems in the same spectrum) then occurred[2], as one device may transmit, while others in the area are idle. Moreover, radio systems can dynamically use and release spectrum wherever and whenever they are available ("spectrum agile radios"). This dynamic spectrum access by spectral agile radios helps to minimize unused spectral bands ("white spaces"). There are two different cognitive radio strategies[3]. When the secondary users can use the primary user's band only if not currently used by the owner (PU), the scheme is known as overlay. The existence of the primary user can be obtained through spectrum sensing. In the second approach, the secondary users are allowed to avail the band even with the primary user existence, but should control their interference powers to a tolerable threshold to not harm the primary users. This scheme is known as underlay. This paper adapts the second which, obviously, provides more spectrum efficacy [3, 4]. The main challenge for the cognitive user is to control their interference levels not to exceed the limit where it may introduce harmful to the primary user. For this reason, interference should be carefully controlled under the assumption of imperfect channel estimation and under the probabilities of getting false alarms and/or miss detections in channel sensing process. The cognitive user should also guarantee its own quality of service requirements by transmitting at certain power for desired rates and by limiting the delay encountered by the transmission in the buffers[5]. 2 Wireless channel conditions vary over time due to changing environment and mobility. The channel fading coefficients are possible to be estimated imperfectly through training techniques, which is critical for the successful deployment of cognitive radio systems in practice. In addition to channel estimation, activities of primary users should be detected through channel sensing. Hence, more challenging scenario may face the developers. There are certain interdependencies between these tasks of channel estimation and sensing. Mistake in channel sensing may lead to errors in the estimation of the channel coefficients, if the primary users are in the network but not detected, the channel estimate may be worse. Studying the transmission performance of cognitive radio in a practical scenario in which SUs perform channel sensing, channel estimation, and operate under *QoS* requirements is the main motivations for the recent researchers. Some early researches in the channel estimation was studied by an analytical approach to the design of pilot-assisted techniques [6, 7]. *Pilot-Assisted Transmission* (PAT) in which known training symbol(s) is multiplexed with the data symbols, may be used to estimate the channel state and to adapt the receiver parameters accordingly [8–10].

For practical wireless networks, consideration the delay *QoS* requirement deterministically is unrealistic because of the time-varying feature of wireless channels. Discussing the statistical case for delay-*QoS* is become guarantees.

Effective capacity is an effective technique in evaluating the capability of a time-varying wireless channel to support data transmissions. The concept of effective capacity has been introduced to supporting *QoS* requirements[11]. The effective capacity is the dual concept of the effective bandwidth, and can be defined as the maximum constant arrival rate that can be supported by the time-varying service process where the delay *QoS* requirement of the system is satisfied[5,12]. The authors in [13] studied the effective capacity of cognitive radio network in the existence of statistical *QoS* constraints assuming the availability of perfect channel side information at the two cognitive radio sides. In this paper, we investigate the concept of the effective capacity in cognitive radio channels, and identify the performance limits under imperfect channel estimation and quality of service constraints. The cognitive radio initially perform channel sensing, then the channel fading coefficients are estimated in the training phase of the transmission. Finally, data transmission is performed. The activity of primary users is modeled by a Markov process. By this work, we jointly evaluate and optimize the training symbol and data symbol powers and transmission rates of the cognitive users. The rest of the paper is organized as follows. In Section II, The cognitive channel model is given. Section III formulates expressions for the probabilities of primary user activities by discussing the channel sensing phase. Section IV discusses the channel training with pilot symbols and derives the MMSE channel estimation technique. In Sections V and VI, data transmission phase and its performance is studied, and a state transition model for cognitive radio transmission is introduced. In Section VI, we formally define the effective capacity in terms of QoS constraints and identify the optimum throughput that the SU can achieve. We provide numerical results in Section VII, and conclude this paper in Section VIII.

II. CHANNEL MODEL AND ASSUMPTIONS

Fig. 1 depicts the proposed frame model for the cognitive transmission. Initially, the secondary user performs channel sensing which lasts m seconds of a frame of total duration T seconds. We assume that pilot symbols are employed in the system to facilitate the sensing of channel fading coefficients. This will make the receiver able to track the time varying channel. Since the MMSE estimate depends only on the training energy and not on the training duration [14], it can be claimed that transmission of a single pilot at every T seconds is optimal[14, 15]. Instead of increasing number of pilot symbols, a single symbol with relatively high power is used as a pilot, with this, a decrease in the duration of the data transmission can be avoided. Consequently, it is assumed that the transmission is over time-selective flat fading channel in which fading remains constant in each frame. Transmitting one pilot symbol is enough in each frame. Both powers of pilot and data symbols, and transmission rates are related to the channel sensing results. Let S_b and r_b be the average transmission power and rate if the primary user is detected as busy, respectively, While, they are S_d and r_d , if the channel is detected

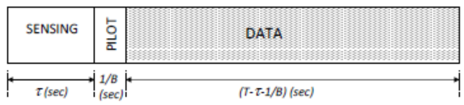

Fig. 1. Transmission frame model consisting of channel sensing, a single symbol as a pilot, and data transmission.

as idle. The secondary transmitter should terminate the transmission, *i.e.*, $S_b = 0$, when the detection process senses the existence of the primary users. The input-output relation between the cognitive transmitter and receiver in the i^{th} symbol duration can be expressed as

$$y_i = \begin{cases} h_i x_{1i} + n_i, & \text{if PU is inactive,} \\ h_i x_{2i} + n_i + \zeta_i, & \text{if PU is active} \end{cases} \quad (1)$$

where x_{1i} and x_{2i} is the secondary transmitted signal when the channel is idle and busy respectively. y_i denotes the channel output signal, h_i represents the fading coefficient between the cognitive transmitter and receiver, modeled as zero-mean Gaussian distributed with variance σ_h^2, $\{n_i\}$ is random noise samples at the cognitive receiver, that are zero-mean Gaussian distributed with variance σ_n^2 for all i. The term ζ_i represents the sum of active primary users' signals received at the cognitive receiver with a variance of σ_ζ^2.

III. Spectrum Sensing Based on Energy Detection

Among different spectrum sensing schemes [16] for reliably identifying the spectrum holes, E*nergy Detection* incurs a very low implementation cost and is hence widely used[16]. It has a good resistance against fast time varying radio environment where none a priori knowledge about the primary users is available (non-coherent detector). In order to identify the presence of primary users with unknown frequency locations, energy detector serves as the optimal sensing scheme since they only need to measure the power of the received signal[16, 17].

We first present the spectrum sensing model. The spectrum occupation status can be modeled the following two hypotheses: Spectrum sensing is to decide between the following two hypotheses:

$$\mathcal{H}_0: \quad z_i = n_i \quad i = 1, 2 \cdots mB,$$

$$\mathcal{H}_1: \quad z_i = n_i + \zeta_i \quad i = 1, 2 \cdots mB.$$

Since the bandwidth is B, we have mB symbols in a duration of m seconds. By the assumption that $\{\zeta_i\}$ signal samples are $i.i.d.$, the optimal detector response for this hypothesis problem is given in [18] by

$$\mathcal{Z} = \frac{1}{mB} \sum_{i=1}^{mB} |z_i|^2 \lessgtr_{\mathcal{H}_1}^{\mathcal{H}_0} \delta, \quad (2)$$

where δ is a pre-designed threshold. The cognitive radio assumes that the primary system is in operation if $\mathcal{Z} \geq \delta$ *i.e.*, $\mathcal{H}_1$. Otherwise, it assumes $\mathcal{H}_0$. Assuming mB is sufficiently high, $\mathcal{Z}$ can be approximated, using Central Limit Theorem, as a Gaussian random variable with mean and variance

$$\mathbb{E}[\mathcal{Z}] = \begin{cases} \sigma_n^2 & \text{if PUs are inactive} \\ \sigma_\zeta^2 + \sigma_n^2 & \text{if PUs are active,} \end{cases} \quad (3)$$

and

$$\sigma_{\mathcal{Z}}^2 = \begin{cases} \sigma_n^4/(mB) & \text{if PUs are inactive} \\ (2(\sigma_\zeta^4 + \sigma_n^4) - (\sigma_\zeta^2 - \sigma_n^2)^2)/(mB) & \text{if PUs are active,} \end{cases} \quad (4)$$

respectively. The probabilities of detection, false alarm and missing of energy detection (The miss detection occurs when the primary is in operation but the cognitive radio fails to sense it) are given as follows [19]

$$P_d = Pr\{\mathcal{Z} > \delta|\mathcal{H}_1\} = Q\left(\frac{\delta - \sigma_\zeta^2 - \sigma_n^2}{\sqrt{2(\sigma_\zeta^4 + \sigma_n^4) - (\sigma_\zeta^2 - \sigma_n^2)^2/(mB)}}\right), \quad (5)$$

$$P_f = Pr\{\mathcal{Z} > \delta|\mathcal{H}_0\} = Q\left(\frac{\delta - \sigma_n^2}{\sqrt{\sigma_n^4/(mB)}}\right), \quad (6)$$

$$P_m = Pr\{\mathcal{H}_0|\mathcal{H}_1\} = 1 - P_d, \quad (7)$$

where $Q(\cdot)$ represents the complementary distribution function of the standard Gaussian[20].

Regarding the channel sensing result, the cognitive radio network has the four cases listed bellow:

1) Correct detection: with two possible cases
   - Channel is busy, detected as busy, (BB).
   - Channel is idle, detected as idle, (DD).
2) Miss detection: channel is busy, detected as idle(BD).
3) False alarm: Channel is idle, detected as busy (DB).

IV. PILOT POWER ANALYSIS

In Pilot Aided Transmission (PAT), a known symbol is embedded in the data transmitted stream to facilitate the receiver to estimate the channel fading coefficients[21]. The cognitive transmitter sends one pilot symbol after the processing of channel sensing to make the receiver able to estimate the channel coefficients. Obviously, this estimation will be affected by channel sensing results.

As mentioned, by the assumption of constant frame fading, one pilot symbol is adequate to provide estimations. The first m seconds of a frame duration T is reserved for sensing process, while one sending a single pilot is optimal[15] because instead of increasing the number of pilot symbols, a single pilot with relatively high power can be used. This increases the duration of the data transmission. After channel sensing and pilot symbol transmission phases, the rest $(T-m)B-1$ symbols are remaining for data transmitting. The average input power in each frame can be written as

$$S_l = \sum_{i=mB+1}^{TB} \mathbb{E}\left[\,|x_{li}|^2\right]; \quad i=0,1,\cdots, \quad l=1,2. \tag{8}$$

where x_{li} is defined in (1). Thereby, the total power assigned to the pilot and data symbols, in a frame is limited by S_b when the channel is busy, or by S_d when the channel is idle. For the possible two cases mentioned above in which the channel is busy, the cognitive transmitter transmits with an average power S_b, while when the primary user is transmitting but suffering form interference due to the transmission of the cognitive users, *i.e.*, BD case, the cognitive transmitter transmits with an average power S_d. It is assumed, here, that depending on the capabilities of the transmitters and the energy resources they are equipped with, there exists peak constraints on both average powers, say: S^m, *i.e.*, ($S_b \leq S^m$ and $S_d \leq S^m$).

Additionally, in order to mitigate the average interference and protect the primary users, the following constraint on S_b and S_d must be imposed:

$$P_d S_b + P_m S_d \leq S^m. \tag{9}$$

where P_d and $P_m = (1-P_d)$ is the detection and miss-detection probability defined in (5) and (7) respectively. Now, the average interference experienced by the primary user can be expressed as

$$\begin{aligned}\mathbb{E}\{P_d S_b|h_{cp}|^2 \quad &+ \quad P_m S_d|h_{cp}|^2\} \\ &= \quad (P_d S_b + P_m S_d)\,\mathbb{E}\{|h_{cp}|^2\} \leq I^m\end{aligned} \tag{10}$$

where h_{cp} denotes the fading coefficient between the cognitive transmitter and primary receiver, and I^m is as the average interference constraint. Note that h_{cp} is not known at the cognitive transmitter and hence the cognitive transmitter cannot adapt its transmission according to it. However, if the statistics of this coefficient ($\mathbb{E}\{|h_{cp}|^2$) is known, then in order to satisfy (10), the cognitive transmitter can choose $S^m = \frac{I^m}{\mathbb{E}\{|h_{cp}|^2\}}$.

The pilot symbol power is also related to the sensing result. Let the power of pilot symbol be $S_{pb} = \mu_b S_b$ If the the primary user is detected, while, it is $S_{pd} = \mu_d S_d$ when primary user is not detected. where μ_b and μ_d are fractions of the total power assigned to the pilot symbol when channel is detected as busy and idle, respectively. Since we assume that the fading coefficients $\{h_i\}$ stay constant within each frame, the received signal in the pilot phase inside a certain frame i (*i.e.*, y_{pi}) can be written as

$$y_{pi} = \begin{cases} h_i(S_{pb})^{1/2} + n + \zeta_i & \text{for BB case} \\ h_i(S_{pd})^{1/2} + n_i & \text{for DD case} \\ h_i(S_{pd})^{1/2} + n_i + \zeta_i & \text{for BD case} \\ h_i(S_{pb})^{1/2} + n_i & \text{for DB case,} \end{cases} \tag{11}$$

where $p = (mB+1)i$ is the sample index of the pilot in the frame i.

If the receiver employs minimum mean-square error (MMSE) estimator to obtain the estimate fading coefficients, then the estimated fading coefficients for each scenario can be found using MMSE estimation [15, 22] as follows.

$$\hat{h}_i = \begin{cases} \frac{\sqrt{S_{pb}}\sigma_h^2}{S_{pb}\sigma_h^2+\sigma_n^2+\sigma_\zeta^2} y_{pj} & \text{for BB and DB cases} \\ \frac{\sqrt{S_{pd}}\sigma_h^2}{S_{pd}\sigma_h^2+\sigma_n^2} y_{pj} & \text{for BD and DD cases.} \end{cases} \tag{12}$$

It is essentially to know that the MMSE estimates given above are related to the channel sensing results. See Appendix A for more details. $\hat{h}_i$ in (12) is the estimate channel fading, which is a circularly symmetric, complex, Gaussian random variable with zero mean and variance $\sigma^2_{\hat{h}_i}$, *i.e.*, $\hat{h}_i \sim \mathcal{CN}(0, \sigma^2_{\hat{h}_i})$. It can be expressed as $\hat{h}_i = \sigma_{\hat{h}_i} w$, where w is a standard complex Gaussian random variable, $w \sim \mathcal{CN}(0, 1)$. Thus the fading coefficient can now be expressed as follows[23]

$$\hat{h}_i = h_i + \epsilon_i, \tag{13}$$

where ϵ_i is the estimate error in the i^{th} fading coefficient h_i, and $\epsilon_i \sim \mathcal{CN}(0, \sigma^2_{\epsilon_i})$[6, 23, 24].

Now, the input-output relationship for data phase in (1) can be rewritten as

$$y_i = \begin{cases} (\hat{h}_i - \epsilon_i)x_{1i} + n_i + \zeta_i & \text{if channel is busy,} \\ (\hat{h}_i - \epsilon_i)x_{2i} + n_i & \text{if channel is idle,} \end{cases} \tag{14}$$

The estimation of the channel variance is[21]

$$\sigma^2_{\hat{h}} = \begin{cases} \frac{S_{pb}\sigma^4_h}{S_{pb}\sigma^2_h + \sigma^2_n + \sigma^2_\zeta} & \text{for BB case} \\ \frac{S_{pd}\sigma^4_h}{S_{pd}\sigma^2_h + \sigma^2_n} & \text{for DD case} \\ \frac{S_{pd}\sigma^4_h}{(S_{pd}\sigma^2_h + \sigma^2_n)^2}(S_{pd}\sigma^2_h + \sigma^2_n + \sigma^2_\zeta) & \text{for BD case} \\ \frac{S_{pb}\sigma^4_h}{(S_{pb}\sigma^2_h + \sigma^2_n + \sigma^2_\zeta)^2}(S_{pb}\sigma^2_h + \sigma^2_n) & \text{for DB case,} \end{cases} \tag{15}$$

where S_p may equal to S_{pb} or S_{pd} depending on channel sensing result.

The variance of the estimation error σ^2_ϵ can be written as

$$\sigma^2_\epsilon = \sigma^2_{\hat{h}} - \sigma^2_h$$

, assuming that there is no correlation between the error and its estimation. We have omitted the frame index in above equation because of the block fading assumption, and because of the assumed identicalness property of the fading coefficient and its estimates random variables in each frame.

V. Data Transmission Phase

Finding the capacity of the channel in (14) is not easy task, a lower bound capacity is generally obtained by considering the estimate error ϵ as another source of Gaussian noise, *i.e.*, by considering the term $(n_i - \epsilon_k x_{li})$, $l = 1, 2$, in (14) as Gaussian distributed noise uncorrelated with the input[5].

The channel can be modeled as a two Morkov chain states(*i.e.*, ON and OFF), for the state when target transmission rate is greater than or less than the instantaneous rate that the channel can support, respectively. These two states are possible in each of the four cases discussed above. Hence, totally there are eight states (2×4) as delineated in Fig. 2.

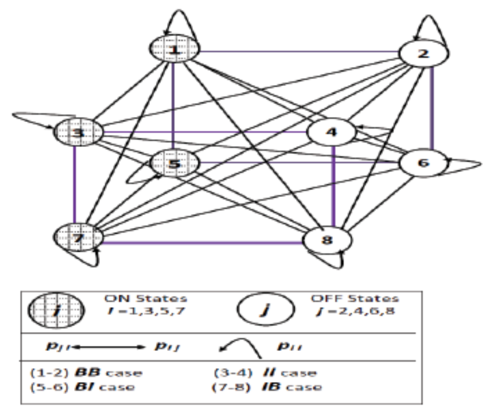

Fig. 2. The 8 states transition model for the cognitive radio channel.

Considering the channel estimation results and interference generated by the primary user ζ, we have the following lower bounds instantaneous channel capacities in the i^{th} frame for the four scenarios described above:

$$C^l_{ki} = C_o \log_2(1 + \eta_{ki}|w_i|^2), \qquad k = 1, 2, 3, 4 \tag{16}$$

where $C_o = \frac{(T-m)B-1}{T}$, and

$$\eta_{ki} = \begin{cases} \frac{S_{db}\sigma^2_{\hat{h}_i}}{S_{db}\sigma^2_{\epsilon_i} + \sigma^2_n + \sigma^2_\zeta} & k = 1\text{ ; BB case} \\ \frac{S_{dd}\sigma^2_{\hat{h}}}{S_{dd}\sigma^2_{\epsilon_i} + \sigma^2_n} & k = 2\text{ ; DD case} \\ \frac{S_{dd}\sigma^2_{\hat{h}}}{S_{dd}\sigma^2_{\epsilon_i} + \sigma^2_n + \sigma^2_\zeta} & k = 3\text{ ; BD case} \\ \frac{S_{db}\sigma^2_{\hat{h}}}{S_{db}\sigma^2_{\epsilon_i} + \sigma^2_n} & k = 4\text{ ; DB case,} \end{cases} \tag{17}$$

C^l_{ki} is the i^{th} frame's lower band capacity of each scenario k, which are obtained by assuming $\epsilon_i x_i$ and ζ_i as worst case noises whereas these noises are considered as Gaussian distributed [14]. S_{db} is the **d**ata symbol power when the channel is detected as **b**usy while S_{dd} is the **d**ata symbol power when the channel is detected as i**d**le. These two powers are related to the cognitive average powers, as $S_{db} = S_b(1 - \mu_b)/TC_o = S_b(1 - \mu_b)/((T - m)B - 1)$, and $S_{dd} = S_d(1 - \mu_d)/((T - m)B - 1)$.

Since $w \sim \mathcal{CN}(0, 1)$, the magnitude $|w|$ will have the Rayleigh distribution and the squared magnitude $|w|^2$ will have Exponential distribution with unity mean.

The transmitter will transmit the information at the desired rate unconcerned with the channel conditions. We assume that the transmitter will send its data at fixed rate r_b if the channel is sensed as busy, and at r_d if it is sensed as idle. If these rates are below the instantaneous capacity values, *i.e.*, when $r_b < C_1^l, C_4^l$ or $r_d < C_2^l, C_3^l$ (the index k is dropped again for convenience), the transmission can be assessed to be in the ON state and, so, the target rates can be achieved. While , if $r_b \geq C_1^l, C_4^l$ or $r_d \geq C_2^l, C_3^l$, the channel is in the OFF state, where reliable communication can not be achieved and outage event occurs.

The activity of the primary user between the frames can be also modeled as a two-state Markov model. **B**usy state indicates that the primary user occupied the channel, and i**D**le state indicates the absent of the primary users in the channel, as can be seen in Fig. 3. Switching from busy state to idle state and from idle state to busy state is with probability b and d, respectively. The state transition is assumed to occur every T seconds.

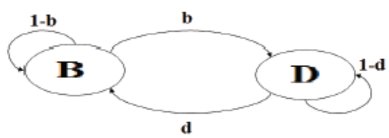

Fig. 3. Primary user activity between two states: **B**usy and i**D**le

Taking into account the four possible cases related to the channel sensing results jointly with the reliability of the transmissions states, the cognitive radio transmission can be represented by state transition model as **P**(8×8) transition matrix. The entries transition probabilities depend on channel coefficients, sensing probabilities, transmission rates, and the two state Markov model declared in Fig. 3

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 & \cdots & p_7 & p_8 \\ p_1 & p_2 & \cdots & p_7 & p_8 \\ \acute{p}_1 & \acute{p}_2 & \cdots & \acute{p}_7 & \acute{p}_8 \\ \acute{p}_1 & \acute{p}_2 & \cdots & \acute{p}_7 & \acute{p}_8 \\ p_1 & p_2 & \cdots & p_7 & p_8 \\ p_1 & p_2 & \cdots & p_7 & p_8 \\ \acute{p}_1 & \acute{p}_2 & \cdots & \acute{p}_7 & \acute{p}_8 \\ \acute{p}_1 & \acute{p}_2 & \cdots & \acute{p}_7 & \acute{p}_8 \end{bmatrix} \quad (18)$$

Table (I) summarizes the entries of the matrix **P**, where

TABLE I
THE TRANSITION PROBABILITIES OF MATRIX **P**

$l = 1, 2, 5, 6$	$n = 3, 4, 7, 8$
$p_{l1} = (1-b)P_d e^{-\lambda_1} = p_1$	$p_{n1} = dP_d e^{-\lambda_1} = \acute{p}_1$
$p_{l2} = (1-b)P_d(1-e^{-\lambda_1}) = p_2$	$p_{n2} = dP_d(1-e^{-\lambda_1}) = \acute{p}_2$
$p_{l3} = b(1-P_f)e^{-\lambda_2} = p_3$	$p_{n3} = (1-d)(1-P_f)e^{-\lambda_2} = \acute{p}_3$
$p_{l4} = b(1-P_f)(1-e^{-\lambda_2}) = p_4$	$p_{n4} = (1-d)(1-P_f)(1-e^{-\lambda_2}) = \acute{p}_4$
$p_{l5} = (1-b)P_m e^{-\lambda_3} = p_5$	$p_{n5} = dP_m e^{-\lambda_3} = \acute{p}_5$
$p_{l6} = (1-b)P_m(1-e^{-\lambda_3}) = p_6$	$p_{n6} = dP_m(1-e^{-\lambda_3}) = \acute{p}_6$
$p_{l7} = bP_f e^{-\lambda_4} = p_7$	$p_{n7} = (1-d)P_f e^{-\lambda_4} = \acute{p}_7$
$p_{l8} = bP_f(1-e^{-\lambda_4}) = p_8$	$p_{n8} = (1-d)P_f(1-e^{-\lambda_4}) = \acute{p}_8$

λ_i in the table is defined as:

$$\lambda_i = \begin{cases} \frac{2^{(r_b/C_o)}-1}{\eta_i}, & \text{i=1,4;} \\ \frac{2^{(r_d/C_o)}-1}{\eta_i}, & \text{i=2,3.} \end{cases} \quad (19)$$

The details are provided in Appendix B.

According to the entries of the matrix **P** displayed in Table (I), the rank of this matrix is 2.

Note that all p.s and $\acute{p}$.s are functions of λ_i which in tern function of all the parameters in (17).

VI. EFFECTIVE CAPACITY OPTIMIZATION FOR COGNITIVE USER

A. Preliminary on Effective Capacity

The Effective Capacity (E_C) (or Effective Bandwidth) theory is a powerful approach to evaluate the capability of a wireless channel to support data transmissions with diverse statistical quality of service (QoS) guarantees[11, 12, 25, 26]. It is defined, as the dual concept of effective bandwidth, as the maximum constant arrival rate that the channel can support while meeting the QoS requirement[12].

In particular, the statistical QoS guarantee can be characterized by a metric called QoS exponent denoted by θ, $0 < \theta < \infty$[11]. The QoS exponent θ characterizes the exponentially decaying rate of the violation probability against the queue-length threshold[12]. With the pair (Effective Capacity E_C and QoS exponent θ), it can be observed that there is tradeoffs between the QoS requirement and the system rate. Higher θ represents more stringent delay QoS requirements, and vice versa.

The delay, which is a QoS measure, can be described through the probability that the occupancy of the buffer is higher than a specific value, say x, so the QoS exponent can be formulated as

$$\theta = -\lim_{x \to \infty} \frac{\log Pr\{L > x\}}{x}, \tag{20}$$

where L is the cognitive queue length and follows the equilibrium queue-length distribution of the buffer at the source [11, 26]. When $\theta \to 0$, the user does not impose any delay constraints on the service process. On the other hand, $\theta \to \infty$ implies that any delay is not tolerable, and thus the effective capacity reduces to the minimum supportable service rate.

From (20), for large x,(x^m), the buffer violation probability can be approximated as

$$Pr\{L > x^m\} \approx \exp(-x^m \theta). \tag{21}$$

Smaller θ corresponds to looser constraints, and larger θ implies more strict QoS constraints.

B. Framework for Effective Capacity Optimization

The goal here is to analyze the maximum capacity that the cognitive radio channel of the given state transition model of Fig. 2 can sustain under constraints specified by the QoS exponent of the connection imposed in the form of delay violation probabilities.

The effective capacity for a given θ is defined in [11, 27] as

$$E_c = -\lim_{t \to \infty} \frac{1}{\theta t} \log \mathbb{E}(\exp(-\theta R(t))) \tag{22}$$

where $R(t) = \sum_{i=1}^{t} r(i)$ is the time-accumulated service process. $r(i)$ is discrete time stationary and ergodic stochastic service process. $\mathbb{E}(.)$ is the expectation operator with respect to the random variable r.

It can be noticed that the service rate is $r(i) = r_b T$ if the cognitive user is in state ON_1 or ON_7 at time i (the subscribe points to the state number). Similarly, the service rate is $r(i) = r_d T$ in states ON_3 and ON_5. In the remaining states $(OFF_j, j = 2, 4, 6, 8)$, the target transmission rates is greater than the instantaneous channel capacities and, so, communication can not be achieved. This leads to vanish all the service rates in these four even states.

Equation (22) can be solved using the technique given in [25] as follows

$$E_c = \frac{1}{\theta} \log \rho(\mathbf{M}) = \frac{1}{\theta} \log \rho(\mathbf{D.P}), \tag{23}$$

where $\rho(\mathbf{M})$ function is the *spectral radius* of the matrix $\mathbf{M}$, $\mathbf{D} = diag(d_1(\theta), \cdots, d_N(\theta))$ is a diagonal matrix with elements equal to the moment generating functions of the processes in N states [25] (here, we have 8 states). *Spectral radius of a matrix is the maximum of the absolute values of its eigenvalues, i.e.,* $\rho(A) \stackrel{def}{=} \max_i(|\omega_i|)$, ω_i's are the eigenvalues[28]. $\mathbf{P}$ in (23) is the transition matrix given in (18). Note that, in our assumptions, the transmission rates are deterministic and constants in each state, thus, the possible rates are: Tr_b, Tr_d, and 0 for which the moment generating functions $e^{T\theta r_b}, e^{T\theta r_d}$ and 1 respectively. Therefore, $\mathbf{D} = diag(e^{T\theta r_b}, 1, e^{T\theta r_d}, 1, e^{T\theta r_d}, 1, e^{T\theta r_b}, 1)$. So the matrix $\mathbf{M}$ can be filled:

$$\mathbf{M} = \mathbf{D.P} = \begin{bmatrix} \tau_b p_1 & \tau_b p_2 & \cdots & \tau_b p_7 & \tau_b p_8 \\ p_1 & p_2 & \cdots & p_7 & p_8 \\ \tau_d \acute{p}_1 & \tau_d \acute{p}_2 & \cdots & \tau_d \acute{p}_7 & \tau_d \acute{p}_8 \\ \acute{p}_1 & \acute{p}_2 & \cdots & \acute{p}_7 & \acute{p}_8 \\ \tau_d p_1 & \tau_d p_2 & \cdots & \tau_d p_7 & \tau_d p_8 \\ p_1 & p_2 & \cdots & p_7 & p_8 \\ \tau_b \acute{p}_1 & \tau_b \acute{p}_2 & \cdots & \tau_b \acute{p}_7 & \tau_b \acute{p}_8 \\ \acute{p}_1 & \acute{p}_2 & \cdots & \acute{p}_7 & \acute{p}_8 \end{bmatrix}, \tag{24}$$

where $\tau_b = e^{T\theta r_b}$, and $\tau_d = e^{T\theta r_d}$ are moment generating functions of the possible rates when the channel is busy and idle respectively.

It is easy to note that the matrix $\mathbf{M}$ has also a rank of 2. The characteristic polynomial of the matrix shorten to:

$$Q(\omega) = \omega^2 - C_7 \omega + C_6, \tag{25}$$

where the nonzero-eigenvalues ω can be found by just solving the quadratic equation (25). See Appendix C for details.

The effective capacity in (22) can be optimized by choosing the maximum values of r_b and r_d over the optimized power allocation constraints. This maximization is firstly done by choosing the maximum value of the eigenvalue of the matrix ($\mathbf{D.P}$) which maximize the function $\rho(\mathbf{M})$ in (23), then another optimization should be done over the entire variables which leads to the optimal effective capacity formula.

$$
\begin{aligned}
E_c^{opt} &= \max -\frac{1}{\theta T B}\log\Bigg\{\frac{1}{2}\Big(\tau_b(p_1+\acute{p}_7)+\tau_d(p_5+\acute{p}_3)+p_2+p_6+\acute{p}_4+\acute{p}_8\Big)+\frac{1}{2}\Bigg(\Big(\tau_b(p_1+\acute{p}_7)+\tau_d(p_5+\acute{p}_3)\\
&+p_2+p_6+\acute{p}_4+\acute{p}_8\Big)^2-4\Big(\tau_b^2(\acute{p}_7\,p_1-\acute{p}_1\,p_7)+\tau_d^2(p_5\,\acute{p}_3-p_3\,\acute{p}_5)+\tau_b(\acute{p}_7\,p_2+\acute{p}_7\,p_6-\acute{p}_1\,p_4-\acute{p}_2\,p_7\\
&-\acute{p}_5\,p_7-\acute{p}_6\,p_7-\acute{p}_1\,p_8+\acute{p}_8\,p_1)+\tau_d(\acute{p}_3\,p_2-\acute{p}_2\,p_3-p_4\,\acute{p}_5+p_5\,\acute{p}_4-p_3\,\acute{p}_6+p_6\,\acute{p}_3-\acute{p}_5\,p_8+\acute{p}_8\,p_5)\\
&+\tau_b\tau_d(\acute{p}_3\,p_1+\acute{p}_7\,p_5-\acute{p}_1\,p_3)-\acute{p}_2\,p_4+\acute{p}_4\,p_2+\acute{p}_4\,p_1-p_4\,\acute{p}_6+p_6\,\acute{p}_4-\acute{p}_2\,p_8-\acute{p}_6\,p_8+\acute{p}_8\,p_6+\acute{p}_8\,p_2\Big)\Bigg)^{\frac{1}{2}}\Bigg\}\\
S.t.\quad & 0 < S_b, S_d \le S^m\\
& 0 < \mu_b, \mu_d \le 1\\
& r_b, r_d \ge 0\\
& P_d S_b + P_m S_d \le S^m\\
& (P_d S_b + P_m S_d)\mathbb{E}(|h_{cp}|^2) \le I^m
\end{aligned} \tag{26}
$$

The effective capacity expression in (26) is obtained by choosing the largest value of the eigenvalues of the matrix $\mathbf{M}$ for a given sensing duration m, detection threshold δ, and QoS exponent θ. One can note that If the sensing results are perfect with no errors, *i.e.*, the detection probability $P_d = 1$, and so ($P_m = P_f = 0$), the transition probabilities in matrix $\mathbf{P}$, $p_5 = p_6 = p_7 = p_8 = \acute{p}_5 = \acute{p}_6 = \acute{p}_7 = \acute{p}_8 = 0$ in the effective capacity expression in (26). As declared in [29], problem (26) is always convex, and an analytical optimized solution is possible whenever the generating function has an analytical expression[29, 30]. In the following Section, we investigate the impact of several parameters on the effective capacity through numerical example, postponing the analytical solution to a future work.

VII. Numerical Example

In this section, we numerically illustrate the impact of the sensing duration m, detection threshold δ, and other factors on the effective capacity. We set all variances to unity($\sigma_h = \sigma_n = \sigma_\zeta = 1$), we also assume the symbol rate $B = 10000 symbol/sec$, and the frame duration $T = 0.25s$, this means that we have 2500 symbol in the frame. Unless they are not variable, time allocated for sensing is set to $5ms$, and QoS exponent θ is assumed to be 10%. The maximum average power constraint $S_m = 20dB$. The fraction assigned to the pilot symbol is 10% either the channel is busy or idel (*i.e.*, $\mu_b = \mu_d = 0.1$). Finally, to simplify the objective function of the effective capacity, we assume that the transition probabilities of the two-state Markov model in Fig. 3, b & d, such that $b = 1 - d$, where $b = 0.8$. We further assume that in each frame, primary user activity does not change, while it may change independently from one state to another across the frames.

In Fig. 4, the normalized effective capacity is plotted versus the delay QoS exponent (θ) for various interference-limit values. We observe that the capacity increases as θ decreases. However, the gain in the effective capacity decreases for higher values of θ. The figure shows that in the case with loose QoS restrictions, *i.e.*, lower values of θ, the capacity benefits significantly, whereas in the case with higher values of θ, *i.e.*, $\theta = 10(1/bit)$, about 70% reduction in the capacity is noticed.

Fig. 5 dedicates the effective capacity of the cognitive user versus the interference limit that the primary user can tolerate, I^m for various QoS exponent values. The figure reveals that the capacity gain that can be achieved under strict peak interference constraint is much lower than the one under released interference constraint. Also, as the conclusion in the previous figure, for specific I^m value, the capacity increases as θ becomes lower which means loose QoS restrictions.

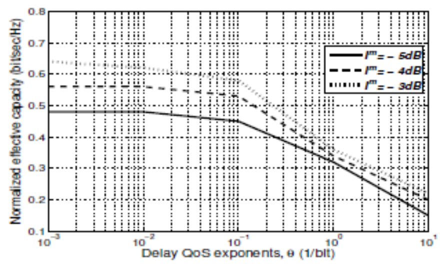

Fig. 4. Normalized effective capacity versus delay QoS exponents, θ, for various interference-limit.

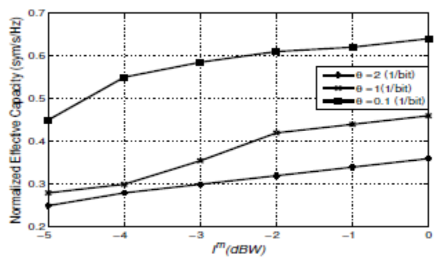

Fig. 5. Normalized effective capacity versus Interference-limit for various QoS exponent values, θ.

Fig. 6 studies the effect of the channel sensing duration (m). It can be seen that for short time reserved for sensing process, the cognitive user is more probable to get false alarm to detect the primary user, whereas the detection probability is certainty for long sensing duration.

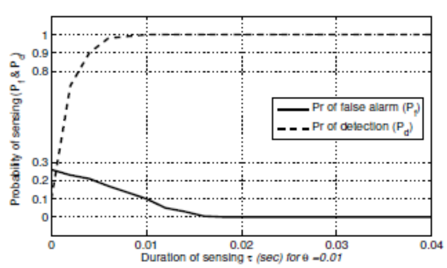

Fig. 6. Probabilities of sensing $(P_d \& P_f)$ versus Channel sensing duration.

In Fig. 7, we display the effective capacity (normalized value) as a function of the detection probability, for different values of S_m. As expected, with increasing S_m, the effective capacity value increases. It can be seen from the figure that the maximum effective capacity points are achieved at P_d is close to 0.9. As P_d further increases and approaches 1, the cognitive users start to consider the channel as busy all the time and hence the performance degradation is occurred, this is because of not being able to take advantage of idle channel states. The impact of the average power on the capacity is also clear in the figure, more average power means more relax constraints, which leads to more capacity.

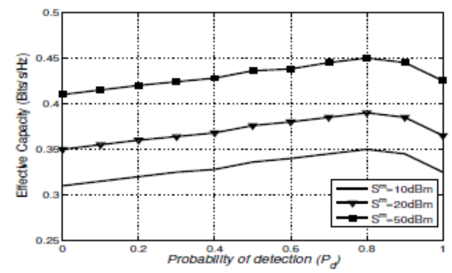

Fig. 7. Effective capacity versus probability of detection P_d for different values of S_m.

In Fig.8, we plot the effective capacity as a function of the energy detection threshold value δ for two different sensing durations m. At the same axis, we compare the probabilities of detection and false alarm in Fig. 9. First,

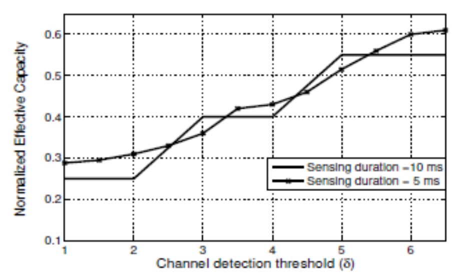

Fig. 8. Effective Capacity versus detection threshold δ.

Fig. 7 shows that the effective capacity is increasing with increasing the detection δ threshold. However, at the same time, as δ increases, the probability of false alarm and the probability of detection are getting smaller as can be seen in Fig 9. For instance, when $\delta \simeq 2.5$, P_f starts diminishing, which in turn increases the effective capacity values significantly. If δ is increased beyond 3, we observe that P_d starts decreasing, causing increasing disturbance to the primary users. The secondary user assumes that the channel is idle in the case of miss detection and transmits at a higher power level, this leads to increase in the effective capacity. This increase occurs at the cost of increased interference to the primary users. Furthermore, we can note that having a larger m decreases the effective capacity values outside the range of δ values at which transitions in the false alarm and detection probabilities occur. This can be interpreted as m increases, $(T-m)$, time available for data transmission, reduces.

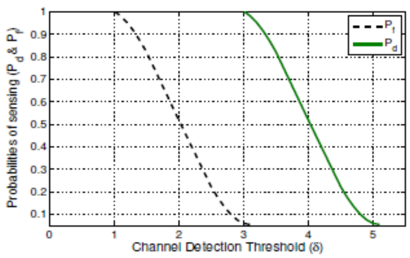

Fig. 9. Probabilities of sensing ($P_d \& P_f$) versus detection threshold δ .

VIII. Conclusion

In this paper, we have analyzed the effective capacity of cognitive radio channels taking into account QoS constraints, imperfect channel information, and transmission power limitations. First, a system model is introduced in which the cognitive transmitter initially senses the channel in order to detect the activity of the primary users. It then sends a pilot symbol for channel estimation followed by data transmission. An energy detector is adopted to perform channel sensing, which incurs a very low implementation cost and is widely used. The estimation of the channel fading coefficients is performed via the aid of a pilot transmission in the training phase. The minimum mean square error estimator $MMSE$ is assumed to be employed at the receiver. Through the study, the interrelation between channel sensing and estimation has been investigated. We have observe that degradation in the channel estimation is a result of faulty sensing. The cognitive transmitter is assumed to transmit data at fixed powers and rates according to the channel sensing results. For the cognitive user, we have constructed a state-transition model taking into our account the reliability of the transmission, channel sensing results, and the primary user activity in the channel. We have formulated the transition probabilities for this model. A closed form for the effective capacity is obtained as a function of exponent delay constraint. Through numerical example, we have examined the impact of delay constraint, interference limit, channel sensing duration , threshold, and sensing probabilities on the effective capacity. A lot of insightful observations and investigations are drawn in Section VII.

Appendix A

The signal received by the cognitive receiver in the training phase is

$$y = \begin{cases} \sqrt{S_p}h + n & \text{if PUs are inactive(channel is idle)} \\ \sqrt{S_p}h + n + \zeta & \text{if PUs are active((channel is busy)} \end{cases} \tag{A-1}$$

S_p represents to the pilot's power, which is equal to S_{pb} if sensing result is busy and equal to S_{pd} if sensing result is idle. It is assumed that n and ζ are zero mean independent Gaussian random variables with variances σ_n^2 and σ_ζ^2, respectively. Hence, the overall variance of the noise is σ_n^2 or $\sigma_n^2 + \sigma_\zeta^2$ depending the sensing results. Noticing that the cognitive receiver does not have exact information about the occupancy status of the primary user, but it has predictions via sensing probabilities, the inclusive noise variance, σ^2, is random taking these two values.

Based on this uncertainty, the MMSE estimate can be found as follows:

$$\begin{aligned} \hat{h} = \mathbb{E}[h|y] &= Pr(\sigma^2 = \sigma_n^2 + \sigma_\zeta^2 | y)\mathbb{E}[h|y, \sigma^2 = \sigma_n^2 + \sigma_\zeta^2] \\ &+ Pr(\sigma^2 = \sigma_n^2 | y)\mathbb{E}[h|y, \sigma^2 = \sigma_n^2] \qquad \text{(A-2)} \\ &= Pr(\sigma^2 = \sigma_n^2 + \sigma_\zeta^2 | y)\frac{\sqrt{S_p}\sigma_h^2}{S_p\sigma_h^2 + \sigma_n^2 + \sigma_\zeta^2} y \\ &+ Pr(\sigma^2 = \sigma_n | y)\frac{\sqrt{S_p}\sigma_h^2}{S_p\sigma_h^2 + \sigma_n^2} y \qquad \text{(A-3)} \end{aligned}$$

Here, we use the property of conditional expectation [31]

$$\mathbb{E}[A|B] = \mathbb{E}[\mathbb{E}[A|B, C]|B]$$

Using Bayes' rule the conditional probabilities expressions are as follow

$$\begin{aligned} Pr(\sigma^2 = \sigma_n^2 + \sigma_\zeta^2 | y) &= \frac{Pr(\sigma^2 = \sigma_n^2 + \sigma_\zeta^2) f(y|\sigma^2 = \sigma_n^2 + \sigma_\zeta^2)}{f(y)} \\ Pr(\sigma^2 = \sigma_n^2 | y) &= \frac{Pr(\sigma^2 = \sigma_n^2) f(y|\sigma^2 = \sigma_n^2)}{f(y)} \end{aligned} \tag{A-4}$$

y is conditionally Gaussian distributed with zero mean and variance σ^2 as the relations (A-1) points. the conditional distribution function of y, *i.e.*, $f(y)$ is defined as [32]

$$f(y|\sigma^2 = \sigma_n^2) = \frac{1}{\pi(S_p\sigma_h^2 + \sigma_n^2)} e^{-\frac{|y|^2}{S_p\sigma_h^2+\sigma_n^2}} \quad \text{(A-5)}$$

$$f(y|\sigma^2 = \sigma_n^2 + \sigma_\zeta^2) = \frac{1}{\pi(S_p\sigma_h^2 + \sigma_n^2 + \sigma_\zeta^2)} e^{-\frac{|y|^2}{S_p\sigma_h^2+\sigma_n^2+\sigma_\zeta^2}}, \quad \text{(A-6)}$$

The probability of the noise variance is related to the channel sensing result. Let us assume that the channel is sensed as busy, so, $Pr(\sigma^2 = \sigma_n^2)$ means that there are no primary users in the channel and hence channel is idle. Since our assumption, channel is sensed as busy. Therefore, $Pr(\sigma^2 = \sigma_n^2)$ is equal to the conditional probability $Pr\binom{\text{Ch is}}{\text{idle}}|\binom{\text{Ch is}}{\text{sensed busy}}$.

See the derivations form (A-7) to (A-11).

Appendix B

$p_1 = \Pr\{$the channel is being busy and it is detected as busy and $r_b < C_1^l(k)$ in the k^{th} frame given that the channel is being busy and it is detected as busy and $r_b < C_1^l(k-1)$ in the $(k-1)^{th}$ frame$\}$
According to the chain rule in probability theorem, if there are four events: A_1, A_2, A_3 and A_4, then

$$\begin{aligned} Pr(A_1, A_2, A_3|A_4) &= Pr(A_1 \cap A_2 \cap A_3|A_4) \\ &= Pr(A_1|A_4) \times Pr(A_2|A_1 \cap A_4) \\ &\times Pr(A_3|A_1 \cap A_2 \cap A_4) \end{aligned} \quad \text{(B-1)}$$

So
$p_{11} = \Pr\{$channel is busy in i^{th} frame$|$channel is busy in $(i-1)^{th}\}$
$\times \Pr\{$channel is busy in i^{th} frame $|$ channel is busy in $(i)^{th}\}$
$\times \Pr\{r_b < C_1^l(i)|r_b < C_1^l(i-1)\}$
$p_{11} = (1-b)P_d Pr\{r_b < C_1^l(i)|r_b < C_{i-1}^l(i-1)\}$
$p_{11} = (1-b)P_d Pr\{z_i > \lambda_1|z_{i-1} > \lambda_1\}$
$p_{11} = (1-b)P_d Pr\{z_i > \lambda_1\} = (1-b)P_d Pr\{z > \lambda_1\} = p_1$

We omitted the index i in z_i due to the fact that z_i and z_{i-1} are independent due to the block fading assumption.

By the same manner , the transition probabilities from any state to state 1 can be expressed as

$$\begin{aligned} p_{l1} = p_{11} = p_{21} = p_{31} = p_{41} &= (1-b)P_d Pr\{z > \lambda_1\} \\ &= (1-b)P_d e^{-\lambda_1} = p_1 \\ p_{n1} = p_{51} = p_{61} = p_{71} = p_{81} &= dP_d Pr\{z > \lambda_1\} \\ &= dP_d e^{-\lambda_1} = \acute{p}_1 \end{aligned} \quad \text{(B-2)}$$

Using the same modality, the full transition probabilities can be obtained, and they are listed in Table I

Appendix C

Let A be an $n \times n$ matrix, the eigenvalues of the matrix A are the zeroes of its characteristic polynomial, $\det(\omega I - A)$, which can be written as

$$Q(\omega) = \omega^n - C_{n-1}\omega^{n-1} + C_{n-2}\omega^{n-2} - \cdots (-1)^n C_0 \quad \text{(C-1)}$$

It is well known that the coefficients C_{n-1} and C_0 are, respectively, the $trace(A)$(the sum of its diagonal entries) and the $det(A)$. All other coefficients $C_{n-k}, k = 1, 2, \cdots$, can be expressed by the sum of the $k-$*rowed principle minors* of A. A k-rowed principal minor of an $n \times n$ matrix A is the determinant of a $k \times k$ submatrix of A whose entries, a_{ij}, have indices i and j that are the elements of the same $k-$element subset of $1, 2, \cdots, n$.

With *rank* (the dimension of the largest square submatrix of A with nonzero determinant) r, where, $r < n$. All nonzero eigenvalues of A are among the zeros of the polynomial[33]

$$Q(\omega) = \omega^r - C_{n-1}\omega^{r-1} + \cdots (-1)^r C_{n-r} \quad \text{(C-2)}$$

$$\begin{aligned} \omega^2 \;& -\big(+\acute{p}_8 + \tau_b\acute{p}_7 + p_6 + \tau_d p_5 + \acute{p}_4 + \tau_d\acute{p}_3 + p_2 \\ + \;& \tau_b p_1\big)\omega + \big(\tau_d\acute{p}_3 p_2 + \tau_b\tau_d\acute{p}_3 p_1 + \tau_b\acute{p}_7 p_2 + \tau_b\acute{p}_7 p_6 \\ + \;& \tau_b\tau_d\acute{p}_7 p_5 - \tau_b\acute{p}_1 p_4 - \tau_b\tau_d\acute{p}_1 p_3 - \tau_d\acute{p}_2 p_3 + \tau_b^2\acute{p}_7 p_1 \\ - \;& \acute{p}_2 p_4 + \acute{p}_4 p_2\acute{p}_4 p_1 - \tau_d^2 p_3\acute{p}_5 - \tau_d p_4\acute{p}_5 + \tau_d p_5\acute{p}_4 \\ + \;& \tau_d^2 p_5\acute{p}_3 - \tau_d p_3\acute{p}_6 - p_4\acute{p}_6 + p_6\acute{p}_4 + \tau_d p_6\acute{p}_3 \\ - \;& \tau_b^2\acute{p}_1 p_7 - \tau_b\acute{p}_2 p_7 - \tau_b\acute{p}_5 p_7 - \tau_b\acute{p}_6 p_7 - \tau_b\acute{p}_1 p_8 \\ - \;& \acute{p}_2 p_8 - \tau_d\acute{p}_5 p_8 - \acute{p}_6 p_8 + \acute{p}_8 p_6 + \tau_d\acute{p}_8 p_5 + \acute{p}_8 p_2 \\ + \;& \tau_b\acute{p}_8 p_1\big) = 0 \end{aligned} \quad \text{(C-3)}$$

$$
\begin{aligned}
Pr(\sigma^2=\sigma_n^2) &= Pr\begin{pmatrix}\text{Ch is}\\ \text{idle}\end{pmatrix} | \begin{pmatrix}\text{Ch is}\\ \text{sensed busy}\end{pmatrix} = \frac{Pr\binom{\text{Ch is}}{\text{idle}} Pr\left(\binom{\text{Ch is}}{\text{sensed busy}} | \binom{\text{Ch is}}{\text{idle}}\right)}{Pr\binom{\text{Ch is}}{\text{sensed busy}}} \\
&= \frac{Pr\binom{\text{Ch is}}{\text{idle}} \overbrace{Pr\begin{pmatrix}\text{Ch is}\\ \text{sensed busy}\end{pmatrix} | \begin{pmatrix}\text{Ch is}\\ \text{idle}\end{pmatrix}}^{\text{false detection}}}{Pr\binom{\text{Ch is}}{\text{idle}} \underbrace{Pr\left(\begin{pmatrix}\text{Ch is}\\ \text{sensed busy}\end{pmatrix} | \begin{pmatrix}\text{Ch is}\\ \text{idle}\end{pmatrix}\right)}_{\text{false detection}} + Pr\binom{\text{Ch is}}{\text{busy}} \underbrace{Pr\left(\begin{pmatrix}\text{Ch is}\\ \text{sensed busy}\end{pmatrix} | \begin{pmatrix}\text{Ch is}\\ \text{busy}\end{pmatrix}\right)}_{\text{correct detection}}} \qquad \text{(A-7)} \\
&= \frac{\frac{b}{b+d}P_f}{\frac{b}{b+d}P_f + \frac{d}{b+d}P_d} = \frac{bP_f}{bP_f + dP_d} \qquad \text{Channel is sensed busy}
\end{aligned}
$$

Using a similar approach, we can obtain the other cases:

$$Pr(\sigma^2=\sigma_n^2) = \frac{b(1-P_f)}{b(1-P_f)+dP_m} \qquad \text{Channel is sensed idle} \qquad \text{(A-8)}$$

$$Pr(\sigma^2=\sigma_n^2+\sigma_\zeta^2) = \frac{dP_d}{bP_f+dP_d} \qquad \text{Channel is sensed busy} \qquad \text{(A-9)}$$

$$Pr(\sigma^2=\sigma_n^2+\sigma_\zeta^2) = \frac{dP_m}{b(1-P_f)+dP_m} \qquad \text{Channel is sensed idle} \qquad \text{(A-10)}$$

Note that the probability of the channel state can be derived easily from the transition matrix of the two-state Markov chain of Fig. 2.

$$P_r(\text{channel in state } i) = \begin{cases} \frac{d}{b+d}, & i \text{ is busy} \\ \frac{b}{b+d}, & i \text{ is idle} \end{cases} \qquad \text{(A-11)}$$

The solution for the maximum eigenvalue of (C-3) is

$$
\begin{aligned}
\omega \; &= \frac{1}{2}\big(\acute{p}_8+\tau_b\acute{p}_7+p_6+\tau_d p_5+\acute{p}_4+\tau_d\acute{p}_3+p_2+\tau_b p_1\big) \\
+ \; & \frac{1}{2}\Big[\big(\acute{p}_8+\tau_b\acute{p}_7+p_6+\tau_d p_5+\acute{p}_4+\tau_d\acute{p}_3+p_2+\tau_b p_1\big)^2 \\
- \; & 4\Big(\tau_d\acute{p}_3\,p_2+\tau_b\tau_d\acute{p}_3\,p_1+\tau_b\acute{p}_7\,p_2+\tau_b\acute{p}_7\,p_6 \\
+ \; & \tau_b\tau_d\acute{p}_7\,p_5-\tau_b\acute{p}_1\,p_4-\tau_b\tau_d\acute{p}_1\,p_3-\tau_d\acute{p}_2\,p_3+\tau_b^2\acute{p}_7\,p_1 \\
- \; & \acute{p}_2\,p_4+\acute{p}_4\,p_2+\acute{p}_4\,p_1-\tau_d^2 p_3\,\acute{p}_5-\tau_d p_4\,\acute{p}_5+\tau_d p_5\,\acute{p}_4 \\
+ \; & \tau_d^2 p_5\,\acute{p}_3-\tau_d p_3\,\acute{p}_6-p_4\,\acute{p}_6+p_6\,\acute{p}_4+\tau_d p_6\,\acute{p}_3 \\
- \; & \tau_b^2\acute{p}_1\,p_7-\tau_b\acute{p}_2\,p_7-\tau_b\acute{p}_5\,p_7-\tau_b\acute{p}_6\,p_7-\tau_b\acute{p}_1\,p_8 \\
- \; & \acute{p}_2\,p_8-\tau_d\acute{p}_5\,p_8-\acute{p}_6\,p_8+\acute{p}_8\,p_6+\tau_d\acute{p}_8\,p_5 \\
+ \; & \acute{p}_8\,p_2+\tau_b\acute{p}_8\,p_1\Big)\Big]^{\frac{1}{2}} \qquad \text{(C-4)}
\end{aligned}
$$

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