Effective Capacity Analysis for Cognitive Networks under QoS Satisfaction

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Abstract

Spectrum sensing and dynamic spectrum access (DSA) techniques in cognitive radio networks (CRN) have been extensively investigated since last decade. Recently, satisfaction of quality-of-service (QoS) demands for secondary users (SU) has attracted great attention. The SU can not only discover the transmission opportunities, but also cognitively adapts the dynamic spectrum access strategies to its own QoS requirement and the environment variations. In this paper, we study how the delay QoS requirement affects the strategy on network performance. We first treat the delay-QoS in interference constrained cognitive radio network by applying the effective capacity concept, focusing on the two dominant DSA schemes: underlay and overlay. We obtain the effective capacity of the secondary network and determine the power allocation policies that maximize the throughput of the cognitive user. The underlay and overlay approaches may have their respective advantages under diverse propagation environment and system parameters. If the cognitive network can dynamically choose the DSA strategy under different environment, its performance could be further improved. We propose a selection criterion to determine whether to use underlay or overlay scheme under the given QoS constraint and the PUs’ spectrum-occupancy probability. Thus, the throughput of the CRN could be increased.

Performance analysis and numerical evaluations are provided to demonstrate the effective capacity of CRN based on the underlay and the overlay schemes, taking into consideration the impact of delay QoS requirement and other related parameters.

Keywords: Underlay; Overlay; Effective capacity; QoS requirement; Selection criterion.

1. Introduction

Recent advances in spectrum-sharing techniques have enabled different wireless communication technologies to coexist and cooperate towards achieving a better gain from the limited spectrum resources. This has been motivated by the fact that most of the allocated spectrum experiences low utilization [1]. Certain authorities, as Federal Communications Commission (FCC) for radio spectrum regulation, divide the radio spectrum into many frequency bands. Licenses are provided to operators, typically for long time such as one or two decades. Depending on the type of radio service that is then provided by the licensees, frequency bands are often idle in many areas, and inefficiently used. Moreover, radio systems can dynamically use and release spectrum wherever and whenever they are available (“spectrum agile radios”). This dynamic spectrum access by spectral agile radios helps to minimize unused spectral bands (“white spaces”). The main challenge for the cognitive users (CUs) is to control their interference levels and to guarantee their own quality of service requirement by transmitting at the desired rates and limiting the delay experienced by the data in the buffers [2].

Among several dynamic spectrum access (DSA) schemes, overlay, underlay, and sensing-based spectrum sharing, are three dominant schemes in cognitive radio networks. Under the overlay scheme [3,4], the CUs can only access the PUs’ spectrum that are idle, i.e., not currently used by PUs, where the activities of the PUs are monitored through spectrum sensing. For the underlay scheme [4-6] the CUs are allowed to use the PUs’ spectrum even when PUs are active, but needed to control the transmit power of CUs to make the interference to PUs to a tolerable level. The sensing-based spectrum sharing [7,8] is a combination of underlay and overlay, in which CUs should adapt their transmit power to the
spectrum sensing results. While the sensing-based approach usually imposes high implementation complexity.

Recently, the problem of quality-of-service (QoS) satisfaction for cognitive users has attracted lots of research attention. The functions of cognition radio networks have been broadened such that the CUs can not only detect the transmission opportunities under the specific DSA approach, but also cognitively adapt the dynamic spectrum access strategies to the QoS requirement and the channel variations. In this paper, we focus on the QoS driven underlay and overlay schemes.

Resource allocation in cognitive network has been investigated extensively. The joint bandwidth and power allocation has been considered in [6], and the optimal resource allocation schemes are investigated under all possible combinations of the average/peak transmit and interference power constraints. In [6,9], the underlay and overlay schemes are compared. Capacity analysis indicates that the overlay scheme offers higher network throughput. Moreover the authors in [9] indicate that in the presence of the primary users, the interference temperature constraint limits network capacity in the overlay scheme more than its underlay capacity counterpart. The impact of the secondary service on the primary systems in the overlay and underlay schemes is studied in [10] using stochastic geometry approach. The authors conclude that by combining the underlay and overlay schemes, the negative impacts on the primary service could be decreased. Sensing-based spectrum sharing is introduced in [9], in which a heuristic algorithm to find the optimal power allocation is proposed. Two different power allocation strategies for the secondary service transmitter based on the spectrum sensing output are introduced.

These works have not taken into consideration the requirement on QoS constraints in mixed access CRNs, thus the developed schemes may cause performance degradation for CUs. In this paper, the concept of effective capacity proposed in [11], which is the maximum constant arrival rate that can be supported by the given time-varying service process is applied. In [12,13], the maximization of effective capacity is studied given that the delay QoS is satisfied. The underlay scheme is assumed in [6]. While in [13] the analysis of the effective capacity is mainly based on channel estimation and sensing results of the overlay strategy. Efficient resource allocation with statistically guaranteed delay QoS for the hybrid DSA schemes to achieve maximum capacity of the CUs is needed. We present a systematic approach which suggests the best access scheme for a given system requirement. The mixed strategy proposed in this paper is similar in spirit to the sensing-based spectrum sharing proposed in [7] except that, in this paper, the effective capacity is adopted instead of ergodic capacity and the general case of Gamma distribution is assumed for the channel fading. Also in [7], just overlay scheme is analyzed. In this study, both DSA schemes are studied. We further introduce a new selection criterion to obtain the best access scheme.

For the overlay strategy, we consider the spectrum sensing errors and observe that an optimal sensing time exists under a given QoS constraint. This optimal sensing time increases as the QoS constraint is more stringent. Moreover, how the transmit power and interference power of both underlay and overlay strategies vary with the primary user activities is investigated. It is found that the transmit power of the overlay strategy and the interference power of the underlay strategy always remain unchanged. However, the transmit power of the underlay strategy and the interference power of the overlay strategy vary with PU’s spectrum-occupancy probabilities. Specifically, as the spectrum-idle probability increases, both the transmit power of the underlay strategy and the interference power of the overlay strategy are reduced.

Both underlay and overlay approaches may have their respective advantages under diverse propagation environment and system parameters. For example, the less busy the primary channel with loose QoS requirement, it is preferred to use the overlay scheme for higher achievable capacity regardless of its higher complexity in implementation. While, when the system is more harsh (i.e., low idle periods and more stringent QoS constraint), underlay scheme is preferred to maintain the achievable capacity performance. We propose a selection criterion to make the cognitive network dynamically choose the DSA strategy under different environments.

The rest of this paper is organized as follows. The system model, assumptions, and the concept of the statistical delay QoS requirement and its reflections on the effective capacity are presented in Section 2. In Section 3 the effective capacity optimization and optimal resource allocation for the underlay scheme are studied. In Section 4, spectrum sensing, effective capacity, and power allocation for the overlay case are studied in three subsequent subsections. Then we propose an access strategy selection criterion in Section 5. Performance analysis and numerical results are presented in Section 6. Section 7 concludes the paper.
2. System Model and Effective Capacity Definition

2.1 System Model and Assumptions

A typical CRN model is assumed [7], in which a cognitive wireless network coexists with a primary wireless network by sharing B Hz spectrum band as shown in Fig 1. We assume that there is no direct signalling between the primary and cognitive networks. The cognitive network includes a CU transmitter $T_S$ and a CU receiver $R_S$. The primary network includes a PU transmitter $T_P$ and a PU receiver $R_P$.

![Fig 1: The additive interference channel for a pair of primary and cognitive links with channel gain coefficients: $g_{pp}$, $g_{ss}$, $g_{sp}$, $g_{ps}$.

The primary transmitter adjusts its transmit power $S_p$ based only on its own transmission requirement. The secondary transmitter transmits with variable power $S_s$ which should not exceed a maximum value of $S_{sm}$. All channel power gains are assumed to be stationary and ergodic independent random processes. These system gains are grouped in the vector $G = [g_{ss}, g_{sp}, g_{pp}, g_{ps}]$ for a specific realization as shown in Fig 1. All channel power gains are assumed to be independent and identically distributed (i.i.d.), and follow Gamma distribution. Gamma distribution fits the experimental data [14], and it has been considered as an adequate model to characterize wireless channel fading such as slow fading (shadowing) or even fast fading [14,15]. It is a general case of exponential distribution, and it closely approximates the lognormal distribution. It is also assumed that the cognitive transmitter has perfect information of interference channel gains. These information can be obtained by direct feedback from the primary receiver or indirect feedback from a third-party such as a band manager which interposes between the primary and cognitive users [16]. It can also be obtained through periodic sensing of pilot signal from the primary receiver assuming the channel reciprocity.

The corresponding probability density functions (pdf) are $f_{g_{ss}}$, $f_{g_{sp}}$, $f_{g_{pp}}$, and $f_{g_{ps}}$, which are given as

$$f_X(x) = \frac{x^{\mu-1}e^{-x/k}}{\Gamma(\mu)k^{\mu}}; \quad x \geq 0,$$

where $\Gamma(\cdot)$ is the Gamma function which is defined as

$$\Gamma(z) = \begin{cases} (z - 1)! = \prod_{i=1}^{z-1} i, & \text{if } z \text{ is positive integer}; \\ \int_0^\infty e^{-tx}t^{z-1}dt, & \text{if } z \text{ is real or complex}. \end{cases}$$

In (1), $\mu$ is known as the shape parameter of the distribution, and $k$ is the scale parameter. The average $E[X] = \mu k$, and $\text{Var}[X] = \mu k^2$. Without loss of generality, we assume the scale parameter $k = 1$. The background noise at both receivers are modeled as AWGN independent circularly symmetric complex Gaussian (CSCG) random variable with zero-mean and variance $\sigma^2_s$ and $\sigma^2_p$ respectively. For all the used abbreviations below, the common subscripts s and p refer to the cognitive user and primary user respectively, while the superscripts u, o, {0,1} and * refer to the underlay, overlay, state number and optimal value respectively. The maximum interference at the primary receiver should be kept below a threshold value $I_{th}$. This value is a system parameter which can be specified by the primary network operator or by the
In this paper, resource allocation policies will be developed for both the underlay and overlay based CRNs with statistical delay QoS guarantees. As mentioned above, in the underlay scheme, the secondary user is required to always satisfy the interference constraint. Therefore, even in circumstances when the primary user is not transmitting, the cognitive user has to adjust its transmission power based on the interference threshold constraint. Consequently, the derived resource allocation schemes should not be only a function of system gain vector $G$, but also will be varied with different QoS requirement. In order to clearly illustrate our concept, we assume that each frame at the data link layer of the CU transmitter has the same time duration. The frames are stored at the transmit buffer and split into bit-streams at the physical layer. The CU transmitter employs adaptive modulation and power control based on the statistical QoS constraint and the system gain vector $G$, which can be perfectly derived by the CU transmitter.

The factors that will impact the resource allocation of the cognitive network include: (1) the average transmit and interference power constraints; (2) the statistical delay QoS requirement; (3) the primary network activities; (4) the DSA strategy used by the cognitive network; and (5) the interference caused by the PU transmitter on the cognitive network. The existing literatures studied the underlay strategy, while the primary network activities are ignored. In this paper, the PUs’ spectrum-occupancy probability will be taken into consideration even for the underlay strategy. It is assumed that the primary network will choose whether to use the spectrum or not at the beginning of each frame. As the spectrum-occupancy status of the primary network can be viewed as the two hypothesis tests from the CU’s perspective, we denote the probability that the primary network does not occupy the spectrum as $P_i$ (idle probability), and the probability that the spectrum is occupied as $R_b$ (busy probability). We assumed that these probabilities will not change during the frame interval.

### 2.2 Delay QoS Guarantees and Channel Capacity

Quality of service (QoS) guarantees in terms of delay constraint play a critically important role in modern wireless networks. Non-real-time services such as data, aim at maximizing the throughput with a loose delay constraint. While for real-time services, such as multimedia and video conference, the QoS requirement needs a stringent delay-bound to achieve high spectral efficiency. Some other services falling in between such as web browsing and paging system, which are delay-sensitive but the delay QoS requirements are not as stringent as those of real-time services. Therefore, the diverse services impose totally different delay QoS constraints, which bring great challenges to the design of future wireless networks.

In wireless communications, the most scarce radio resources are power and spectral bandwidth. As a result, extensive research has been devoted to the techniques that can enhance the spectral efficiency of wireless systems. These techniques use the concept of Shannon capacity based on the information theory[17]. Power and rate adaptation has been widely considered as one of the key solutions to improve the spectral efficiency. Water-filling algorithm is one of well known scheme that maximizes spectral efficiency[18], in which more power is assigned to a channel which is in good condition and less power when the channel becomes worse. When the channel quality is below a certain threshold, no power is allocated. Additionally, total channel inversion algorithm has a different idea of power and rate adaptation[18], in which more power is assigned to a channel in a deep fading state and less power for the good channel. This is to keep a constant signal-to-noise ratio, such that a constant rate service process can be obtained. Apparently, water-filling is better than total channel inversion since the former provides higher spectral efficiency[19]. It is important to note that Shannon theory does not place any restrictions on complexity and delay. However, a natural question is whether the former is also better than the latter in terms of QoS guarantees? In order to answer the above question, it is necessary to take the QoS metrics into account when applying power and rate adaptation.

The dual concepts of effective bandwidth and effective capacity give us a powerful approach to evaluate the statistical QoS performance from the networking perspective. The effective-bandwidth theory has been extensively studied in the early 90’s with the emphasis on wired asynchronous transfer mode (ATM) networks [20]. This theory enables us to analyze network statistics such as queue distributions, buffer overflow probabilities, and delay-bound violation probabilities, which are important for statistical QoS guarantees. In [11], the authors proposed an interesting concept, namely effective capacity. The effective capacity approach is particularly convenient for analyzing the statistical QoS performance of wireless transmissions where the service process is driven by the time-varying wireless channel. In this paper, by integrating information theory with the effective capacity, we investigate the impact of QoS constraint on the power and rate adaptation over cognitive radio networks. The problem is how to maximize the capacity subject to a given delay QoS constraint. There exists a tradeoff between the capacity and the QoS requirement, the higher capacity gain
comes at the price of sacrificing QoS provisioning, and vice versa. When the QoS constraint becomes loose, the optimal power control converges to the water-filling scheme, where Shannon (ergodic) capacity can be achieved. On the other hand, when the QoS constraint becomes stringent, the optimal power-control converges to the total channel inversion such that the system operates at a constant service rate.

The authors in [11] and [21] showed that the probability of the queue length of the transmit buffer exceeding a certain threshold \( q \), decays exponentially as a function of \( q \). If we define \( Q \) as the stationary queue length, then the delay rate of the tail distribution of the queue length \( Q \) can be written as

\[
\theta = -\lim_{q \to \infty} \log q \Pr(Q \geq q).
\]

(2)

For large threshold (say \( q^\text{th} \)), the following approximation for the buffer violation probability can be made

\[
\Pr(Q \geq q^\text{th}) \approx e^{-\theta q^\text{th}},
\]

where \( 0 < \theta < \infty \) is a constant called QoS exponent. (see [11] for details).

The above equation states that the probability of the queue length exceeding a certain threshold \( q^\text{th} \) decays exponentially fast as the threshold \( q^\text{th} \) increases.

Furthermore, when the focus is on delay-bound violation probability, an expression similar to the above equation can be obtained as [22]

\[
\Pr(D \geq d^\text{th}) \approx e^{-\theta \varepsilon d^\text{th}},
\]

(4)

where \( D \) and \( d^\text{th} \) denote the delay and delay bound, respectively, and \( \varepsilon \) is the source arrival rate determined as \( \varepsilon = q^\text{th}/d^\text{th} \) [23]. The effective bandwidth function intersects with the effective capacity function at the value of \( \varepsilon \) (see [22] for details).

From Eqs. (3) and (4), we can see that the parameter \( \theta \) plays an important role for the statistical QoS guarantees, which indicates the decaying-rate of the QoS violation probability.

In practical applications, the value of \( \theta \) depends on the statistical characterization of the arrival and service processes, bounds on delay or buffer lengths, and target values of the delay or buffer length violation probabilities.

A smaller \( \theta \) corresponds to a slower decay rate, which implies that the system can only provide a looser QoS guarantee, while a larger \( \theta \) leads to a faster decay rate, which means that a more stringent QoS requirement can be supported. In particular, when \( \theta \to 0 \), the system can tolerate an arbitrarily long delay, which corresponds to the scenario studied in information theory. On the other hand, when \( \theta \to \infty \), the system cannot tolerate any delay, which corresponds to an extremely stringent delay-bound. Due to its close relationship with statistical QoS demands, \( \theta \) is called the QoS exponent [11].

### 2.3 Effective Capacity Concept

Based on the concept of QoS exponent, the effective capacity is defined as the maximum constant arrival rate that a given service process can support for which the QoS exponent \( \theta \) is fulfilled. Analytically, the effective capacity can be formally expressed as follows.

Let the sequence \{\( R[i], i = 1, 2, \cdots \)\} denote a discrete-time stationary and ergodic stochastic service process and \( S[t] = \sum_{i=1}^{t} R[i] \) be the sum of the service process over time sequence of \( i = 1, 2, \cdots, t \). Then, the effective capacity of the service process, denoted by \( C(\theta) \), where \( \theta > 0 \), is defined as [11]

\[
C(\theta) = -\lim_{t \to \infty} \frac{1}{\theta t} \log \mathbb{E}[e^{-\theta S[t]}],
\]

(5)

where \( \mathbb{E}[\cdot] \) denotes the expectation. When the sequence \{\( R[i], i = 1, 2, \cdots \)\} is an uncorrelated process, the effective capacity formula turns to

\[
C(\theta) = -\frac{1}{\theta} \log \mathbb{E}[e^{-\theta R[i]}].
\]

(6)

Notice that the effective capacity can be considered as the maximal throughput under the constraint of QoS exponent \( \theta \).
In the following sections, we will apply this metric for cognitive radio network and design the corresponding resource allocation algorithms.

3 Optimal Resource Allocation For Underlay Scheme

In this section, we aim at obtaining the optimal resource allocation which can maximize the throughput of the cognitive network under a given statistical delay QoS guarantee determined by QoS exponent $\theta$.

When the CU applies the underlay scheme, the spectrum sensing is not needed. The CU can use the whole frame duration for transmission with adjusted transmit power no matter whether the primary network occupies the spectrum or not. The cognitive network with the underlay scheme has two system states for each frame, which are listed as follows:

- **State 0**: The channel is idle, i.e., it is not occupied by the primary network, with a probability $P_i$.
- **State 1**: The channel is busy, i.e., it is currently used by the primary network, with a probability $P_b$.

We denote the service rates of these two states as $R_{i,0}$ and $R_{i,1}$ at CU, respectively. Based on Shannon information theory, the achievable service rates of the two system states can be written as

$$R_{i,0} = TB\log\left(1 + \frac{g_{ss}^{u,0}}{\sigma_z^2}\right), \quad \text{and} \quad R_{i,1} = TB\log\left(1 + \frac{g_{ss}^{u,1}}{\sigma_z^2}\right),$$

(7)

where $g_{ss}^{u,0}$ and $g_{ss}^{u,1}$ are the transmit power of the CU transmitter when the spectrum is idle and busy, respectively. The effective capacity formula of the cognitive network for each state mentioned above can be written as (see Appendix 8 for details)

$$P_{c,u}^{0}(\theta) = -\frac{1}{\theta}\log_b\left(P_i E_{g_{ss}}[e^{-\theta R_{i,0}}]\right)$$

(8)

$$P_{c,u}^{1}(\theta) = -\frac{1}{\theta}\log_b\left(P_b E_z[e^{-\theta R_{i,1}}]\right),$$

(9)

where, $E_{g_{ss}}$ is the expectation operator over the random variable $g_{ss}$, and $E_z$ is the expectation operator over the ratio of the random variables $g_{ss}$ and $g_{ps}$ which will be discussed shortly.

Since in the underlay case, the cognitive user does not preform spectrum sensing, it is not possible to differentiate state 0 or state 1. Hence, we can impose the cognitive user to operate in the state that provides higher capacity. Therefore, the aim now is to solve (8) and (9) individually, and make the cognitive user to choose a state that provides better performance.

The effective capacity (8) can be written as

$$P_{c,u}^{0}(\theta) = -\frac{1}{\theta}\log\left(P_i E_{g_{ss}}\left[\frac{R_{i,0}}{1 + R_{i,0}}\right]\right) = -\frac{1}{\theta}\log\left(P_i E_{g_{ss}}\left[1 + \frac{g_{ss}^{u,0}}{\sigma_z^2}\right]^{-\alpha^u}\right),$$

(10)

where the term $\alpha^u = (TB\theta/\ln2)$ can be named as the normalized QoS exponent. It can characterize the statistical delay QoS requirement since it is only a function of $\theta$.

By evaluating (10), the following formula can be obtained

$$P_{c,u}^{0} = -\frac{1}{\theta}\log\left[\frac{\Gamma(\alpha^u - \mu_1)}{\Gamma(\alpha^u)} (\gamma^u)^{\mu_1} 1_{\text{F}_1}(\mu_1; 1 + \mu_1 - \alpha^u; \gamma^u) + \frac{\Gamma(\mu_1 - \alpha^u)}{\Gamma(\mu_1)} (\gamma^u)^{\mu_1} 1_{\text{F}_1}(\alpha^u; 1 + \alpha^u - \mu_1; \gamma^u)\right],$$

(11)

where $\gamma^u = \frac{\sigma_z^2}{g_{ss}^{u,0}}$, $\mu_1 = \mu$, and the function $1_{\text{F}_1}$ is called Confluent Hypergeometric function of first kind, defined as [24]:

$$1_{\text{F}_1}(a, b, z) = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{(b)_n n!}, \quad \text{where} \quad (a)_n = \begin{cases} a(a + 1)(a + 2) \cdots (a + n - 1), & n \neq 0; \\ 1, & n = 0. \end{cases}$$

(12)

To evaluate (9), we may refer to the statistical fact that says: the ratio of two independent Gamma distributed random variables with shape parameters $\mu_1$ and $\mu_2$, respectively is Beta Prime distributed random variable with parameters $\mu_1$
and $\mu_2$ [25]. Let us define the random variables $X$ and $Y$ as $X = g_x$, and $Y = (g_p S_p + \sigma_x^2)/S_s$. Since power channel fading is assumed to be Gamma distributed, $Y$ is also Gamma distributed with mean $\mu_2$. The pdf of $Y$ can be expressed as

$$f_Y(y) = \frac{y^{\alpha-1}e^{-\gamma}}{\Gamma(\gamma)}$$

where $\mu_2 = \left(\frac{\mu_1 + \gamma + \sigma_x^2}{\sigma_x^2}\right)^{\gamma}$. Then, the pdf of the ratio random variable $Z = \frac{X}{Y}$ has the following Beta Prime distribution

$$f_Z(z) = \frac{z^{\gamma-1}(1+z)^{-\gamma-1} \mu_2}{\beta_1}$$

Here, $\mu_1 = \mu$ is the expected value of power channel fading. In (14), $\beta_1(\mu_1, \mu_2)$ is the Beta function which is related to Gamma function as $\beta_1(\mu_1, \mu_2) = \frac{\Gamma(\mu_1)\Gamma(\mu_2)}{\Gamma(\mu_1 + \mu_2)}$. Substituting $R_u^{u,1}$ defined in (7) into (9), we get

$$E_u^{u,1}(\theta) = -\frac{1}{\theta} \log_e \left( P_b E_z \left( e^{-TB \log_e \left( 1 + \frac{\mu g_x}{\sigma_s g_x + \sigma_y^2} \right)} \right) \right) = -\frac{1}{\theta} \log_e [P_b E_z [(1 + z)^{-\mu}]]$$

By evaluating (15) using (14), the following formula can be obtained below,

$$E_u^{u,1} = -\frac{1}{\theta} \log_e[P_b \frac{\Gamma((\alpha+\mu_2)/(\alpha+\mu_2))}{\Gamma((\alpha+\mu_1 + \mu_2)/(\alpha+\mu_1 + \mu_2))}]$$

The effective capacity formula (11) and (16) are derived using the properties of gamma and hypergeometric functions listed in [24].

Now, the main objective is to find the optimal power allocation for the two power $\{S_s^{u,0}, S_s^{u,1}\} = \left\{\frac{\sigma_y^2}{\lambda^2}, \frac{\mu_1 + \gamma + \sigma_x^2}{\mu_2}\right\}$ to maximize the effective capacity of the cognitive network subject to the upper bounded transmit and interference powers. From (11), we can formulate an expression for the optimal effective capacity for the cognitive network channel in the state 0, and write the optimization problem as follows

$$\max \gamma = -\frac{1}{\theta} \log_e[P_b \Gamma((\alpha+\mu_1)/(\alpha+\mu_1)) (Y)^\alpha \left( \Gamma((\alpha+\mu_2)/(\alpha+\mu_2)) \right) \Gamma((\alpha+\mu_1 + \mu_2)/(\alpha+\mu_1 + \mu_2))]
\left[ F_1(\mu; 1 + \mu_1 - \alpha^u, Y^u) + \frac{\Gamma((\mu_1 - \alpha^u)/(\mu_1))}{\Gamma((\mu_1))} (Y^u)^{\alpha^u} \left( F_1(\mu_u; 1 + \mu_1 - \mu_1; Y^u) \right) \right],$$

S.t. $0 \leq P_b S_s^{u,0} \leq S_s^m$, $P_b E[g_x S_s^{u,0}] \leq I_{th}$. (17)

The CU transmitter should satisfy transmit power constraint, which appears in the first constraint in (17). At the same time, since the transmissions of the cognitive network will interfere the primary network, in order to protect the QoS of the primary network, the average interference power constraint is imposed in the second constraint of (17).

Using the fact that $\log(.)$ is a monotonically increasing function, the solution to the maximization problem (17) can be mapped to the following minimization problem

$$\min \gamma = -\frac{1}{\theta} \log_e[P_b \Gamma((\alpha+\mu_1)/(\alpha+\mu_1)) (Y)^\alpha \left( \Gamma((\alpha+\mu_2)/(\alpha+\mu_2)) \right) \Gamma((\alpha+\mu_1 + \mu_2)/(\alpha+\mu_1 + \mu_2))]
\left[ F_1(\mu; 1 + \mu_1 - \alpha^u, Y^u) + \frac{\Gamma((\mu_1 - \alpha^u)/(\mu_1))}{\Gamma((\mu_1))} (Y^u)^{\alpha^u} \left( F_1(\mu_u; 1 + \mu_1 - \mu_1; Y^u) \right) \right],$$

S.t. $0 \leq P_b S_s^{u,0} \leq S_s^m$, $P_b E[g_x S_s^{u,0}] \leq I_{th}$. (18)

The corresponding optimal power allocation can be obtained as (see the details in Appendix B).

$$[ C_{u1}^u(\lambda^u) \lambda^u + F_1(\mu; 1 + \mu_1 - \alpha^u, Y^u) + \frac{Y^u}{1+\mu_1 - \alpha^u \lambda^u} + \frac{\lambda^u}{1+\mu_1 - \mu_1} + \frac{\lambda^u}{1+\mu_1 - \mu_1} \gamma^u]\bigg\{ F_1(\alpha^u; 1 + \alpha^u - \mu_1; Y^u) \bigg\} = \sigma_x^2 (\lambda^u + \gamma^u)$$

where $\lambda^u$ and $\gamma^u$ are the optimal Lagrangian multipliers for the underlay scheme that satisfy the constraints in (18), and

$$C_{u1}^u = \Gamma(\mu_1 - \mu_2)/\Gamma(\mu_1), \quad C_{u2}^u = \Gamma(\mu_1 - \mu_2)/\Gamma(\mu_1).$$

After finding the optimal Lagrangian multipliers $\lambda^u$ and $v^u$ using Algorithm 1, (19) can be solved numerically to find the corresponding power allocation $\gamma^u$, then $S_{s}^{u,0} = \frac{\gamma^u}{\gamma^u}$. Similar to the work done in the state 0, the optimization problem to adapt $S_{s}^{u,1}$ can be formulated to the following minimization problem

$$
\min_{\mu_2} \frac{\Gamma(\alpha^u+\mu_2)}{\Gamma(\alpha^u+\mu_1+\mu_2)}(\mu_1 + \mu_2) \quad \text{S.t.} \quad 0 \leq P_b S_{s}^{u,1} \leq S_{s}^{m}, \quad P_b E[S_{s}^{u,1}] \leq 1^{th},
$$

and the corresponding optimal power allocation of the state 1 can be obtained as (see also the details in Appendix B)

$$
(\mu_2^*)^2 \frac{\Gamma(\alpha^u+\mu_2)}{\Gamma(\alpha^u+\mu_1+\mu_2)}(\Psi(\alpha^u + \mu_2) + \Psi(\mu_1 + \mu_2) - \Psi(\mu_1 + \mu_2)) = (\mu_1 S_p + \sigma^2)(\lambda^u + v^u \mu_1)
$$

where the optimal value $\mu_2^*$ is the value of $\mu_2$ corresponding to $S_{s}^{u,1}$, i.e., $\mu_2^* = (\mu_1 S_p + \sigma^2)/S_{s}^{u,1}$. The function $\Psi$ in (22) is known as polygamma function and defined as [24]

$$
\Psi(z) = \int_0^\infty \frac{t^{z-1}e^{-t}}{t} dt = -\gamma_0 \sum_{k=0}^\infty \frac{1}{k+z} \quad z \neq 0, -1, -2, \ldots,
$$

where $\gamma_0 \approx 0.57721$ is the Euler-Mascheroni constant.

The cognitive transmitter will be allocated the power that maximize the effective capacity. The optimal power allocation for the underlay cognitive radio $S_{s}^{u}$ which maximizes the effective capacity can be determined by

$$
S_{s}^{u} = \arg \max_{S_{s}^{u,0}, S_{s}^{u,1}} \left( E_{C}^{u,0}(S_{s}^{u,0}), E_{C}^{u,1}(S_{s}^{u,1}) \right).
$$

As a result, the optimal effective capacity of the underlay scheme is

$$
P_{C}^{u,\text{opt}}(\theta, S_{s}^{u}) = \max\left( E_{C}^{u,0}(\theta, S_{s}^{u}), P_{C}^{u,1}(\theta, S_{s}^{u}) \right).
$$

Algorithm 1: The pseudocode of the optimal Lagrangian multipliers calculations for underlay scheme.

Initialization: $\lambda^{u,0}, v^{u,0}$

REPEAT …

1- Compute: $S_{s}^{u,0,j}$, and $S_{s}^{u,1,j}$ by (19) and (22)

2- Compute: subgradient of $\lambda^{u,j}$, and $v^{u,j}$ by (B-7).

3- Update: $\lambda^{u,j+1}$, and $v^{u,j+1}$ by $\lambda^{u,j+1} = \lambda^{u,j} + \epsilon_1 \cdot L_s(\lambda^{u,j+1})$ and $v^{u,j+1} = v^{u,j} + \epsilon_2 \cdot L_s(v^{u,j+1})$

UNTIL $\lambda^{u,i}$, and $v^{u,i}$ converge.

where $L_s(\cdot)$ is the subgradient of the dual function for the multiplier calculated by (B-7). $\epsilon_1$ and $\epsilon_2$ are the stepsize for each multiplier. They have been chosen to equal to $(1 + c)/(\alpha + c)$, where $c$ is some adaptive constant and $\alpha$ is the number of iteration. This variable stepsize is shown to give a fast convergence [26]. See Fig 3(a).
4 Optimal Resource Allocation For Overlay Scheme

In this section, we aim at deriving optimal resource allocation scheme for the overlay scheme. As discussed previously, while applying the overlay scheme, the CUs need to utilize the spectrum sensing to identify the spectrum occupancy status before accessing the spectrum and can only use the vacant spectrum for transmissions. Therefore, for a frame of $T$ seconds duration, the first $\tau$ seconds will be used to sense the channel, and the remaining $(T-\tau)$ seconds will be exploited for data transmission. The resulting frame structure is shown in Fig 2.

![Fig 2: The transmission frame structure.](image)

4.1 Spectrum Sensing Model

According to [7,8,27], the spectrum occupation status can be determined between the following two hypotheses:

- detected as idle: $y(i) = n(i)$, $i = 1,2,\ldots,\tau B$,
- detected as busy: $y(i) = n(i) + n_p(i)$, $i = 1,2,\ldots,\tau B$, (26)

where $y(i)$ is the signal received by the CU receiver, $n_p(i)$ is the interference on the CU generated by the transmit signal of the PU. Since the bandwidth is $B$, we have $\tau B$ symbols in a duration of $\tau$ seconds (for simplicity, we assume that $\tau B$ is an integer). Assuming that $\{n_p(i)\}$ samples are i.i.d. signal and modeled as zero-mean Gaussian distributed with variance of $\sigma_{n_p}^2$. The optimal detector response for this hypothesis problem is given in [28] as

$$
\mathcal{Y} = \frac{1}{\tau B} \sum_{i=1}^{\tau B} |y_i|^2 E_{\text{busy}} \delta,
$$

(27)

where $\delta$ is a pre-designed threshold. The cognitive radio assumes that the primary system is in operation if $\mathcal{Y} \geq \delta$. Otherwise, it is idle. Assuming $\tau B$ is sufficiently high, $\mathcal{Y}$ can be approximated, using Central Limit Theorem [29], as a Gaussian random variable with mean and variance

$$
\mathbb{E}[\mathcal{Y}] = \begin{cases} 
\sigma_n^2 + \sigma_{n_p}^2 & \text{if PU is inactive} \\
\sigma_n^2/(\tau B) & \text{if PU is active},
\end{cases}
$$

and

$$
\sigma_{\mathcal{Y}}^2 = \begin{cases} 
(\mathbb{E}[|n_p|^4] + \sigma_n^4 + 2\sigma_n^2 \sigma_{n_p}^2 - \sigma_{n_p}^4) / (\tau B) & \text{if PU is inactive} \\
\sigma_n^4/(\tau B) & \text{if PU is active},
\end{cases}
$$

(28)

respectively, where $\mathbb{E}[|n_p|^4]$ is the forth moment of the received signal $n_p$. Note that $\mathbb{E}[|n_p|^4] = 2\sigma_n^4$ for the CSCG assumption. The probability of false alarm of the energy detector is given as follows [30]

$$
P_f = \Pr\{\mathcal{Y} > \delta | \text{PU inactive}\} = Q\left(\frac{\delta - \sigma_n^2}{\sqrt{\sigma_n^4/(\tau B)}}\right),
$$

(29)

where $Q(\cdot)$ is the complementary cumulative distribution function of the standard Gaussian. If we assume that the primary signal $n_p$ has complex-valued PSK waveform [30], the probability of detection of the energy detector can be written as

$$
P_d = \Pr\{\mathcal{Y} > \delta | \text{PU active}\} = Q\left(\frac{\delta - \sigma_{\text{PU}}^2 - \sigma_n^2}{\sigma_n \sqrt{(2\sigma_{\text{PU}}^2 + \sigma_n^2)/(\tau B)}}\right),
$$

(30)
Due to the unavoidable sensing errors, the cognitive network has four states, which are listed as follows:

1. State 0: Channel is idle, detected as idle, with probability: \((1 - P_s)P_i\)
2. State 1: Channel is busy, detected as idle, with probability: \((1 - P_p)P_b\)
3. State 2: Channel is idle, detected as busy, with probability: \(P_sP_i\)
4. State 3: Channel is busy, detected as busy, with probability: \(P_dP_b\)

As defined in the overlay strategy, the cognitive network can access the spectrum only when the channel is sensed as idle. Therefore, the service rates of the cognitive network for the above four states, denoted by \(R^{0,0}, R^{0,1}, R^{0,2}, \) and \(R^{0,3}\), are:

\[
R^{0,0} = (T - \tau) \log_2 \left( 1 + \frac{S^{0,0}_s \gamma_s}{\sigma_s^2} \right), \quad \text{and} \quad R^{0,1} = (T - \tau) \log_2 \left( 1 + \frac{S^{0,1}_s \gamma_s}{\rho s \gamma_s + \sigma_s^2} \right)
\]

and \(R^{0,2} = R^{0,3} = 0\), respectively, where \(S^{0,0}_s\) and \(S^{0,1}_s\) are the transmit power of the CU transmitter for the state 0 and 1, respectively.

The effective capacity of the overlay scheme can be written as:

\[
E_c^s(\theta) = -\frac{1}{\theta} \log_2 ((1 - P_s)P_i E_{\text{w}}(e^{-\theta R^{0,0}}) + (1 - P_p)P_b E_{\text{w}}(e^{-\theta R^{0,1}}) + P_sP_i + P_dP_b).
\]

The derivation is similar to the case of the underlay scheme and it is omitted here for the sake of brevity.

Using the same idea presented in the underlay case, (32) can be expanded as:

\[
E_c^s(\theta) = -\frac{1}{\theta} \log_2 ((1 - P_s)P_i E_{\text{w}} [e^{-(T - \tau)B \log_2 \left( 1 + \frac{S^{0,0}_s \gamma_s}{\sigma_s^2} \right)}] + (1 - P_p)P_b E_{\text{w}} [e^{-(T - \tau)B \log_2 \left( 1 + \frac{S^{0,1}_s \gamma_s}{\rho s \gamma_s + \sigma_s^2} \right)}] + P_sP_i + P_dP_b),
\]

where \(E_{\text{w}}\) is the expectation over the ratio \(\frac{S^{0,0}_s \gamma_s}{\sigma_s^2}\). Let \(\alpha^o = ((T - \tau)B\theta/\ln2)\) as the normalized QoS exponent. This parameter, \(\alpha^o\) can characterize the statistical delay QoS requirement, since at certain sensing time it is only a function of \(\theta\). Note that \(\alpha^o\) is related to \(\alpha^u\) as \(\alpha^u = \frac{(T - \tau)}{\Gamma(\theta)}\alpha^o\). (33) can be written as:

\[
E_c^s(\theta) = -\frac{1}{\theta} \log_2 ((1 - P_s)P_i E_{\text{w}} \left[ \left( 1 + \frac{S^{0,0}_s \gamma_s}{\sigma_s^2} \right)^{-\alpha^o} \right] + (1 - P_p)P_b E_{\text{w}} [(1 + w)^{-\alpha^o}] + P_sP_i + P_dP_b),
\]

By evaluating (34), a solution of the effective capacity can be expressed as:

\[
E_c^s = -\frac{1}{\theta} \log_2 [(1 - P_s)P_i \left( \frac{\Gamma(\alpha^o + \mu_1)}{\Gamma(\mu_1)} \right) \left( \frac{\Gamma(\mu_1 + \mu_2)}{\Gamma(\mu_1)} \right) F_1(\mu_1; 1 + \mu_1 - \alpha^o, \gamma^o) + \Gamma(\mu_1 + \mu_2) \left( \frac{\Gamma(\mu_1 + \mu_2)}{\Gamma(\mu_1)} \right) F_1(\mu_1 + \mu_2; 1 + \alpha^o - \mu_1; \gamma^o) + (1 - P_p)P_b \left( \frac{\Gamma(\alpha^o + 2 \mu_3)}{\Gamma(\mu_1 + \mu_2 + 2 \mu_3)} \right) F_1(\mu_1 + \mu_2 + 2 \mu_3; 1 + \alpha^o - \mu_1; \gamma^o)] + P_sP_i + P_dP_b),
\]

where \(\gamma^o = \frac{\sigma_s^2}{S^{0,0}_s}\) and \(\mu_3 = \frac{(\rho s \gamma_s + \sigma_s^2)}{S^{0,0}_s}\). The derivation is omitted here due to the lack of space.

### 4.2 Optimal Power Allocation for Overlay Scheme

The power allocation of the cognitive radio network, \(\{S^{0,0}_s, S^{0,1}_s\}\) should be carefully assigned to optimize the overall capacity of the channel. Furthermore, the sensing period \(\tau\), which plays a great role on the system performance, should also be considered in this optimization process.

The cognitive network needs to meet transmit power constraint, which can be expressed as:

\[
(T - \tau)B((1 - P_s)PS^{0,0}_s + (1 - P_p)P_BS^{0,1}_s) \leq S^m
\]

Due to the existence of sensing errors, the interference from the CU transmitter to the PU receiver is inevitable and it exists only when miss detection \((1 - P_d)P_b\) occurs. Thus, the cognitive network should also meet the following constraints:

\[
(T - \tau)B((1 - P_s)PS^{0,0}_s + (1 - P_p)P_BS^{0,1}_s) \leq S^m
\]
interference constraint

\[(1 - P_d)P_b E[g_{sp} S_{s,0}^{0.1}] \leq 1^\text{th},\]  

where \( E[g_{sp}] = \mu\) as it is assumed above.

Our objective is to derive the optimal resource allocation \(\{S_{s,0}^{0.1}, S_{s,1}^{0.1}, \tau\}\) to maximize the effective capacity under the transmit and interference power constraints in (36) and (37) respectively.

The optimization problem can be formulated as

\[
\begin{align*}
\text{max} & \quad \gamma \alpha \mu_1 \mu_2 \tau \\
\text{s.t.} & \quad (36) \text{ and } (37) \text{ hold, and } 0 < \tau < T/2,
\end{align*}
\]

where

\[
C_{11} = \Gamma(\alpha^0 - \mu_1)/\Gamma(\alpha^0), \quad C_{12} = \Gamma(\mu_1 - \alpha^0)/\Gamma(\mu_1), \quad C_0 = P_1 P_b + P_2 P_b,
\]

The variables \(\gamma^0\) and \(\mu_3\) are related to the original optimized variables \(S_{s,0}^{0.0}\) and \(S_{s,0}^{0.0}\) as \(\gamma^0 = \frac{\sigma_s^2}{\sigma_s^2}\) and \(\mu_3 = (\mu_1 S_p + \sigma_s^2)/S_{s,0}^{0.1}\) respectively. The optimized sensing period variable \(\tau\) in (38) is embedded in the variable \(\alpha^0\). Furthermore, we assume that the sensing time should not last more than half of the frame period \(T\). The authors in [30] conclude that for multi slot spectrum sensing, in all cases, the optimal throughput is obtained when the sensing period is less than \(T/8\).

The problem (38) can be solved by two steps. In the first step, we try to obtain the optimal power allocation under a given sensing time \(\tau\). In the second step, we can obtain the optimal sensing time by exhaustive search given the power allocation.

Similar to the underlay case, the solution of the maximization problem (38) can be mapped to the following minimization problem

\[
\begin{align*}
\text{min} & \quad L_{\gamma^0,\mu_3,\tau} (1 - P_d)P_b \left( C_{11} \gamma^0 \mu_1 + 1 + \mu_1 - \alpha^0; \gamma^0 \right) + C_{12} \gamma^0 \mu_1 + 1 + \alpha^0 - \mu_1; \gamma^0 \right) \\
\text{s.t.} & \quad 0 \leq (T - \tau) B \left( (1 - P_2) P_b S_{s,0}^{0.0} + (1 - P_d) P_b S_{s,1}^{0.0} \right) \leq S_{s,0}^{0.0}, \quad (1 - P_d) P_b \mu_3 S_{s,0}^{0.1} \leq 1^\text{th}.
\end{align*}
\]

The corresponding optimal power allocation for a given \(\tau\) can be written as

\[
\begin{align*}
&\left( C_{11} \mu_1 \gamma^0 \right) \gamma^0 \mu_1 + 1 + \mu_1 - \alpha^0; \gamma^0 \right) + \gamma^0 \mu_1 + 1 + \alpha^0 - \mu_1; \gamma^0 \right) \\
&+ C_{12} \gamma^0 \mu_1 + 1 + \mu_1 - \mu_1; \gamma^0 \right) + \gamma^0 \mu_1 + 1 + \alpha^0 - \mu_1; \gamma^0 \right) \\
&= (T - \tau) B \sigma_s^2 \left( \lambda^0 \gamma^0 + \nu^0 \mu_1 \right)
\]

\[
\begin{align*}
&\left( \mu_3 \right)^2 \frac{\Gamma(\alpha^0 + \mu_2)}{\Gamma(\alpha^0 + 1 + \mu_2)} \left( \Psi(\alpha^0 + \mu_3^2) + \Psi(\mu_1 + \mu_3^2) - \Psi(\mu_3) \right) \\
&= (T - \tau) B \left( \mu_3 S_p + \sigma_s^2 \right) \left( \lambda^0 + \nu^0 \mu_1 \right)
\end{align*}
\]

where \(\gamma^0 = \frac{\sigma_s^2}{\sigma_s^2}\) and the multipliers, \(\lambda^0\) and \(\nu^0\) are the optimal Lagrangian multipliers satisfying the constraints in (39). The optimal value \(\mu_3^2\) in (41) is the value of \(\mu_3\) corresponding to \(S_{s,0}^{0.1}\), i.e., \(\mu_3 = (\mu_1 S_p + \sigma_s^2)/S_{s,0}^{0.1}\). Algorithm 4.2 listed below illustrates the pseudocode to find the optimal multipliers.

Different sensing time will result in different performance. Thus, determining the optimal sensing time is a critically important task in the resource allocation scheme. Therefore, after deriving the optimal power allocation scheme at any given \(\tau\), the optimal sensing time, denoted by \(\tau^*\), can be found by one-dimensional exhaustive search [31]. Hence the
optimal effective capacity of the overlay scheme is

$$E_{C}^{o,\text{opt}}(\theta) = E_{C}^{o}(\theta, S_{o}^{a}, S_{o}^{1}, \tau^{'}) \quad (42)$$

Algorithm 2 The pseudocode for finding the optimal Lagrangian multipliers and resource allocation for overlay scheme.

1. Initialization: $\lambda_{0,0}, \nu_{0,0}$

2. FOR $\tau = 0: T/2$

3. REPEAT …

4. (1) Compute: $S_{o}^{a,0,0}$ and $S_{o}^{a,1,0}$ by (40) and (41).

5. (2) Compute: subgradient of $\lambda_{i,j}$ and $\nu_{i,j}$ by (B-8).

6. (3) Update: $\lambda_{i,j+1}$ and $\mu_{i,j+1}$ by (B-8) $\lambda_{i,j+1} = \lambda_{i,j} + \varepsilon_{1} \cdot L_{b}(\lambda_{i,j+1})$ $\nu_{i,j+1} = \nu_{i,j} + \varepsilon_{2} \cdot L_{s}(\nu_{i,j+1})$

7. UNTIL $\lambda_{i,j}$ and $\nu_{i,j}$ converge.

8. END FOR

9. Optimal Resource Allocation: $\tau^{'} = \arg\max \ E_{c}^{o}(\theta)$

10. $(S_{o}^{a,0,1}, S_{o}^{a,1,1}) = \{\text{power allocation of (40) and (41)} \mid \tau^{'} \}$

$\varepsilon_{1}$ and $\varepsilon_{2}$ are the stepsize for the multipliers. They have been chosen to be dynamic values which are updated every iteration by the same approach given in Algorithm 1. This accelerates the convergence as can be seen in Fig 3(b).

5 Underlay-Overlay Selection criterion

In the previous sections, we have obtained optimal power allocation for both underlay and overlay schemes. However, underlay and overlay schemes have different features. For example, the underlay strategy does not need to perform spectrum sensing but the interference power constraint should be satisfied all the time. While a portion of time is assigned to sensing process for the overlay case, the cognitive network only needs to meet the interference power requirement when the PU is detected to be active. Therefore, the underlay and overlay approaches may have their respective advantages under diverse propagation environment and system parameters. If the cognitive network can dynamically choose the DSA strategies, the performance of the whole network could be further improved.

In this section, we aim to develop a selection criterion for the cognitive network based on PUs’ spectrum-occupancy probability and the QoS requirement. The main reason for choosing these two metrics can be explained as follows. First, when the cognitive and primary networks are given, the maximum transmit power of the CU and the interference threshold that can be tolerated by the PU are determined and will remain unchanged for a relative long time period. Second, the PUs’ traffic load, which affects the spectrum access activity of the primary network and thus the performance of the cognitive network, is time-varying and can be well reflected by the PUs’ idle probability. The heavier traffic load will cause higher spectrum-occupancy probability and vice versa. Accordingly, the PUs’ spectrum-occupancy probability should be included in the selection criterion. Third, QoS guarantee is critically important to the wireless communication system and especially for the cognitive networks. As different applications and services may have diverse QoS requirement, it should also be taken into consideration.

Now, we first take a closer look at the relationship between the QoS requirement and the QoS exponent. For a given discrete service rate process, the effective capacity $E_{c}$ can be calculated from (6). The corresponding constraints for queue length-bound violation and delay-bound probabilities are determined from (3) and (4) by

$$\Pr\{Q > q^{th}\} \approx e^{-k_{q}q^{th}} \leq P_{Q}, \quad (43)$$

and
\[ \Pr[D > d^\text{th}] \approx e^{-\theta d^\text{th}} \leq P_D, \]

(44)

where \( P_Q \) and \( P_D \) are the maximum queue length-bound violation and delay probabilities that are allowed by the system, respectively.

The QoS requirements depend on the type of service, for example for audio, \( d^\text{th} = 50\text{ms} \) and the violation probability \( P_D = 10^{-2} \), while for video communication \( d^\text{th} = 150\text{ms} \) and \( P_D = 10^{-3} \) [22]. According to these requirements, the limits of the value of \( \theta \) can be specified. Consequently, the QoS exponent \( \theta \) should satisfy

\[ \theta \geq -\frac{\ln P_Q}{q^\text{th}}, \quad \text{and} \quad \theta \geq -\frac{\ln P_D}{e^\text{th}}. \]

(45)

Note that \( \epsilon \) is defined in Subsection 2.2.

Substituting the optimal resource allocation schemes \( \{S_x^{u,0}, S_x^{u,1}\} \) and \( \{S_x^{o,0}, S_x^{o,1}, \tau^*\} \) for the underlay and overlay schemes given in (19), (22), and (40), (41) respectively, into (11) and (35), respectively, we can obtain the maximum effective capacity for the underlay and overlay schemes, denoted by \( E_C^{u,\text{opt}}(\theta) \) and \( E_C^{o,\text{opt}}(\theta) \), respectively.

The methodology of the DSA selection criterion can be summarized by considering a two-dimensional plane, where the \( x \)-axis represents the QoS exponent \( \theta \) and the \( y \)-axis denotes the idle probability \( P_I \). Then, we can divide the plane into two regions, denoted by \( \mathbb{R}^\alpha \) and \( \mathbb{R}^\beta \), respectively. The point \((\theta, P_I)\) in \( \mathbb{R}^\alpha \) satisfies \( E_C^{u,\text{opt}}(\theta) \geq E_C^{o,\text{opt}}(\theta) \), and the point \((\theta, P_I)\) in \( \mathbb{R}^\beta \) satisfies \( E_C^{o,\text{opt}}(\theta) \geq E_C^{u,\text{opt}}(\theta) \). Given the delay bound and its violation probability, the corresponding QoS exponent \( \theta \) can be calculated. If the point \((\theta, P_I)\) falls into \( \mathbb{R}^\alpha \), the cognitive network should choose the underlay scheme. On the other hand, if \((\theta, P_I)\) falls into \( \mathbb{R}^\beta \), the cognitive network should select the overlay scheme.

6. Numerical Results and Performance Analysis

In this section, numerical results are presented to evaluate the performance of the proposed power allocation strategies for both underlay and overlay schemes. In the calculation, the frame duration is set to \( T = 50\text{ms} \), the sampling frequency is 100 K samples/s. Unless otherwise is stated, the probability of the channel to be idle is set to \( P_I = 0.4 \) as a target value. Other parameters are listed in the corresponding figures.

First, we evaluate the convergence of our algorithm. \textbf{Fig 3(a)} and \textbf{Fig 3(b)} show the iterative algorithms for determining the optimal Lagrangian multipliers for underlay and overlay schemes respectively. The maximum secondary transmit power is set to 10dBW, i.e., \( S_p = 10\text{W} \). The interference threshold and the primary transmit power are set to 0dBW and \( S_p = 10\text{dBW} \) respectively. As shown in \textbf{Fig 3}, the Lagrangian multipliers can quickly converge to their optimal values when choosing dynamic stepsizes stated in Algorithm 1. For overlay scheme, the optimal sensing time needs to be determined from one-dimensional exhaustive search by Algorithm 2, which will increase the algorithm complexity. As it can be seen in \textbf{Fig 3(b)}, the overlay system requires a higher number of iterations for convergence.

\textbf{Fig 3: Tracking the optimal Lagrangian multipliers for both underlay and overlay schemes.} \( 1^\text{th} = 0\text{dBW}, \ S_p = 10\text{dBW}, \ S_s = 10\text{dBW}, \) and \( \theta = 10^{-3} \)
Fig 4 presents the normalized effective capacity (which is defined as the effective capacity divided by $TB$ and thus has the unit of “bits/sec/Hz”) of both schemes as a function of the QoS exponent $\theta$. In Fig 4(a), we can observe that, in underlay scheme, the QoS exponent $\theta$ plays a critically important role in the maximum throughput of the cognitive network. When $\theta$ is small (i.e., the QoS constraint is loose), the cognitive network can realize higher throughput. On the contrary, when $\theta$ is large (i.e., the QoS constraint is stringent), the cognitive network can only support lower arrival rates. Figure Fig 4(b) illustrates the normalized effective capacity as a function of delay exponent and the impact of the probability of the channel being idle, $P_i$ for the overlay scheme. We can observe that the effective capacity is a decreasing function of $\theta$ and is an increasing function of the probability ($P_i$). In loosely QoS constraint system, the effective capacity variation is insignificant, while for more stringent QoS constraint, the capacity degrade is dramatically. Higher $P_i$ means the cognitive transmitter assumes that the channel is idle with a higher probability. The CU exploits the situation and transmits with a higher power level, which in turn, gains more capacity. While at lower $P_i$, the cognitive transmitter assumes the channel is busy with a higher probability (since $P_b = 1 - P_i$) and thus, it reduces the transmit power to comply with the interference constraint.

![Fig 4: Normalized effective capacity versus exponent delay for underlay and overlay schemes. $S_{\text{out}} = 10\text{dBW}$.](image)

To compare the effective capacities for the proposed adaptive and non-adaptive schemes, Fig 5 shows the effective capacity as a function of QoS exponent $\theta$ for adaptive and non-adaptive schemes. The top two curves are for overlay and the two lower curves for underlay. Solid curves show the result for adaptive scheme and dashed curves for non-adaptive scheme. It is shown that the effective capacities under the adaptive schemes are always higher than the fixed allocated power policies. Moreover, it can be noticed that the rate that the performance degrades in the adaptive algorithms is faster than that of the non adaptive algorithm. This means that the adaptive algorithm is more sensitive to the QoS delay exponent variation.
The transmit power of the primary network $S_p$ is another critical parameter that affects the performance of the cognitive network. 

**Fig 6** shows the effective capacity versus the transmit power of the primary user $S_p$ for both schemes with different QoS exponent $\theta$.

We can observe that the performance of the cognitive network degrades as $S_p$ increases. This phenomenon can be explained as a larger $S_p$ will cause more severe interference to the CU receiver. This causes the CU transmitter need more power to overcome the negative impact brought by the interference of the PU sender. However, as shown in Figure 5, when the QoS exponent is small (i.e., $\theta = 10^{-3}$), the performance loss of the underlay cognitive network is not obvious. On the contrary, the performance of the cognitive network degrades more obvious when the QoS exponent is large (i.e., $\theta = 0.1$). The reason for this observation is that a small $\theta$ denotes loose QoS requirement and the power allocation becomes water-filling as we explained in Subsection 2.2, thus the power resource can be more efficiently utilized. However, large $\theta$ means stringent QoS requirement, which results the cognitive network transmit with constant rate. Therefore, more power is used to overcome the more serious interference caused by larger $S_p$ and the upper bounded power resource is less efficiently utilized.

**Fig 6**: Normalized effective capacity versus primary transmit power with different $\theta$, $P_i = 0.4$, $I_{th} = 0\text{dBw}$. 

Furthermore, the above figures (Figs 4-6) show that the overlay scheme provides higher capacity than the underlay.
scheme which is consistent with the results reported in most previous literature, for example, see \[4,42\]. It can be also concluded that the overlay scheme is more susceptible to $\theta$ than the underlay scheme, this is more observable in high stringent delay systems.

**Fig 7** investigates how the secondary user’s transmit power and the interference power suffered by the primary user of both schemes vary with $P_i$. **Fig 7(a)** illustrates the secondary user’s transmit power versus the QoS exponent $\theta$ under different spectrum-idle probabilities $P_i$. It can be observed that the transmit power of the overlay scheme remains constant for all $\theta$ and does not vary with $P_i$, which means that the transmit power constraint given by (36) is always active and the upper bounded power resource can be fully utilized.

![](image1.png)

**Fig 8**: Secondary transmit and interference power for both schemes for the proposed optimal power allocation. $S_m = 10\text{dBw}$ and $S_p = 10\text{dBw}$.

On the contrary, the transmit power of the underlay schemes varies with $P_i$. The larger $P_i$ is, the less the utilized transmit power is. However, the underlay based cognitive network can achieve better performance with larger $P_i$. Consequently, as $P_i$ increases, the underlay scheme can derive better performance but consume less transmit power, which denotes that the transmit power resource is more efficiently utilized. Moreover, it can be seen that the transmit power of the underlay scheme always converges to the average interference power constraint ($I_{th} = 5\text{dBw}$). This is because the underlay based cognitive network will gradually converge to an interference-power constrained system as the QoS constraint becomes more stringent. **Fig 7(b)** shows the average interference power versus $\theta$ for different $P_i$. Different from **Fig 7(a)**, we can observe that the interference power constraint of the underlay based cognitive network is always satisfied for different $\theta$ and $P_i$. However, the interference power caused by the overlay schemes varies with $\theta$ and $P_i$. Specifically, the interference power of the overlay strategy increases as $\theta$ becomes large and decreases as $P_i$ increases. Such phenomena can be explained as follows. First, when $\theta$ becomes larger, the CU will transmit with constant rate, and more transmit power will be allocated to the worse channel conditions. Due to the imperfect spectrum sensing, the CU transmitter uses larger power for transmission when the miss-detection happens and thus will cause large interference power. Second, when $P_i$ increases, the actual probability of miss-detection $(1 - P_i)(1 - P_d)$ decreases and thus the interference power caused by the overlay scheme is reduced. Moreover, we can find from Figure 7 that when $P_i$ is large, the interference power of the overlay scheme is always lower than the threshold and thus the overlay scheme is a transmit-power constrained system. While when $P_i$ is small, the characteristics of the overlay scheme vary with $\theta$. The overlay based system is a transmit-power constrained system, for small $\theta$, and both transmit and interference power constraints are active for large $\theta$.

**Fig 8** shows the effective capacity versus sensing time $\tau$ for different $P_i$. We can observe that the effective capacity is a concave function with respect to $\tau$. In addition, it is also noticed that the optimal sensing time for each curve changes with $P_i$. The cognitive user needs more sensing time to achieve optimal capacity when $P_i$ is low.
Fig 8: Effective capacity of the overlay scheme versus sensing time for different $P_1$, $S_s = S_p = 10\text{W}$, $\theta = 0.1$, and $I_{th} = 0\text{dB}$

In Fig 9, each curve divides the plane into two regions. The region below the curve is $R^u$, where the point $(\theta, P_1)$ satisfies $E_C^{u,\text{opt}}(\theta) \geq E_C^{o,\text{opt}}(\theta)$, and the region above the curve is $R^o$, where the point $(\theta, P_1)$ satisfies $E_C^{o,\text{opt}}(\theta) \geq E_C^{u,\text{opt}}(\theta)$. Based on the figure, the access strategy selection criterion proposed above can be performed. If the point $(\theta, P_1)$ falls below the curved border, the cognitive network should choose the underlay scheme. On the contrary, the cognitive network should choose the overlay strategy if $(\theta, P_1)$ falls above the border. If the point falls exactly on the border, which means $E_C^{o,\text{opt}} = E_C^{u,\text{opt}}$, it gives the same effective capacity for both schemes. Considering the higher implementation complexity of the overlay scheme, the preferred choice is then the underlay scheme.

Fig 9: The probability $P_1$ versus the QoS exponent $\theta$ for different interference threshold. Showing the border line of the optimal capacity for both schemes.

Furthermore, Fig 9 presents the border curves under different interference thresholds. We can observe from the figure that the area of the region $R^u$ increases when the interference threshold $I_{th}$ becomes larger. Generally, from the analysis and the comparison of the two different DSA schemes, it can be deduced that the selection of the efficient strategy depends on several factors. It is a function of the channel occupancy probability, as well as a function of the interference threshold allowed. The choice of the scheme has also to be compromised between complexity and the achievable performance. By observing the above analysis and results, we can notice that the less busy the primary channel with loose QoS requirement (i.e., high $P_1$ and small $\theta$), it is preferred to use the overlay scheme for higher achievable capacity. On the other side, when the system is more harsh (i.e., low $P_1$ and large $\theta$), underlay scheme is preferred to maintain the achievable capacity performance.
7 Conclusions

In this paper, we integrated the concept of effective capacity into information theory and developed the optimal resource allocation for both underlay and overlay DSA schemes. We studied the impact of the delay-QoS constraint on the network performance, and considered the PUs’ spectrum-occupancy probability. Analytical results for the effective capacity for both schemes were derived. Optimal power allocations to achieve maximum effective capacity were also obtained.

The underlay and overlay schemes have their respective advantages under different QoS requirement and idle probability. We proposed a selection criterion to determine whether to use underlay or overlay scheme under the given QoS constraint and the primary user’s spectrum occupancy probability to optimize effective capacity. Moreover, it is concluded that the transmit power of the underlay strategy and the interference power of the overlay strategy vary with the PUs’ spectrum-occupancy probability, but the transmit power of the overlay strategy and the interference power of the underlay strategy always remain unchanged. The overlay scheme is more susceptible to $\theta$ than the underlay scheme. This is more observable in high stringent delay constraint systems.

Numerical results were presented to give more interesting observations and insights of system performance.

Appendices

Appendix A

As the PU active probability is taken into consideration for the underlay scheme, the solution of the effective capacity of (6) can be found as [21], [23]

$$E_C^u(\theta) = -\theta \log_e (\rho(P_u \cdot D^u)),$$

where $D^u$ is a diagonal matrix given by

$$D^u = \text{diag}[d^u_1(\theta), d^u_2(\theta)] = [\mathbb{E}[e^{-\theta R^u_0}], \mathbb{E}[e^{-\theta R^u_1}]],$$

and $\rho(\cdot)$ function is the spectral radius of a matrix, which is defined as the maximum of the absolute values of its eigenvalues, i.e., $\rho(A) \overset{\text{def}}{=} \max_i |\omega_i|$, $\omega_i$'s are the eigenvalues [34]. $P_u$ in [A-1] denotes the transition probability matrix of the underlay based cognitive radio network.

The system has two system states. As the primary network independently selects whether to access the spectrum or not at the beginning of each frame, the two system states are time-uncorrelated. Thus the entries of the $(2 \times 2)$ transition probability matrix can be written as

$$
\begin{align*}
P^u_{00} &= p^u_{10} = P_i, \\
P^u_{01} &= p^u_{11} = P_b.
\end{align*}
$$

[A-2]

Since the transition probability matrix $P^u$ has a unit rank, the spectral radius of matrix $(P^u \cdot D^u)$ can be easily calculated by

$$\rho(P^u \cdot D^u) = \text{trac}(P^u \cdot D^u) = P_i \mathbb{E}[e^{-\theta R^u_0}] + P_b \mathbb{E}[e^{-\theta R^u_1}]$$

[A-3]

Substituting the above equation into (A-1), the formula in (8) is obtained.

Appendix B

The objective function in (18) is nonnegative weighted sum of the functions $f_1^u(x)$ and $f_2^u(x)$ defined in (20). According to the definition of Hypergeometric function given in (12), this function is a polynomial function with nonnegative exponents, and thus it is strictly convex. The function $f_3^u(y)$ in (20) is monotonically increasing function for $y > 0$ as shown in Fig 10. As a result, we can conclude that the objective function in (18) is convex [26,31].
The function \( \mathcal{L} \) is needed. Assuming that the value of the Lagrangian multipliers for the \( P_i \), \( x_i \), and \( y_i \) be written as 

\[
\max_{\lambda \geq 0, \nu \geq 0} \min_{x, y} \mathcal{L}(x, y, \lambda, \nu)
\]

where \( \lambda \) and \( \nu \) are the Lagrangian multipliers associated with the transmit power and interference power constraints, respectively, and \( E[S] = \mu x \). Then, the Lagrange dual problem can be formulated as 

\[
\max_{\lambda \geq 0, \nu \geq 0} \min_{x, y} \mathcal{L}(x, y, \lambda, \nu)
\]

where \( \lambda \) and \( \nu \) are related to \( \alpha \) and \( \mu \) in (20) respectively.

Performing the partial derivatives of the unconstrained problem \( \mathcal{L}(x, y, \lambda, \nu) \) with respect to each variable and setting it to zero, this leads to a series of subproblems. Solving this system of equations yields the optimal solution \( \{x^*, y^*\} \) of the subproblem

\[
\begin{align*}
C_1 f_1'(x^*) + C_2 f_2'(x^*) + \lambda P_i + \nu \mu I &= 0 \\
C_i f_i'(y^*) + \lambda P_i + \nu \mu I &= 0 \\
P_i x^* + P_b y^* - S^m &= 0 \\
P_i \mu x^* + P_b \mu y^* - I^1 &= 0
\end{align*}
\]

where, \( f' \) is the first derivative of the function \( f \) with respect to the relevant variable. The last two linear equations, (B-5), (B-6) are linearly dependent, and hence, infinite solutions can be obtained. While, solutions for \( x^* \) and \( y^* \) can be obtained from (B-3) and (B-4) respectively. Plugging \( \{x^*, y^*\} \) into (B-2), we can solve the dual problem and find the optimal Lagrangian multipliers, denoted by \( \{\lambda^*, \nu^*\} \).

Finally substituting \( \{\lambda^*, \nu^*\} \) into \( \{x^*, y^*\} \), we can derive the optimal power allocation \( \{x^*, y^*\} \) given by (19) and (22).

In order to use the iteration method illustrated in Algorithm 1 to derive the power allocation \( \{x^*, y^*\} \), the subgradient of the Lagrange dual function is needed. Assuming that the value of the Lagrangian multipliers for the \( i^{th} \) iteration are \( \{\lambda^i, \nu^i\} \) and the corresponding optimal power allocation are \( \{x^i, y^i\} \), the subgradient for the dual function, denoted by \( \{\mathcal{L}^i_{\lambda}, \mathcal{L}^i_{\nu}\} \), can be written as

\[
\begin{align*}
\mathcal{L}^i_{\lambda} &= P_i x^i + P_b y^i - S^m \\
\mathcal{L}^i_{\nu} &= P_i \mu x^i + P_b \mu y^i - I^1
\end{align*}
\]

The same can be done for the overlay scheme.
\[ \mathcal{L}^{i,j}_A = (T - \tau)B((1 - P_i)P_y^i + (1 - P_d)P_b^i) - S \]
\[ \mathcal{L}^{o,i}_v = (1 - P_d)P_b^i - \mu \]

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