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Fundamental Bound State Description of light Atoms and the fine structure constant α ~ 1/137

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Abstract.

Based on a QED like Lagrangian, in which the fermions are dressed by photons, a "complete" bound state description of $p-e^-$ and e^+-e^- systems is presented, in which no external parameter is needed. With a quantum condition linear in the radius, the deduced binding energies are consistent with Coulomb energies and radii in general agreement with other work. The sum of partial coupling strengths is in agreement with the electric coupling constant $\alpha \sim 1/137$, showing that this important quantity can be understood from first principles.

PACS/ keywords: 3.50.Kk, 11.15.-q, 31.15.Ne/ Bound state description of light atoms, based on a modified QED Lagrangian with fermions dressed by bosons. Description of hydrogen and positronium bound states. Integrated coupling strength consistent with the fine structure constant $\alpha \sim 1/137$.

1. Introduction

The study of fundamental forces is the only way to gain insight into the basic structure of matter and the origin of the universe. For a realistic description of these forces a quantum field theory is needed, in which **all** parameters can be derived from first principles (completeness). Such a theory is required for a logic understanding of nature. Quantum electrodynamics (QED) is close to a complete theory. Only with one parameter, the coupling constant $\alpha \sim 1/137$ determined precisely from experimental data, QED gives rise to a quantitative description of spectra of light atoms (by use of the Coulomb potential), fine and hyperfine structure splittings and Lamb shift as well as magnetic moments of leptons. Nevertheless, QED has to be regarded as an effective theory, in which α cannot be derived from first principles. Therefore, a complete version of QED must exist, in which **all** parameters (also the coupling constant) are constrained by basic boundary conditions. In this theory one should understand, why a value of $\alpha \sim 1/137$ has been chosen by nature.

In the description of atoms the effective character of QED is clearly visible in the structure of the Coulomb potential, a bound state potential of fermions. But a free particle bound state in the vacuum (with potential as well as kinetic energy) cannot be composed of fermions only. Its kinetic energy gives rise to rotation, which would be spurious, if the fermion recoil could not be absorbed by other particles (photons). Therefore, in a complete theory the fermions have to be dressed (accompanied) by photons, see ref. [1].

A double bound state structure of fermions and bosons leads naturally to a finite theory (with finite radial moments), since in such a system both fermions and bosons can be described by radial wave functions, which are normalized. Finite dimensions of atomic states are conform with Bohr's model of the atom, but also with

the observation of covalent radii [2]. In the present paper such a theory is discussed, but it is not intended to replace QED entirely by a new theory (many observations may be simpler described in an effective theory) but to show that a complete theory exists, which yields a satisfactory understanding of the electric coupling constant α.

2. Formalism

The Lagrangian with fermions dressed by boson fields may be written in the form

$$\mathcal{L} = \frac{1}{\tilde{m}^2}(\bar{\Psi} D_\nu)\ i\gamma^\mu D_\mu\ (D^\nu \Psi)\ -\ \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\ , \tag{1}$$

where $\tilde{m}$ is the mass parameter and Ψ are charged fermion fields, $\Psi = \Psi^+$ and $\bar{\Psi} = \Psi^-$. Vector boson fields A_μ with coupling g to fermions are contained in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$. The second term of the Lagrangian represents the Maxwell term with Abelian field strength tensors $F^{\mu\nu}$ given by $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, which gives rise to both electric and magnetic coupling.

This Lagrangian includes naturally higher order boson and fermion fields. In the past two arguments have been brought forward against this type of Lagrangian: the necessary $1/\tilde{m}^2$ factor should give rise to uncontrolled divergences in standard (infinite) gauge theories; further, a Lagrangian with higher order fermion fields will lead to nonphysical solutions [3]. However, both arguments do not apply to the present case: the Lagrangian (1) leads to a finite theory due to the necessary normalization of the boson wave functions (as mentioned above). In such a theory the mass parameter $\tilde{m} = m_1 m_2/(m_1 + m_2)$ cannot lead to convergence problems; in contrary, it is needed to establish the consistency of the theory. Further, non-physical solutions can be excluded by strict geometrical and energy-momentum constraints.

By inserting $D^\mu = \partial^\mu - igA^\mu$ and $D_\nu D^\nu = \partial_\nu \partial^\nu - ig(A_\nu \partial^\nu + \partial_\nu A^\nu) - g^2 A_\nu A^\nu$ in eq. (1), the first part of $\mathcal{L}$ gives rise to a number of terms, which contain boson and fermion fields and/or their derivatives. All terms containing the derivative of the fermion fields $\partial^\nu \Psi$ are related to a complex dynamics of the system. For stationary solutions only two terms of the Lagrangian contribute

$$\mathcal{L}_{2g} = \frac{-ig^2}{\tilde{m}^2}\ (\bar{\Psi} A_\nu)\ \gamma^\mu \partial_\mu\ (A^\nu \Psi) \tag{2}$$

and

$$\mathcal{L}_{3g} = \frac{-g^3}{\tilde{m}^2}\ (\bar{\Psi} A_\nu)\ \gamma^\mu A_\mu\ (A^\nu \Psi)\ . \tag{3}$$

From the Lagrangians (2) and (3) fermion matrix elements have been derived, based on generalized Feynman diagrams, see e.g. ref. [4]. These have been used in the general form $\mathcal{M}^f = <g.s.|\ K(p'-p)\ |g.s.> \sim \bar{\psi}(p')\ K(q)\ \psi(p)$, where $\psi(p)$ is a fermion wave function $\psi(p) = \frac{1}{\tilde{m}^{3/2}} \Psi(p_1)\Psi(p_2)$ and $K(q)$ a kernel related to the boson structure of the Lagrangian. In the present case the latter is given by $K(q) = \frac{1}{\tilde{m}^5}\ [O^3(q_i)\ O^3(q_j)]$, in which $O^3(q_i)$ represents a product of boson fields or derivatives given by the square brackets in eqs. (2) and (3). Using $\alpha = g^2/4\pi$ this leads to matrix elements of the form

$$\mathcal{M}_{2g} = \frac{\alpha^2}{\tilde{m}^5}\ \bar{\psi}(p')\ A_\nu(q_4') A^\nu(q_3')\ \gamma_\mu \gamma^\rho \partial_\mu \partial^\rho\ A_\sigma(q_2') A^\sigma(q_1')\ \psi(p) \tag{4}$$

and

$$\mathcal{M}_{3g} = \frac{-\alpha^3}{\tilde{m}^5}\ \bar{\psi}(p')\ A_\nu(q_4')A^\nu(q_3')\ \gamma_\mu\gamma^\rho A_\mu(q_2)A^\rho(q_1)\ A_\sigma(q_2')A^\sigma(q_1')\ \psi(p)\ . \tag{5}$$

One may compare these matrix elements to similar ones derived from the first order QED Lagrangian $\mathcal{L}_{f.o.} = \bar{\Psi}\ i\gamma_\mu D^\mu\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. By writing similarly $\mathcal{M} = \bar{\psi}(p')\ K(q)\ \psi(p)$, with $K(q) = \frac{1}{\tilde{m}}\ [O^1(q_2)\ O^1(q_1)]$ one obtains for the case $\partial\Psi = 0$ only one (boson-exchange) matrix element $\mathcal{M}_{f.o.} = \frac{-\alpha}{\tilde{m}}\ \bar{\psi}(p')\ \gamma_\mu\gamma^\rho A_\mu(q_2)A^\rho(q_1)\ \psi(p)$. Since the boson-fields $A^\mu(q_i)$ are relativistic, they overlap only momentarily and do not form a stable potential. Only in the non-relativistic limit (which is not realized for strongly bound atomic states) one can write $\mathcal{M} = \bar{\psi}(p')\ V(q)\ \psi(p)$, where $V(q) \sim \alpha \cdot 1/q^2$ is the Coulomb potential.

The two theories show two essential differences: 1. The "boson-exchange" matrix element $\mathcal{M}_{3g}$ has a more complex structure than $\mathcal{M}_{f.o.}$ with additional boson fields, needed to balance the fermion motion. 2. A second matrix element $\mathcal{M}_{2g}$ is present, which does not exist in first order theories. This term leads to a dynamical stabilization of the system (discussed below).

From these matrix elements bound state potentials can be deduced. First, by using a gauge condition $\partial^2 A^\nu = 0$ the bosonic part $(\partial_\nu A^\nu(q_2'))(\partial_\sigma A^\sigma(q_1'))$ in eq. (4) can be replaced by $\frac{1}{2}\partial^2(A_\nu(q_2')A^\sigma(q_1'))$. Then (analogue to the fermion wave functions) normalized boson (quasi) wave functions $W_\mu^\nu(q') = \frac{1}{\tilde{m}}A_\mu(q_j')A^\nu(q_i')$ of scalar $(\mu = \nu)$ and vector $(\mu \neq \nu)$ structure are introduced. Further, a boson-exchange interaction is obtained with a form $V_\mu^\nu(q) = \frac{1}{\tilde{m}}A_\mu(q_2)A^\nu(q_1)\ (\mu \neq \nu)$, which is similar to that in first order QED.

By equal time requirement the fermion and boson vectors can be reduced by one dimension, yielding boson wave functions[1] of scalar and vector structure $w_s(q')$ and $w_v(q')$ and an interaction $v_v(q)$. This yields

$$\mathcal{M}_{2g} = \frac{\alpha^2}{2\tilde{m}^3}\ \bar{\psi}(p')\ w_s(q')\ \partial^2 w_s(q')\ \psi(p) \tag{6}$$

and

$$\mathcal{M}_{3g} = \frac{-\alpha^3}{\tilde{m}^2}\ \bar{\psi}(p')\ w_{s,v}(q')v_v(q)w_{s,v}(q')\ \psi(p)\ . \tag{7}$$

The bosonic part of eq. (7) can also be written in the form of a matrix element, in which the wave functions $w(q')$ are connected by $v_v(q)$

$$\mathcal{M}^g = \frac{-\alpha^3}{\tilde{m}^2}\ w_{s,v}(q')\ v_v(q)\ w_{s,v}(q'). \tag{8}$$

In the evaluation of these matrix elements one can rely on the Hamiltonian formalism by relating kinetic and potential energies by $(T+V)\psi = E\psi$, where the binding energy E is determined by use of the virial theorem. Further, we assume that all quantities described in momentum space can be transformed to r-space by Fourier transformation. Energies, masses and momenta are given in energy units (using c=1), whereas the constant $\hbar c$ is used for radius-momentum conversion.

Going to r-space the fermion matrix element (6) can be written by

$$\mathcal{M}_{2g} = \bar{\psi}(r)\ V_{2g}(r)\ \psi(r)\ , \tag{9}$$

in which $V_{2g}(r)$ is a potential, which can be derived from a boson Hamiltonian of a form

$$-\frac{\alpha^2(\hbar c)^2}{2\tilde{m}}\left(\frac{d^2w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\right) + V_{2g}(r)\ w_s(r) = E_i\ w_s(r)\ . \tag{10}$$

[1] with dimension $[GeV]$.

This leads to

$$V_{2g}(r) = \frac{\alpha^2(\hbar c)^2}{2\tilde{m}} \left(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\right)\frac{1}{w_s(r)} + E_o \ . \tag{11}$$

A connection to the vacuum is made by assuming $E_o = E_{vac} = 0$. This potential is of large importance, since it leads to dynamical stabilization of the system: during the overlap of boson fields fermion-antifermion pairs are locked and form a stable system, which cannot decay. The stabilizing potential $V_{2g}(r)$ shows a quite linear rise towards larger radii, see below, very similar to the empirically introduced "confinement" potential needed in hadron potential models [5].

Further, the matrix element (7) can be written in r-space by

$$\mathcal{M}_{3g} = \bar{\psi}(r)\ V_{3g}(r)\ \psi(r)\ , \tag{12}$$

in which the potential $V_{3g}(r)$ has the form of a folding potential

$$V_{3g}(r) = -\frac{\alpha^3\hbar c}{\tilde{m}} \int dr'\ w_{s,v}(r')\ v_v(r-r')\ w_{s,v}(r') \tag{13}$$

with boson wave functions $w_{s,v}(r)$ and an interaction $v_v(r)$. As mentioned above, this potential can also be considered as boson matrix element, in which the bosons are "bound" in the potential $v_v(r)$. It is important to note that due to the condition $E_o = 0$ for $V_{2g}(r)$ both potentials (11) and (13) yield absolute binding energies, for which a constant should not be added (this is different from other potentials, as e.g. the harmonic oscillator potential).

The structure of $\mathcal{M}_{3g}$ gives rise to two states (scalar and vector) with similar fermion and boson wave functions $\psi_{s,v}(r) \sim w_{s,v}(r)$. The fermion wave functions are orthogonal, leading to the constraint

$$\int r^2 dr\ \psi_s(r)\psi_v(r) = \int r^2 dr\ w_s(r)w_v(r) =< r_{w_s,w_v} >= 0\ . \tag{14}$$

$< r_{w_s,w_v} >= 0$ indicates that there is no spurious motion of bosons. This condition is satisfied for

$$w_v(r) = w_{v_o}\ [w_s(r) + \beta R\ \frac{dw_s(r)}{dr}]\ , \tag{15}$$

where w_{v_o} is obtained from the normalisation $2\pi \int r dr\ w_v^2(r) = 1$ and βR is given by $\beta R = -\int r^2 dr\ w_s(r) / \int r^2 dr\ [dw_s(r)/dr]$. Because of the derivative structure, $w_v(r)$ has a smaller root mean square radius than $w_s(r)$. Therefore, a natural geometric condition requires that the interaction for this state takes place inside the bound state volume of $w_s^2(r)$. This leads to the geometrical boundary condition

$$|V_{3g}^v(r)| \simeq c\ w_s^2(r)\ . \tag{16}$$

The conditions (14) and (16) require a form of the boson wave function of the scalar state

$$w_s(r) = w_{s_o}\ exp\{-(r/b)^\kappa\}\ , \tag{17}$$

where w_{s_o} is fixed by the normalisation $2\pi \int r dr\ w_s^2(r) = 1$. The interaction in eq. (13) has a form $v_v(r) \sim -\hbar c\ \frac{w_{s_o}}{w_{v_o}} w_v(r)$. However, the derivative part of $w_v(r)$ gives rise to a small attraction at large radii. This can be avoided by a small reduction of the derivative part of a few %. But a better consistency of the geometric boundary condition (16) is obtained by simply reducing the repulsive part of the interaction to zero.

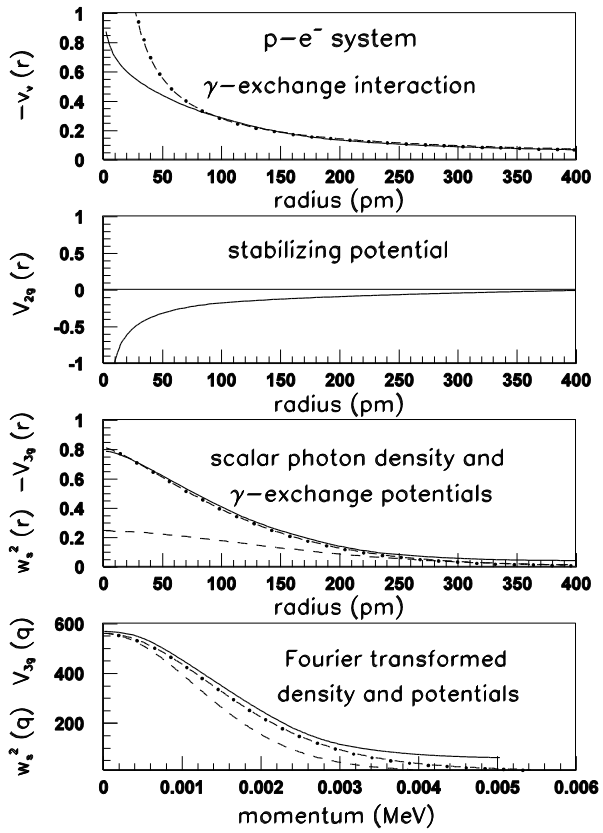

Figure 1: **Radial dependence of a self-consistent solution for a $p-e^-$ bound state with $< r_{w_s}^2 >^{1/2} =$ 210 pm. Upper part: Relative interaction $v_v(r)$ in comparison with the Coulomb potential given by dot-dashed line. Second part: Stabilizing potential $V_{2g}(r)$. Lower two parts: Boson density (dot-dashed line) and boson-exchange potentials (dashed and solid lines) shown in r- and q-space.**

In the entire formalism there are only three parameters, the slope and shape parameters b and as well as the coupling constant α, which have to be determined from boundary conditions as discussed below. Concerning the application of the potentials (11) and (13) for scalar and vector states, $V_{3g}(r)$ has a different form for each state, whereas $V_{2g}(r)$ is shared between scalar and vector states in the ratio 1:3, yielding $V_{2g}^s(r) = \frac{1}{4}V_{2g}(r)$ and $V_{2g}^v(r) = \frac{3}{4}V_{2g}(r)$.

In addition to the states without angular momentum (s-states) two other states exist with angular momentum L=1 (p-states), for which similar forms of their wave functions can be assumed. In atomic systems all L=0 and L=1 states give rise to degenerate singlet and triplet states. However, fine and hyperfine structure splittings of these states are observed, which are in the hydrogen atom 5-6 orders of magnitude smaller than the binding energies. These splittings as well as very small shifts (as the Lamb shift) are satisfactorily described in QED and are not considered in the present analysis.

The general structure of the bound state solutions is shown in fig. 1 for a system with root mean square radius

$< r_{w_s}^2 >^{1/2}$ of about 180 pm. In the upper part the radial dependence of the interaction $v_v(r)$ is compared to the $1/r$ dependence of the Coulomb potential, which shows that there are no divergences for $r \to 0$ and ∞ in the present description. Below the potential $V_{2g}(r)$ is displayed, which shows a quite linear increase at larger radii. In the third part the radial dependence of boson density $w_s^2(r)$ and potentials $V_{3g}^{s,v}(r)$ is shown, which indicates that relation (16) is reasonably well fulfilled. Since both states lead to s-states with large momentum spread, a significant mixing of both states is expected. In the lowest part the Fourier transformed density (dot-dashed line) and potentials (solid and dashed lines) are given, which show a satisfactory matching of relation (16) also in momentum space.

Binding energies have been calculated by using the virial theorem in the radial form $E_f^{ng} = 4\pi[\int r^2 dr\ \psi^2(r) V_{ng}(r)$ $\frac{1}{2}\int r^3 dr\ \psi^2(r)\frac{d}{dr}V_{ng}(r)]$, where the fermion wave functions $\psi(r)$ are normalized by $4\pi\int r^2 dr\ \psi^2(r) = 1$. In addition, $V_{3g}(r)$ can be interpreted as "bound state" of bosons. The corresponding binding energies E_g have been calculated by $E_g = 2\pi[\int r dr\ w^2(r) v_v(r) - \frac{1}{2}\int r^2 dr\ w^2(r)\frac{d}{dr}v_v(r)]$. The masses (due to binding) of the scalar and vector states are defined by $M_f^{s,v} = |E_{2g}^{s,v}| + |E_{3g}^{s,v}|$, whereas the total mass is given by $M_{tot} = M_f + m_1 + m_2$, where m_1 and m_2 are the participating fermion masses.

2.1 Boundary conditions

An important implication of the coupling to the vacuum is energy-momentum conservation, which requires that the (negative) binding energies of bosons E_g and fermions E_f are compensated by the corresponding root mean square momenta

$$0 =< q_g^2 >^{1/2} + E_g \tag{18}$$

and

$$0 =< q_f^2 >^{1/2} - x\ M_f\ , \tag{19}$$

where $x = \sqrt{2\tilde{m}/M_f}$. In eq. (19) the effect of the mass parameter and the binding energies in both potentials $V_{2g}(q)$ and $V_{3g}(q)$ are taken into account. The average momentum square for bosons is given by $< q_g^2 >= \int q^3 dq\ V_{3g}(q) / \int q dq\ V_{3g}(q)$, whereas that for fermions is $< q_f^2 >= \int q^4 dq\ \psi^2(q) V_{3g}(q) / \int q^2 dq\ \psi^2(q) V_{3g}(q)$. An important consequence of energy-momentum conservation is that all distributions of momentum q_g and q_f can to be identified with similar ones of energy or mass.

In addition, we require momentum matching, implying that the recoil momenta for bosons $< q_g^2 >_{rec}^{1/2}$ and fermions $< q_f^2 >_{rec}^{1/2}$ cancel each other

$$< q_g^2 >_{rec}^{1/2} - < q_f^2 >_{rec}^{1/2} = 0\ . \tag{20}$$

All conditions (18) - (20) have to be fulfilled for scalar and vector states and can be taken as a strict consistency check of the assumed wave functions.

Another condition can be derived from the structure of the potential $V_{2g}(r)$, which may be written in a different form (this can be seen from dimensional arguments)

$$V_{2g}(r) = \frac{\alpha^2 (M_s/2) < r_{w_s}^2 >}{2} \left(\frac{d^2 w_s(r)}{dr^2} + \frac{2}{r}\frac{dw_s(r)}{dr}\right)\frac{1}{w_s(r)}\ . \tag{21}$$

Dividing both potentials (11) and (21) leads to a mass-radius constraint

$$Rat_{2g} = \frac{(\hbar c)^2}{\tilde{m}(M_s/2) < r_{w_s}^2 >} = 1 \ . \tag{22}$$

This condition is very powerful and relates all parameters, κ, b and α.

3. Application to atomic systems

This formalism has been applied to the basic atomic bound state systems $p - e^-$ and $e^+ - e^-$, which have been studied previously in the Bohr model, with the Schrödinger equation, the Dirac equation and in QED (using effective potentials).

3.1 $p - e^-$ bound states

For this system $\tilde{m} = m_1 m_2/(m_1 + m_2) = 0.511$ MeV. By using the geometric boundary condition (16) together with the constraints (18) - (20) and (22), all parameters are determined.

First, s-states (without angular momentum L) are discussed. The 1s and 2s states correspond to vector and scalar states in the notation above. A satisfactory description of both states is obtained by using a coupling constant α of 2.14, a shape parameter κ of 1.35 and a slope parameter b of 210 pm. The derived momenta and binding energies (masses) are given in table 1, which show consistent boson and fermion properties. The deduced root mean square radii $< r_{w_{s,v}}^2 >^{1/2}$ are found to be 186 and 100 pm for scalar and vector state, respectively. Since $w_s(r)$ matches the radial form of the vector potential $V_{3g}^v(r)$ by the condition (16), the root mean square radius of the 1s potential $< r_{V_{1s}}^2 >^{1/2}$ is 186 pm, leading to radii at half maximum $R_{1/2}^{1s}$ of 58 pm and $R_{1/2}^{2s}$ of 115 pm, in reasonable agreement with the radii deduced from other work, see table 2.

The momentum distributions of the potentials in fig. 1 are comparable to the difference in average momentum of about 1.3 keV between both states (table 1), indicating that there should be a mixing between scalar and vector states. Using $E_f(1s) = (1-x)E_v + xE_s$ and $E_f(2s) = (1-x)E_s - xE_v$ with a mixing x of about 14 %, the binding energies in table 2 are well reproduced.

3.2 $e^+ - e^-$ bound states

In this system (positronium), $\tilde{m}$ is a factor of two smaller than for the hydrogen atom ($\tilde{m}$= 0.255 MeV). Relation (22) indicates that the binding energies have to be a factor two smaller than in $p - e^-$, whereas the

Table 1: Test of energy-momentum conservation and boson-fermion momentum matching for the strongest bound states. Energies, masses and momenta in keV.

system	$< q_g^2 >^{1/2}$	E_g	$< q_f^2 >^{1/2}$	xM_f
$p - e^-$, (2s)	(2.1 ± 0.1)	-2.1	(2.1 ± 0.2)	2.1
$p - e^-$, (1s)	(3.3 ± 0.2)	-3.5	(3.5 ± 0.3)	3.4
$e^+ - e^-$, (2s)	(1.0 ± 0.1)	-1.0	(1.0 ± 0.1)	1.0
$e^+ - e^-$, (1s)	(1.6 ± 0.1)	-1.7	(1.7 ± 0.2)	1.7

Table 2: Results for $p-e^-$ and e^+-e^- bound state solutions, using α=2.14, κ=1.35 and a mixing between the binding energies of ns and 2ns (but also between 2np and 4np) states of 14 %. Binding energies E_f are given in eV, b and radii in pm.

			$p-e^-$					
sol.	b	E_f(ns)	E_f(2ns, 2np)	E_f(4np)	$R_{1/2}^{ns}$	$R_{1/2}^{2ns}$	n R_{Bohr}	R_{cov}^{*}
1	210	-13.6 (1s)	-3.4 (2s) (2p)	-0.85 (4p)	58	115	53	31±5
2	420	-3.4 (2s)	-0.85 (4s) (4p)	-0.21 (8p)	116	230	106	
3	630	-1.51 (3s)	-0.38 (6s) (6p)	-0.09 (12p)	174	345	158	
4	840	-0.85 (4s)	-0.21 (8s) (8p)	-0.05 (16p)	232	460	212	
5	1050	-0.54 (5s)	-0.14 (10s) (10p)	-0.03 (20p)	290	575	265	
6	1260	-0.38 (6s)	-0.09 (12s) (12p)	-0.02 (24p)	348	690	318	
7	1470	-0.28 (7s)	-0.07 (14s) (14p)	-0.02 (28p)	406	805	371	
8	1680	-0.21 (8s)	-0.05 (16s) (16p)	-0.01 (32p)	464	920	424	
			e^+-e^-					
sol.	b	E_f(ns)	E_f(2ns,2np)	E_f(4np)	$R_{1/2}^{ns}$	$R_{1/2}^{2ns}$	n R_{Bohr}	
1	420	-6.8 (1s)	-1.7 (2s) (2p)	-0.43 (4p)	116	230	106	
2	840	-1.7 (2s)	-0.43 (4s) (4p)	-0.11 (8p)	232	460	212	

* covalent radius from ref. [2].

radii are a factor two larger. As shown in table 1, the energy-momentum conditions (18) and (19) are fulfilled, and also a quantitative account of the binding energies is obtained, as shown in the lower part of table 2.

3.3 Quantum condition and sum of equivalent first-order coupling strengths

Other solutions should exist for larger values of the slope parameter b. However, in elementary systems a discrete spectrum of eigenvalues is expected due to the quantum structure of the theory. In the present case the radius is the only variable, which is determined by the slope parameter b. This requires a quantum condition, in which the slope parameter b_n of the n^{th} solution is related directly to the first solution discussed above

$$b_n = n\ b_1. \tag{23}$$

Indeed, with a value of b of 420 pm (and α and κ unchanged) 2s and 4s states are obtained, in good agreement with experiment (assuming the same mixing between both states). Solutions up to n=8 are given in table 2, which show only very small deviations from the corresponding Coulomb energies of less than 1 %. For all solutions energy-momentum conservation and momentum matching is fulfilled, confirming the full consistency of the theoretical framework.

Solutions for p-states with angular momentum L=1 can be obtained with similar wave functions as for s-states. For solution 1 in table 1 a dominant wave of vector structure yields agreement with the binding energy of the 4p-state at -0.85 eV, whereas the 2p-state binding energy of -3.4 eV is obtained by a wave function dominated by scalar form. In a similar way also the other p-wave solutions in table 1 are obtained. Because of the good

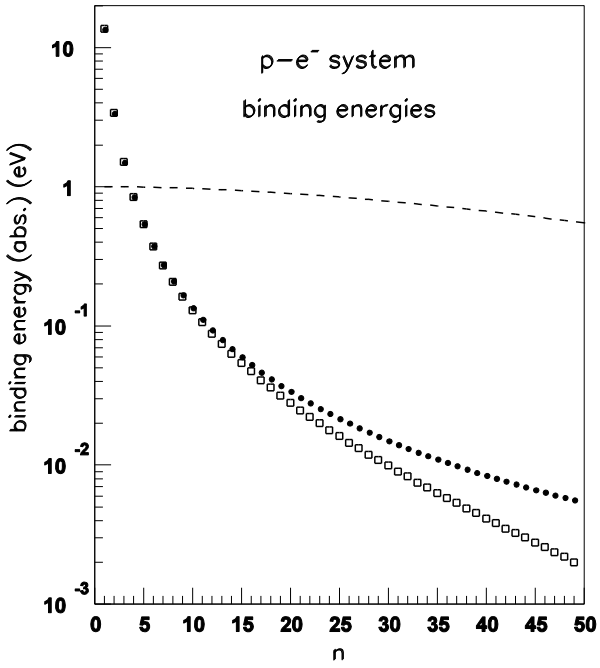

Figure 2: **Binding energies (absolute values) for ns states in the $p-e^-$ system as a function of n. The small points relate to the Coulomb energies, the open squares to the binding energies E_f^n, which yield a sum of partial couplings strengths $\sum \Omega_n \alpha_\Delta^n \sim \alpha$. The correction factors Ω_n to the Coulomb energies are given by dashed line.**

agreement with Coulomb energies, it should be possible to understand the magnitude of the fine structure constant $\alpha \sim 1/137$ from the present approach.

An equivalent first-order coupling constant α_{eq} can be derived from the potentials $V_{3g}^n(r)$ by writing $\alpha_{eq} \sim \sum_{n=1}^{\infty} \alpha_\Delta^n$, where $\alpha_\Delta^n = 2\alpha^3 \ (\int dr V_{3g}^{s,n}(r))/(\int dr\ \hbar/r)$ (the factor 2 is due to s- and p-state contributions). In a first step the potentials $V_{3g}^n(r)$ have been calculated for each n independent of all other solutions n'≠n. This leads to $\alpha_\Delta^n = \alpha_\Delta^1/n$ (α_Δ^1=1.87 10^{-3}), which is proportional to the harmonic series (see above) and yields $\sum_{n=1}^{\infty} \alpha_\Delta^n$ divergent. The corresponding spectrum is identical to the Coulomb energy spectrum, given by solid points in fig. 2.

However, for larger values of n an independence of the potentials from those of solution 1 cannot be expected. Their potential strengths should fall off with the momentum dependence of the potential $V_{3g}^s(q)$ for n=1. Therefore, correction factors Ω_n have been applied to the partial coupling strengths α_Δ^n, which are given by $\Omega_n = V_{3g}^s(q_n)/V_{3g}^s(q_1)$. To cover the entire fall-off of the potential up to large momenta (see fig. 1), values of Ω_n

from n=1-500 have been calculated with a momentum step size $\Delta q \sim 40$ eV, which corresponds to about 2 % of the average momentum $< q_f^2 >^{1/2}$ of the lowest state (with n=1). Since $V_{3g}^s(q_n)$ decreases rapidly for large n (for n=300 the factor Ω_n has already fallen off to a value of about 10^{-6}) this leads to a good convergence of $\sum \Omega_n \alpha_\Delta^n$ with a value of $\sum_{n=1}^{500} \Omega_n \alpha_\Delta^n \sim 7.5\ 10^{-3}$, which is in excellent agreement with $\alpha \sim 7.3\ 10^{-3}$. The correction factors Ω_n are shown by the dashed line in fig. 2, which fall off rather slowly for small n, but decrease rapidly for large values of n.

The corresponding potentials $\Omega_n V^n(r)$ result in binding energies for ns states shown in fig. 2 by open squares, which decrease more rapidly than the Coulomb energies. Up to n$\sim$10 (for which data are available) very small differences between $E_f(ns)$ and $E_{Coul}(ns)$ of ≤ 0.6 % are found, which are difficult to detect experimentally (the Balmer series with transitions $n \to 2$ up to n=9 are still consistent with both theories). However, in the present formalism a consistent analysis of the binding energies is obtained with $\sum \Omega_n \alpha_\Delta^n \sim \alpha_{eq} \sim \alpha$. If the binding energies would not fall off stronger than the Coulomb energies, a finite value of α_{eq} could not be obtained.

4. Summary

By replacing the fermions in QED by fermions dressed with photon fields, a complete (parameter free) description of basic atomic systems is achieved. The dressing of fermions is needed for a physically correct description of free bound states in the vacuum, from which radii of atoms can be deduced. The main points may be summarized as follows:

1. The general arguments against the use of higher order Lagrangians (leading to divergences and ghosts) are not valid for the present Lagrangian, since all matrix elements are finite and non-physical solutions can be eliminated by strict boundary conditions.
2. The correct structure of free particle bound states is obtained without spurious motion. Further, the potential $V_{2g}(r)$ warrants dynamical stability of the system.
3. A quantitative description of the $p - e^-$ and $e^+ - e^-$ systems is obtained without open parameters. The validity of the Coulomb energy spectrum (up to $n \sim 10$) is confirmed.
4. The electric coupling (fine structure) constant $\alpha \sim 1/137$ is reproduced, supporting strongly the validity of the present formalism.

Finally, a complete theory without external parameters is needed also for systems bound by all other fundamental forces. This should lead to a realistic understanding of the properties of these forces and a better insight into the structure of the universe.

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