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Reduced Order Modeling of Linearized Power Electronic Cricuit

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Abstract

In this paper, most of the systems in our daily life consist of power electronic circuits. They are micro circuits which are difficult to be analyze because the circuit consists of nonlinear elements. This paper deals with one of the power electronic circuit, boost converter. The non-linear elements of the converter circuit are linearized. The obtained linearized converter circuit is a higher order model. Designing a controller for this higher order model is complex and simulation time also increases. So, efficient order reduction techniques have been used to reduce the system order. The models are simulated using MATLAB 13.0 simulink software. Simulation results show that hankel was a better method for order reduction depicting same characteristics for higher and lower order models.

Keywords: Order reduction; Boost Converter; Hankel**.**

1. INTRODUCTION

Switching power converters pose several unique problems in the construction of efficient time-domain simulators. Events of interest in a typical power converter cover many orders of magnitude on the time scale, starting from switching transitions in the order of nanoseconds to closed-loop start-up or load transients that may last for seconds[1]. Detailed models that describe physical properties of semiconductor switching devices are used only when results of interests are within one, or at most several switching cycles. Such results include switching losses, lengths of switching transitions, and voltage current overshoots during switching. For majority of other simulation tasks, such as studies of the circuit steady-state waveforms, conversion functions, stability of feedback loops, load, input or reference transients, application of detailed nonlinear models is impractical. This is because simulation time step must be short compared to the switching period, and each simulation step requires computationally intensive iterative solution [2]. If the simulation runs over many switching cycles, simulation time becomes the limiting factor. In order to improve efficiency of time-domain simulation, semiconductor devices are replaced with much simpler models. The simplification is justified by the fact that switching transitions are many orders of magnitude shorter than the total simulation time, and that errors introduced by ignoring details of the switching transitions are insignificant in the results expected from long-term simulations. Numerous methods specifically geared toward efficient long-term simulation of switching power converters have been developed. An ideal switch has zero impedance when on, zero admittance when off, and switches between the two states in zero time. With *n* ideal, single-pole, single-throw switches, the switching converter network reduces to one of two possible switched networks (without switches). Then, approach is to write and solve state-space equations for each of the switched networks, and to establish conditions for transitions of switched networks. These switched networks is of higher order and needs to be reduced to lower order for less complication in designing controller circuits for the switching converters. These are various order reduction methods are available. Among which modal order reduction is one of them. an equivalent circuit is directly generated from the reduced transfer function obtained using MOR based on Pade approximation via the Lanczos process[3].

Model order reduction is a technique for reducing the computational complexity of mathematical models in numerical simulations. Many modern mathematical models of real-life processes pose challenges when used in numerical simulations, due to complexity and large size (dimension), model order reduction aims to lower the computational complexity of such problems [4]. The oldest method is pade approximation method for reducing the higher order to lower order[5]. There is a different mixed order technique also available for reducing the order. In that one of the mixed methods is routh-pade approximation [6]. One of the new order reduction technique considered in the paper is Hankel reduction method. Hankel reduction is a stochastic realisation theory. Several methods based on Hankel matrix have been used for deriving lower order state models from a given complex system described by its transfer function matrix or state model. The minimal realization can be achieved in fixed number of operations on the Hankel matrix. The method is applicable to linear SISO and MIMO dynamic systems [8].

2. SYSTEM MODELLING

2.1. General Boost Converter

In boost converter, the output voltage is greater than the input voltage – hence the name "boost". A boost converter (step up converter) is a DC to DC power converter that steps up the voltage (while stepping down current) from its input as supply to its output as load. It is a class of switched mode of power supply (SMPS) containing the at least two semi conductors (one is a diode and one is transistor) and at least one energy storage element. A capacitor, inductor or the two I combination to reduce the voltage ripple, filters made of capacitors (sometimes in combination with inductors) are normally added to such a converters output (load side filter) and input (supply side filter). The switch in a boost converter is typically a MOSFET, IGBT or BJT are used as the switches.

The ideal schematic diagram for the boost converter is shown in below.

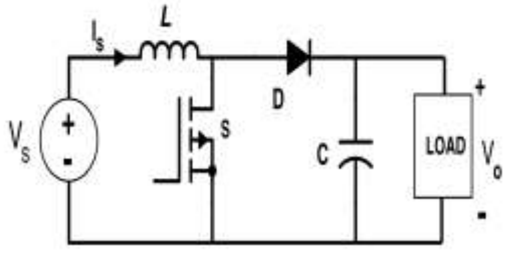

Fig. 1: Schematic Circuit of Boost Converter

2. 2. Detailed High-Order Model

In the ideal boost converter has the linear and nonlinear components. So we can cange the components of nonlinear to linear as shown in below. Power converter model synthesis consists of component models and control laws. First, high-order detailed models of switching-converter components diode, switches are shown below, are set forth. A wide-bandwidth inductor model includes equivalent series resistance, r_L, and lumped shunt parasitic capacitance, c_L. The equivalent series resistance, r_C, and inductance, L_c, of the capacitor are extracted from the hardware prototype using impedance characterization. Switching-component modeling is more challenging, as the resulting model should predict accurately both steady-state characterizations as well as fast dynamics. The MOSFET is represented as a switching state dependent resistance with appropriate drain to source parasitic capacitance, C_{sw}, and wiring inductance, L_{sw}.

These values can be found in MOSFET data sheets. The static V-I characteristics of the diode can be modeled as a diode state-dependent series resistance and an offset voltage source. The capacitance exhibited by semiconductor-metal junctions plays a dominant role in turn-on/-off transients. Therefore, the switching transient dynamics, such as reverse recovery, are accounted for by a diode state-dependent linear capacitor, C_d. The capacitance is higher when the diode is off. A series resistance is considered with this capacitor, rc_d, to damp the reverse recovery current. Wiring inductance and resistance of the diode ($L_d$and $r_{Ld)}$ are also considered. A different variation of this diode model is presented. It should be noted that proposed models in above Fig, are just one form of model development; one can also use alternative piecewise-linear high-fidelity component models

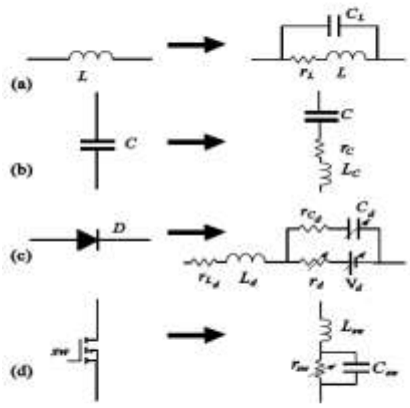

Fig. 2: Highly detailed behavioral component models: (a) Inductor; (b) Capacitor; (c) Diode; (d) MOSFET

In the above figure switching component models and, subsequently, the final converter model depend on the state of switching components. Switching state and timing are either externally determined by a command signal (transistors turn on/off), or internally resolved by meeting appropriate threshold conditions (e.g., diodes). Mathematically, the switching time constraint equation can be expressed as

$$c^j(x^j(t_f^j), u(t_t^j), t_f) = 0$$

The continuous state-space model is determined by partitioning the circuit graph to the spanning tree and link branches, and choosing the inductive link currents and capacitive tree voltages as the state variable. This process is automated in available numerical toolboxes (e.g., automated state model generator). Based on the component models in Fig.2,the state vector consists of inductor currents and capacitor voltages of both bulky and parasitic components

$$x = [i_L^i, v_{C_L}^i, v_C^j, i_{L_c}^j, i_{L_{sw}}^m, v_{C_{sw}}^m, i_{L_d}^n, v_{C_d}^n]^T$$

Where i=1……..k_L, j = 1……..K_C, m = 1……K_{sw}, and n = 1……..K_d, k_L, k_C, k_{SW}, k_d are the number of inductors, capacitors, active switches, and diodes. The input vector is composed of the input voltage sources, load currents, and the diode voltage drops.

$$u = [v_g^1, ..., v_g^{k_g}, i_{load}^1, ...i_{Load}^{k_{iload}}, V_d^1(on), ..., V_d^{k_d}(on)]'$$

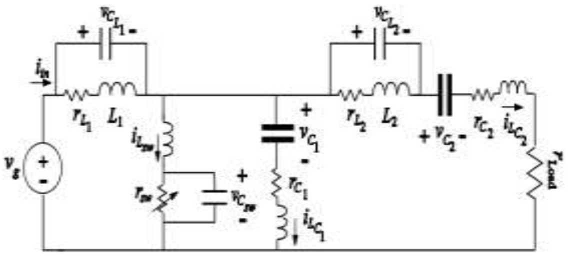

Fig. 3: Detailed High-Order Model

Considering, the state variables and input variables as below

$$x = [i_L^i, v_{C_L}^i, v_C^j, i_{L_c}^j, i_{L_{sw}}^m, v_{C_{sw}}^m, i_{L_d}^n, v_{C_d}^n]^T \dots$$

$$U = [V_S, i_{load}]$$

State space variables are obtained from higher order model as in fig.3. Using block reduction techniques. The state variables are obtained as

$$A = \begin{bmatrix} -\frac{r_L}{L} & 0 & \frac{1}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C} \\ -\frac{1}{C_L} & 0 & 0 & \frac{1}{C_L} & 0 & \frac{1}{C_L} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_{sw}} & 0 & -\frac{1}{L_{sw}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_{sw}} & -\frac{1}{r_{sw}C_{sw}} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{L_d} & 0 & 0 & -\frac{r_{LOAD}+r_{L_d}}{L_d}-\frac{(r_d+r_{C_d})}{(r_{C_d}+r_d)L_d} & -\frac{1}{L_d}-\frac{r_{C_d}}{(r_{C_d}+r_d)L_d} & \frac{r_{LOAD}}{L_d} \\ 0 & 0 & 0 & 0 & 0 & \frac{r_d}{(r_{C_d}+r_d)C_d} & -\frac{1}{(r_{C_d}+r_d)C_d} & 0 \\ 0 & -\frac{1}{L_C} & 0 & 0 & 0 & \frac{r_{LOAD}}{L_C} & 0 & \frac{r_{LOAD}+r_C}{L_C} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{L_{sw}} & 0 \\ 0 & 0 \\ \frac{1}{L_d} & -\frac{r_{C_d}}{(r_{C_d}+r_d)L_d} \\ 0 & \frac{1}{(r_{C_d}+r_d)c_d} \\ 0 & 0 \end{bmatrix}, C = I_{8,} D = [0]_{8x2}$$

The obtained system is a 8^{th} order system. As the realistic model of the system was high in dimension, that a direct simulation or design would be neither computationally desirable nor practically possible in this case. Thus, reduction of system model is highly desirable. From this above state matrices we get the transfer function by substitute the suitable values for the variables. From this state space matrix we get the higher order transfer function. In theoretically the state space transfer function find by using the

$$T/F = C\,(SI - A)^{-1}\,B + D$$

3. ORDER REDUCTION

The development of Model Order Reduction (MOR) in terms of its application in Power system stability, Control system design, designing of reduced order estimators, etc. Recently MOR is developed in the area of power systems and control theory, which studies properties of dynamical systems in application for reducing their complexity, while preserving their input-output behaviour as much as possible. Nowadays, model order reduction has been a flourishing field of research, both in systems and control theory and in numerical analysis. This has a very healthy effect on MOR as a whole system, bringing together different techniques and different ideas.

There are different types of order reduction techniques are available, in that some of the order reduction techniques used in this work are

1. Pade approximation
2. Modal reduction
3. Hankel reduction

3.1. Pade Approximation Reduction

The Pade approximation was introduced by Pade in 1892 and it was extended by wall in 1948.

Consider a function

$f(x) = c_0+c_1x+c_2x^2+c_3x^3+\dots\dots\dots+c_nx^n$

and a rational function $\frac{u_m(x)}{v_m(x)}$ are the m^{th} order polynomial in $m \angle n$. the rational function $\frac{u_m(x)}{v_m(x)}$ is set to be Pade approximation of f(x) if and only if the first (m+n) terms of power series expansion of f(x) and rational function $\frac{u_m(x)}{v_m(x)}$ are identical.

For the function f(x) is to be approximated, let the following Pade approximant can be defined as $\frac{u_m(x)}{v_m(x)}$ =

$$\frac{a_0 + a_1x + a_2x^2 + a_3x^3 + \dots.. + a_mx^m}{b_0 + b_1x + b_2x^2 + b_3x^3 + \dots\dots + b_nx^n},$$

$G_k(s)$ is higher order and

The reduced order transfer function is $R_k(s)$.

$$R_k(s) = \frac{P_k(s)}{Q_k(s)} = \frac{a_0 + a_1s + as^2 + \dots\dots}{b_0 + b_1s + b_ss^2 + \dots\dots\dots\dots}$$

m = order of highest of numerator

n = order of highest of denominator

This is the reduced order transfer function for the considered system in Pade approximation.

3.2. Modal Reduction

Balancing is an important approach for modal reduction of controlled systems which consists of two steps: the first step is to find a transformation that balances the controllability and observability gramians in order to determine which states have the greatest contribution to the input-output behavior. The next step is to perform a Galerkin projection onto the states corresponding to the largest singular values of the balanced gramians for the region of interest in state-space. In order to perform model reduction via balancing, three components are required: a controllability gramian, an observability gramian, and a transformation matrix which balances the system.

Gramians (or covariance matrices) and the transformation are required for balanced model reduction. The routines for unscaled systems are mainly for verifying these routines by comparison against the MATLAB commands for linear systems. In practice, the routines for scaled systems are applied as it needs to be taken into account that a state changing by orders of magnitude can be more important than a state which hardly changes, even though its steady state may have a smaller absolute value. After obtaining a balanced system, it needs to be determined how many states can be reduced and which reduction method to use. The former problem can be solved by a trial and error procedure while taking into account the magnitude of the Hankel singular values of the states to be reduced. The answer to the latter question is that balanced truncation is the method of choice for nonlinear systems as other techniques.

3.3. Hankel Order Reduction

This is a stochastic realization theory with the Hankel matrix present a new procedure for obtaining a reduced-order state variable model for a stationary Gaussian process. Furthermore, we also show that the error in our N-dimensional reduced order model is bounded by the N + 1 singular value of the system's Hankel matrix. The Hankel matrix results have also been used and elsewhere in deterministic and stochastic model reduction. The technique is completely different from others. We use the stochastic realization theory in and solve a different model reduction problem.

Throughout, we follow the standard notation for Hilbert spaces. The orthogonal projection onto a subspace ς is denoted by P_{ς}. The space $L^2 = L^2(0,2\pi)$ and the inner product on L^2 is defined by

$$(h,g) = (h,g)_{L^2} \,\square\, \frac{1}{2}\int_0^{2\pi} h(e^{it})\overline{g(e^{it})}dt \qquad (h, g \in L^2).$$

Moreover, H^2 is the Hardy space of analytic functions in L^2. To be precise, $f \in H^2$ if and only if f is in L^2 and $(f, e^{-int}) = 0$ for all $n > 0$. Throughout, $\Gamma(\psi)$ is the Hankel operator on H^2 with symbol ψ (in L^∞) defined by

$$\Gamma(\psi) f(e^{it}) = P_{H^2}\, \psi(e^{it}) f(e^{-it}) \qquad (f(e^{it}) \in H^2).$$

Throughout, *y(n)* is a purely nondeterministic stationary Gaussian random process. The process *y(n)* can be generated by a stable state variable model of the form

$$x(n+1)=Ax(n)+Bu(n)$$

$$y(n)=Cx(n) \qquad (1)$$

Where A, *B*, and *C* are operators on the appropriate space, and *u(n)* is a Gaussian white noise process such that *x(m)* is independent to *u(n)* for all $m \le n$. The output covariance sequence is given by

$$R_n = E\,(y(n)\,\overline{y(0)}) = CA^nXC^*\ (n \ge 0) \qquad (2)$$

Where *X* is the state covariance satisfying the discrete Lyapunov equation

$$X=AXA^*+BB^* \qquad (O<X<\infty) \qquad (3)$$

System *A, B, C, X* is called a stochastic realization of the covariance sequence *R*, when *(2)* and (3) hold.

Since *y(n)* is purely nondeterministic, there exists unique outer or minimum phase factor θ in H^2 such that

$$R_n = (y(n), y(0)_\upsilon \,\square\, E(y(n)y(0))$$

$$= \frac{1}{2}\int_0^{2\pi} e^{int}\theta(e^{it})\overline{\theta(e^{it})}dt \,\square\, (e^{int}\theta, \theta). \qquad (4)$$

Without loss of generality it is assumed that $R_0 = 1$ or equivalently $\square\theta\square = 1$. Let $y = V^{\infty}_{-\infty}y(n)$ be the Hilbert space generated by the process *y(n)* with the inner product determined by the expectation in (4). By (4) there exists a unitary operator *Y* mapping ϑ onto L^2 such that $Yy(n) = e^{int}\theta$. Therefore, *y(n)* is unitarily equivalent to $e^{int}\theta$. In particular, y(0) can be identified with $\theta(= Yy(0))$. Our strategy is to obtain a reduced order model for the process $e^{int}\theta$ on L^2 .Since $Yy(n) = e^{int}\theta$, this yields a reduced-order model for *y(n)*.

4. PROGRAMMING RESULTS

Circuit Parameters of the Boost Converter System:

V_g = 5 volt, L = 1.316 mH, r_L = 0.14 Ω, C_L = 1 pF, L_{sw} = 20 nH, C_{SW} = 200 pF, r_{SW} (on) = 0.2 Ω, r_{SW} (off) = 2.3 MΩ, L_d = 5 nH, r_{Ld} = 1 mΩ, V_d(on) = 0.61 volt, V_d(off) = 0 volt, r_d(on) = 50mΩ, r_d(off) = 40MΩ, C_d(on) = 15pF, C_d(off) = 100pF, r_{Cd} = 5mΩ, C = 42μF, L_c = 100pH, r_C = 0.38 Ω, R_{load} = 10.5Ω, f_{SW} = 10KHz, Duty = 05.

The circuit parameters are used to find the state space model of the circuit. Step response of the system for both inputs are shown in fig.4 and fig.5

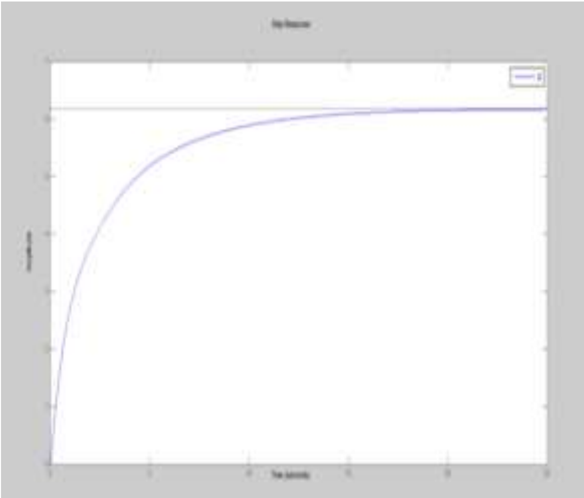

Fig. 4: Original system response for first input.

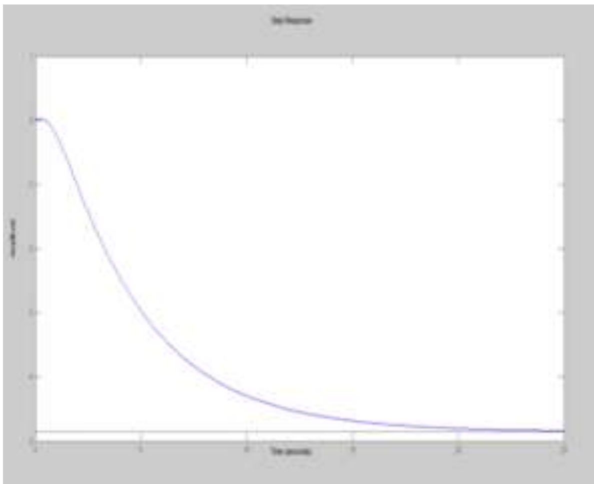

Fig. 5: Original system response for second input.

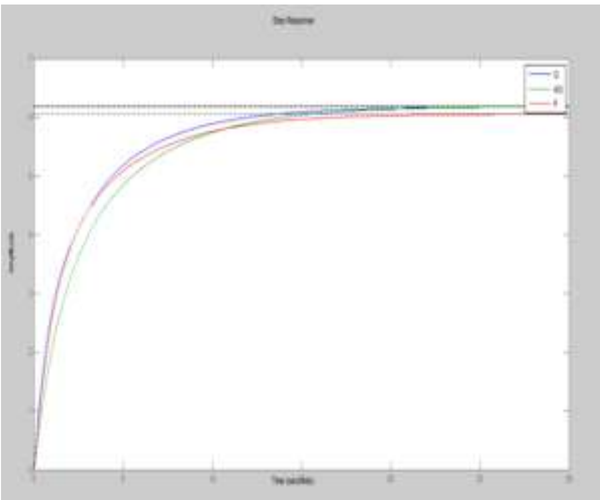

Fig. 6: Response of reduced order models obtained from pade-approximation, modal reduction methods with original system

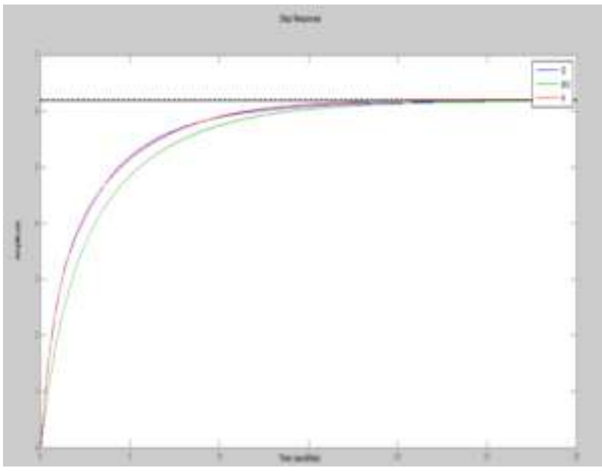

Fig. 7: Response of reduced order models obtained from modal reduction, Hankel reduction methods with original system.

The boost converter circuit considered is of higher order, this system is reduced using pade approximation, modal order reduction and hankel reduction method. The response of the reduced order system obtained from these methods are compared in figs.6&7

5. CONCLUSION

Various controller design techniques have been used to design controller for the boost converter circuit. Hankel based PID controller is best suitable for voltage control of the circuit among all other controllers. The controller

Design is based on reduced order system. Various techniques have been used for order reduction, among which, hankel reduction method was proved to be the reduction method producing reduced model similar to higher order model.

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