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Dust-Ion-Acoustic Shock Waves in a Five Component Dusty Plasma with Positive and Negative Ions

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Abstract

The properties of dust-ion-acoustic (DIA) shock waves in a multi-ion dusty plasmacontaining Maxwellian distributed electrons, inertial positive and negative ions, immobile positively and negatively charged dust particles have been investigated. The standard reductive perturbation method is employed to derive the Burgers equation, which admits shock waves solution. It is observed that the DIA shock structures associated with positive and negative potential shock are formed depending on the plasma parameters (e.g. number densities ratio of electron-to-positive ion, negative ion-to-positive ion, negative dust-to-positive ion, etc.) in such a five component multi-ion dusty plasma. The critical value, above (below) which positive (negative) polarity shock waves are formed, is analyzed numerically.

Keywords: Multi ion; Shock waves; Dusty plasma.

I. Introduction

The linear and nonlinear propagation of electrostatic excitations in dusty plasma have received considerable attention in the last few decades. Such plasmas occur in laboratory [1, 2], astrophysical and space environments [4–6], such a cometary tails, planetary rings and interstellar medium. It has been confirmed that both theoretically and experimentally that a weakly coupled unmagnetized dusty plasma supports two novel eigenmodes, namely, the Dust-ion-acoustic (DIA) mode [7, 8]and Dust-acoustic (DA) mode [1, 9]. The phase speed of DIA mode [7, 8] lies between the ion and electron thermal speed, and the mode occurs on a time scale much shorter than the dusty plasma period. Hence, charged dust grains are regarded as immobile, their effect appears through the modification of the equilibrium quasineutrality. Physically, in the DIA mode, the restoring force comes from the pressure of the inertia less Boltzmann distributed electrons, while the inertia is provided by the ion mass.

In recent years, Rahman et al. [10] have studied the DIA solitary waves and their multi-dimensional instability in a magnetized dusty electronegative plasma with trapped negative ions. Sayed et al. [11] studied DIA solitary waves in a dusty plasma with positive and negative ions. Most of the dusty plasma studies have been confined in considering that the dust grains are negatively charged [9, 12–14]. Therefore, the existence of negative and positive ions following nonthermal distribution which are common to occur in the Earths ionosphere [15], the cometary comae [16], and the upper region of Titans [17]. Very recent it has been found that there some plasma systems, particularly in space plasma environment, where positively charged dust grains are present and also play a significant role [18–21].

There are also direct evidence for the existence of both positively and negatively charged dust particles in different regions of space, viz. cometary tails Chow et al. [20], upper mesosphere Havens et al. [21], Jupiter's magnetosphere Hoyanyi et al. [19], etc.

Armina et al. [22] have considered a four-component dusty plasma containing positively and negatively charged dust grain and Boltzmann distributed electrons and ions. They investigated the possibility for the formation of shock waves (SHWs) and existence of shock structures. Most of the research papers which discussed Dust acoustic solitary waves (DASWs) are based on electrons and ions with Maxwellian-Boltzmann distributions. It is well known that in a dissipative plasma medium, shock waves appear due to the balance between the nonlinearity and the dissipation. Landau damping, kinematic viscosity among the plasma constituents, wave particle interaction, etc. are responsible for arising the dissipation which helps to form the shock structures [23] in a plasma medium. Andersen et al. [24] found the shock structures in Q machine experiment. However, most of theoretical works on the shock waves [25, 26] in a dusty plasma are based on ions with same temperature. In our present work, we considered a five-component unmagnetized dusty plasma system containing Maxwellian distributed electrons, inertial positive and negative ions, immobile positively and negatively charged dust particles.

The manuscript is organized as follows: The governing equations are provided in Sec. II. The Burgers equation is derived in Sec III. The numerical solution of Burgers equation is presented in Sec. IV. Finally, a brief discussion is given in Sec V.

II. Governing Equations

We consider a five-component collisionless unmagnetized dusty plasma consisting of Maxwellian electrons, inertial positive and negative ions and positively and negatively charged static dust. The charge neutrality is thus reads $n_{e0}+n_{n0}+z_{dn}\, n_{dn0} = n_{p0}+z_{dp}\, n_{dp0}$, where n_{s0} is the density of plasma species (where s = e, p, n, dp, dn refer to electron, positive ion, negative ion, positively charged dust, negatively charged dust, respectively). The nonlinear dynamics of the DIA waves in such a dusty plasma system is described by the following one dimensional (1-D) equations:

$$\frac{\partial n_p}{\partial t}+\frac{\partial}{\partial x}\left(n_p u_p\right)=0, \tag{1}$$

$$\frac{\partial n_n}{\partial t}+\frac{\partial}{\partial x}\left(n_n u_n\right)=0, \tag{2}$$

$$\frac{\partial u_p}{\partial t}-u_p\frac{\partial u_p}{\partial x}=-\frac{\partial \phi}{\partial x}+\eta_p\frac{\partial^2 u_p}{\partial x^2}, \tag{3}$$

$$\frac{\partial u_n}{\partial t}-u_n\frac{\partial u_n}{\partial x}=\frac{\partial \phi}{\partial x}+\eta_n\frac{\partial^2 u_n}{\partial x^2}, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2}=\mu_e e^{\phi}-n_p+\mu_n n_n+\mu_{dn}-\mu_{dp}. \tag{5}$$

where n_p is the positive ion number density normalized by its equilibrium value n_{p0}, n_n is the positive ion number density normalized by its equilibrium value n_{n0}, u_p is the positive ion fluid speed normalized by $C_p=(Z_p k_B T_e / m_p)^{1/2}$, u_n is the negative ion fluid speed normalized by $C_n=(Z_n k_B T_e / m_n)^{1/2}$, η_p is the co-efficient of positive ion viscosity normalized by ($m_p n_{p0}\omega_{pp}\lambda_{Dmp}^2$), η_n is the co-efficient of negative ion viscosity normalized by ($m_n n_{n0}\omega_{pn}\lambda_{Dmn}^2$), ϕ is the electrostatic wave potential normalized by T_e / e, the time variable t is normalized by $\omega_p^{-1}=(m_p / 4\pi n_{po} Z_p^2 e^2)^{1/2}$, and the space variable x is normalized by $\lambda_{Dm}=(T_e / 4\pi n_{po} Z_p e^2)^{1/2}$. Here T_e is the electron temperature, k_B is the Boltzmann constant, and e is the magnitude of the electron charge. The parameters used in Eq. (5) are defined as $\mu_e=n_{e0} / Z_p n_{p0}$, $\mu_n=n_{n0} / Z_p n_{p0}$, $\mu_{dp}=n_{dpo} / Z_p n_{p0}$, and $\mu_{dn}=n_{dn0} / Z_p n_{p0}$. The number density of Maxwellian electrons n_e is expressed as

$$n_e=n_{e0}e^{e\phi/T_e} \tag{6}$$

III. Derivation of Burger's Equation

To derive the Burgers equation (BE), we introduce the stretched Coordinates (27) as

$$\zeta=\eth(x-V_p t), \tag{7}$$

$$\tau=\eth^2 t, \tag{8}$$

where V_p is the phase speed of the DIA waves, and ϵ is a smallness parameter measuring the weakness of the dispersion $(0<\epsilon<1)$. We then expand n_p, n_n, u_p, u_n and ϕ in power series of ϵ :

$$n_p = 1+\epsilon n_p^{(1)} + \epsilon^2 n_p^{(2)} + \epsilon^3 n_p^{(3)} + \cdots , \quad (9)$$

$$n_n = 1+\epsilon n_n^{(1)} + \epsilon^2 n_n^{(2)} + \epsilon^3 n_n^{(3)} + \cdots , \quad (10)$$

$$u_p = \epsilon u_p^{(1)} + \epsilon^3 u_p^{(2)} + \epsilon^3 u_p^{(3)} + \cdots , \quad (11)$$

$$u_n = \epsilon u_n^{(1)} + \epsilon^3 u_n^{(2)} + \epsilon^3 u_n^{(3)} + \cdots , \quad (12)$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots, \quad (13)$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, Eqs. (9)-(13) give

$$n_p^{(1)} = \frac{V_p^2}{\phi}^{(1)} , \; n_n^{(1)} = -\frac{V_p^2}{\phi}^{(1)} , \quad (14)$$

$$u_p^{(1)} = \frac{V_p}{\phi}^{(1)} , \; u_n^{(1)} = -\frac{V_p}{\phi}^{(1)} , \quad (15)$$

$$V_p = \frac{1}{\sqrt{\frac{\mu_e}{1+\mu_n}}} . \quad (16)$$

Equation (16) describes the phase speed of DIA waves regarding the dusty plasma under consideration.

To the next higher order of ϵ, we obtain a set of equations, which, after using Eqs. (14)-(16), can be simplified as

$$\frac{\partial n_p^{(1)}}{\partial \tau} - V_p \frac{\partial n_p^{(2)}}{\partial \zeta} + \frac{\partial u_p^{(2)}}{\partial \zeta} + \frac{\partial P}{\partial \zeta} = 0 , \quad (17)$$

$$\frac{\partial n_n^{(1)}}{\partial \tau} - V_p \frac{\partial n_n^{(2)}}{\partial \zeta} + \frac{\partial u_n^{(2)}}{\partial \zeta} + \frac{\partial N}{\partial \zeta} = 0 , \quad (18)$$

$$\frac{\partial u_p^{(1)}}{\partial \tau} - V_p \frac{\partial u_p^{(2)}}{\partial \zeta} + u_p^{(1)} \frac{\partial u_p^{(1)}}{\partial \zeta} + \frac{\partial \phi^2}{\partial \zeta} = \eta_p \frac{\partial^2 u_p^{(1)}}{\partial \zeta^2} , \quad (19)$$

$$\frac{\partial u_n^{(1)}}{\partial \tau} - V_p \frac{\partial u_n^{(2)}}{\partial \zeta} + u_n^{(1)} \frac{\partial u_n^{(1)}}{\partial \zeta} - \frac{\partial \phi^2}{\partial \zeta} = \eta_n \frac{\partial^2 u_n^{(1)}}{\partial \zeta^2} , \quad (20)$$

$$\mu_e \phi^{(2)} + \frac{1}{2}\mu_e \phi^{(1)2} - n_{p2} + \mu_n n_{n2} = 0 . \quad (21)$$

where $P = n_p^{(1)} u_p^{(1)}$ and $N = n_n^{(1)} u_n^{(1)}$. Now combining Eqs.(17)-(21), we obtain a equation of the form:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} = C \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} \quad (22)$$

where

$$A=\frac{V_p^3}{2(1+\mu_n)}[\frac{3}{V_p^4}(1-\mu_n)-\mu_e]\ , \qquad (23)$$

$$C=\frac{\eta_p+\eta_n\mu_n}{2(1+\mu_n)}\ . \qquad (24)$$

Equation (22) is known as Burger's equation. The stationary localized solution of the BE is given by

$$\psi=\psi_m[1-\tanh\left(\frac{\zeta}{\delta}\right)] \qquad (25)$$

where $\phi^{(1)}=\psi$, $\delta=2C/U_0$ is the width, and $\psi_m=U_0/A$ is the amplitude of the shock profile.

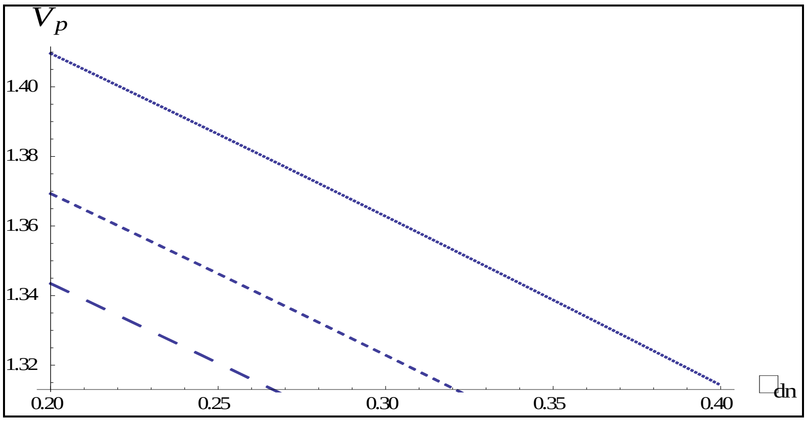


Fig. 1: Variation of phase speed V_p of the shock wave negative dust-to positive ion number density ratio μ_{dn} for different electron-to-positive ion number density ratio μ_e. Other plasma parameters fixed at μ_p =0.5, μ_{dp} =0.4, μ_{dn} =0.4, η_p = 0.6, and η_n = 0.3. The upper (solid) curve is for μ_e = .77, the (dotted) curve is for μ_e = 0.8, and the (dashing) curve is for μ_e = 0.82. Here $\mu_e > \mu_c$, i.e. μ_e is assumed above its critical value.

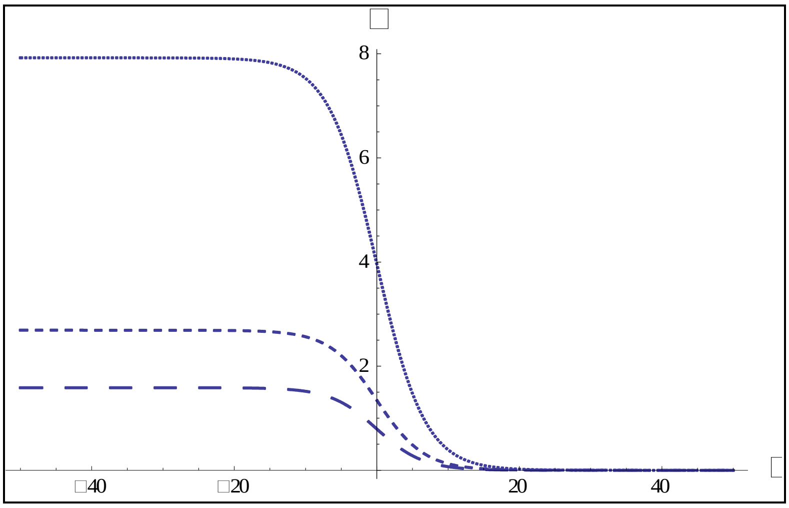


FIG 2: Variation of positive potential shock structures with ξ for different values of μ_e. The other plasma parameters are fixed at μ_p =0.5, μ_{dp} =0.4, μ_{dn} =0.4, η_p = 0.6, and η_n = 0.3. The upper (solid) curve is for μ_e =0.8, the (middle) dotted curve is for μ_e =0.85, and the lower (dashing) curve is for μ_e =0.9. Here $\mu_e > \mu_c$, i.e. μ_e is assumed above its critical value.

IV. Numerical Analysis

It is observed that from equation that the amplitude (width) of the shock waves increases (decreases) as U_0 increases. It is obvious from Eq. (25), A> (<)0, the plasma system under consideration supports compressive (rarefactive) shock waves which are associated with a positive (negative) potential, and no shock waves exist at A=0. It is clear that A is a function of μ_e, μ_n, μ_{dp}, and μ_{dn}. Therefore, the critical value of electron-to-positive ion number density μ_e

(let us say, μ_c) at A($\mu_e = \mu_c$)=0 can be expressed as

$$\mu_c = \frac{1}{4}\left[-4-\mu_{dn}+\mu_{dp}+\sqrt{3}\sqrt{(16-8\mu_{dn}+3\mu_{dn}^2)+8\mu_{dp}-6\mu_{dn}\mu_{dp}+3\mu_{dp}^2}\,\right] \qquad (26)$$

where the shock wave with positive (negative) potential exists above (below) the critical value μ_c. We can find A=0 for a certain (critical) value of μ_e, i.e. A= 0 for $\mu_e = \mu_c$ ⊔ 0.77 [obtained from A($\mu_e = \mu_c$)= 0 for a set of plasma parameters viz. $\mu_{dp} = 0.5$, $\mu_{dn} = 0.44$]. We can also find A=0 for a certain (critical) value of μ_{dn}, i.e. A=0 for $\mu_{dn} = \mu_c$ ⊔ 0.54 [obtained from A($\mu_{dn} = \mu_c$)=0 for a set of plasma parameters viz. μ_e =0.3, μ_{dp} =0.44]. It is clear that $\psi_m = \infty$ at $\mu_e = \mu_c$ and the BE that we have derived is no longer valid at this condition, so the shock waves are found for $\mu_e \neq \mu_c$. Fig. 2 (3) shows the positive (negative) potential shock waves for different values of $\mu_e = 0.5$. Fig. 4 (5) show the positive (negative) amplitude (ψ_m) profiles of shock waves and how it varies with μ_c.

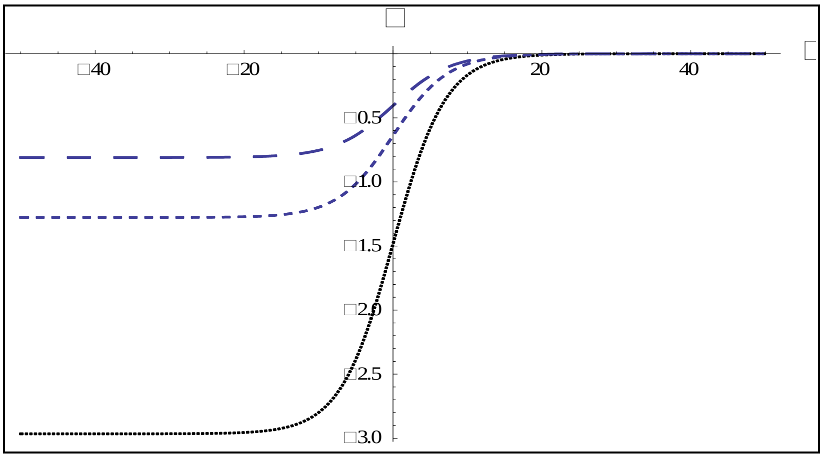


FIG 3: Variation of negative potential shock structures with ξ for different values of μ_e. The other plasma parameters are fixed at μ_p =0.5, μ_{dp} =0.4, μ_{dn} =0.4, η_p = 0.6, and η_n = 0.3. The upper (dashed) curve is for μ_e =0.5, the middle (dotted) curve is for μ_e =0.6, and the lower (solid) curve is for μ_e =0.7. Here $\mu_e < \mu_c$, i.e. μ_e is assumed below its critical value.

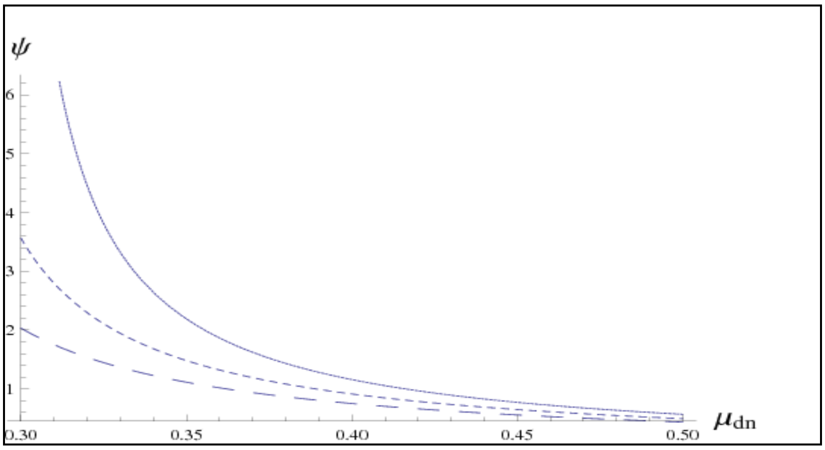


FIG 4: Variation of positive amplitude profile of shock wave with negative dust-to-positive ion number density ratio μ_{dn} for different values of μ_e with plasma parameters μ_p =0.5, μ_{dp} =0.4, η_p = 0.6, and η_n = 0.3. The upper (solid) curve is for μ_e =0.88, the middle (dotted) curve is for μ_e =0.9, and the lower (dashed) curve is for μ_e =0.92. Here $\mu_e > \mu_c$, i.e. μ_e is assumed above its critical value.

V. Discussion

We have considered an unmagnetized five-component dusty plasma system consisting of electrons, positively as well as negatively charged ions and positively as well as negatively charged static dust. The well known reductive perturbation method has been used to derive the BE. The plasma system under consideration supports finite amplitude shock structures, whose basic features (viz. polarity, amplitude, width, speed, etc.) strongly depend on the plasma parameters viz. μ_e , μ_n , μ_{dp} , μ_{dn} , η_p and η_n .

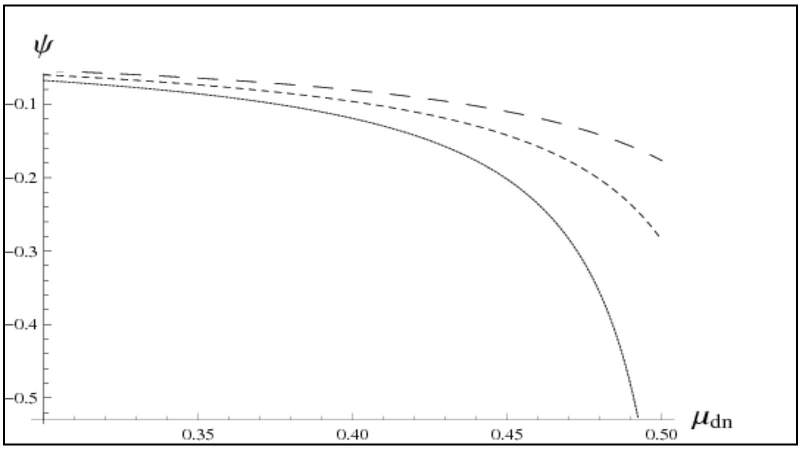
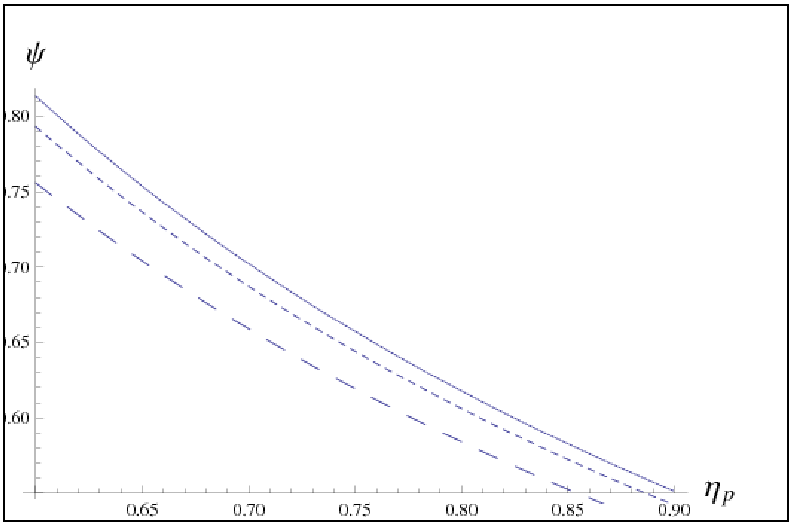
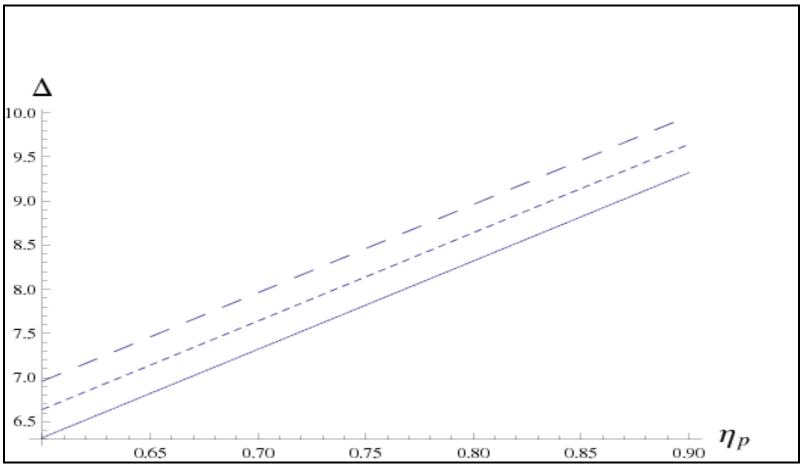


FIG. 5: Variation of negative amplitude profile of shock wave with μ_e and for μ_p =0.5, μ_{dp} =0.4, $\mu_{dn} = 0.4$, $\eta_p = 0.6$, and η_n = 0.3.The upper (dashing) curve is for μ_e = 0.68$, the middle (dotted) curve is for μ_e = 0.7, and the lower (solid) curve is for μ_e = 0.72. Here $\mu_e < \mu_c$, i.e. μ_e is assumed below its critical value.



(a)



(b)

FIG. 6: Variation of (a) ψ with positive ion viscosity η_p and (b) Δ with positive ion viscosity η_p for different values of negative ion viscosity η_n. The other parameters are fixed at $\mu_e > \mu_c$ i. e. μ_e is assumed above its critical value.

The results which have been found from this investigation can be summarized as follows:

1. It is observed that with the increase of dust-to- positive ion number density ratio μ_{dn}, the phase speed (V_p) of DIA waves decreases, while the decreasing values of electron-to-positive ion number density ratio μ_e enhances the phase speed of DIA waves (see Fig. 1).
2. Both polarity (positive and negative potential) shock waves are found to exist in the plasma system under consideration here. For example, the positive potential shock waves are formed above μ_c (critical value of μ_e) and the negative potential shock waves are formed below μ_c (see Fig.2 and 3).
3. It is found that both polarity (positive and negative amplitude of potential) shock waves are found to exist in the plasma system under consideration here. The magnitude of the amplitude of positive (negative) potential shock waves decreases with increasing the electron-to-positive ion number density ratio μ_e. (see Fig. 4 and 5).
4. It is observed that the magnitude of the amplitude of positive potential shock waves decreases with increasing the negative ion viscosity η_n. The width Δ of shock waves increases with the increasing of negative ion viscosity η_n. (see Fig. 6)

In conclusion, the parameters ($\mu_e = 0.1 - 0.9$, $\mu_n = 0.1 - 0.9$, $\mu_{dp} =$ $0.1 - 0.5$, $\mu_{dn} =$ $0.1 - 0.5$, $\eta_p = 0.1 - 0.9$, and $\eta_n = 0.1 - 0.9$), chosen in our numerical calculation are completely relevant to different regions of space plasmas [26].

It may be stressed here that the results of this investigation could be useful for understanding the nonlinear features of electrostatic disturbances in laboratory [28]. It also will help to analyze and interpret spacecraft data on the earth's magnetospheric plasma sheet [29], Jupiter [30] and [31].

This paper should also help to understand the silent features of localized DIA waves in multicomponent laboratory and space dusty plasmas which are composed of the positive and negative ions, maxwellian distributed electrons and positively and negatively charged static dust. We note that observations [17] reveal the presence of both negative and positive ion populations in Titan's ionosphere and our theoretical results for localized DIA waves may be relevant to the formation of structures in an organic-rich aerosol plasma of Titan.

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