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Quantum Gravity Correction to Co-bimaximal Neutrino Mixings

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Abstract.

The quantum gravity may have strong consequence for neutrino oscillation phenemomenon.We found a significant modification of neutrino oscillation due to quantum gravity effects in specific case. We also assume that just above the electroweak scale,neutrino masses are degenerate and their mixing is co-bimaximal.Quantum gravity (Planck scale effects) leads to an effective $SU(2)_L \times U(1)$ invariant dimension-5 Lagrangian involving, neutrino and Higgs fields. On symmetry breaking, this operator gives rise to correction to the neutrino masses and mixing. The gravitational interaction ($M_X = M_{pl}$) demands that the element of this perturbation matrix should be independent of flavor indices. In this paper,we compute the deviation of the three neutrino mixing angles due to Planck scale effects in a co-bimaximal scheme.We found for degenerate neutrinos the modified neutrino mixing angle θ_{12}^{planck}=$\theta_{12} \pm 3.15^o$,θ_{13}^{planck}=$\theta_{13} \pm 0.08^o$ and $\theta_{23}^{planck} = \theta_{23} + 0.12^o$

Keywords:Neutrino Oscillation:Quantum Gravity effect

1 Introduction

A Reivew of the phenomenology of quantum gravity is given in [1.In recent year the subject of quantum gravity phenomenology has rapid growth [2] complementary to theoretical work. One of the main challenge in neutrino physics is to explain the explicit form of the neutrino mixing and mass pattern.We have already known well known neutrino mixing matrix,bimaximal mixing and tribimaximal mixing but now all of the mixing matrices are inapporiate,when conforomded to the recent experiment data,especially a mixing matrix angle $\theta_{13}$is reported by T2K[3] colloaboration and Daya Baya [4] colloaborations. In [5] Ma,proposed a new mixing matrix which is known as Co-bimaximal mixing by assumming mixing angle $\theta_{13} \neq 0^o = 10^o, \theta_{23} = \frac{\pi}{4}$, $tan\theta_{12}^2 = \frac{1-3sin\theta_{13}^2}{2}, \theta_{12} = 34^o$ and Dirac phase$\delta = \pm\frac{\pi}{2}$. Additional effects,which modify the above predections,must exist so that the final predection are close to the experimental determined values.The quantum mechanical phenemenon of neutrinos, propagating in gravitational field has be discussed by many author [6,7] The outline of the article is as follows. In Section 2, we briefly discuss about neutrino oscillation parameter due to Planck scale effects. In Section 3, we discuss about oscillation due to quantum gravity effects. In Section 4, numerical results. In Section 5, we present our conclusions.

2 Neutrino Oscillation Parameter due to Planck Scale Effects

It is imperative to explore all mechanism which can alter Co-bimaximal mixing pattern.A natural source for this change are the correction induced by quantum gravity effects. The effective gravitational interaction of neutrino with Higgs field can be expressed as $SU(2)_L \times U(1)$ invariant dimension-5 operator [8],

$$L_{grav} = \frac{\lambda_{\alpha\beta}}{M_{pl}}(\psi_{A\alpha}\epsilon\psi_C)C^{-1}_{ab}(\psi_{B\beta}\epsilon_{BD}\psi_D) + h.c. \tag{1}$$

Here and every where we use Greek indices α, β for the flavour states and Latin indices i, j, k for the miniass states. In the above equation $\psi_\alpha = (\nu_\alpha, l_\alpha)$is the lepton doublet, $\phi = (\phi^+, \phi^o)$is the Higgs doublet and $M_{pl} = 1.2 \times 10^{19} GeV$ is the Planck mass λ is a 3×3 matrix in a flavour space with each elements $O(1)$. The Lorentz indices $a, b = 1, 2, 3, 4$ are contracted with the charge conjugation matrix C and the $SU(2)_L$ isospin indices $A, B, C, D = 1, 2$ are contracted with $\epsilon = i\sigma_2$, $\sigma_m(m = 1, 2, 3)$are the Pauli matrices. After spontaneous electroweak symmetry breaking the Lagrangian in eq(1) generated additional term of neutrino mass matrix

$$L_{mass} = \frac{v^2}{M_{pl}}\lambda_{\alpha\beta}\nu_\alpha C^{-1}\nu_\beta, \tag{2}$$

where $v = 174GeV$ is the VEV of electroweak symmetric breaking. We assume that the gravitational interaction is"flavour blind" that is $\lambda_{\alpha\beta}$ is independent of α, β indices. Thus the Planck scale contribution to the neutrino mass matrix is

$$\mu\lambda = \mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{3}$$

where the scale μ is

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} eV. \tag{4}$$

We take eq(3) as perturbation to the main part of the neutrino mass matrix, that is generated by GUT dynamics. To calculate the effects of perturbation on neutrino observables. The calculation developed in an earlier paper [9]. A natural assumption is that unperturbed (0^{th} order mass matrix) M is given by

$$\mathbf{M} = U^* diag(M_i) U^{\dagger}, \tag{5}$$

where, $U_{\alpha i}$ is the usual mixing matrix and M_i , the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements U_{e3}. We adopt the usual parametrization.

$$\frac{|U_{e2}|}{|U_{e1}|} = tan\theta_{12}, \tag{6}$$

$$\frac{|U_{\mu 3}|}{|U_{\tau 3}|} = tan\theta_{23}, \tag{7}$$

$$|U_{e3}| = sin\theta_{13}. \quad (8)$$

In term of the above mixing angles, the mixing matrix is

$$U = diag(e^{if1}, e^{if2}, e^{if3})R(\theta_{23})\Delta R(\theta_{13})\Delta^* R(\theta_{12})diag(e^{ia1}, e^{ia2}, 1). \quad (9)$$

The matrix $\Delta = diag(e^{\frac{1\delta}{2}}, 1, e^{\frac{-i\delta}{2}})$ contains the Dirac phase. This leads to CP violation in neutrino oscillation a_1 and a_2 are the so called Majorana phases, which effects the neutrino less double beta decay. f_1, f_2 and f_3 are usually absorbed as a part of the definition of the charge lepton field. Planck scale effects will add other contribution to the mass matrix that gives the new mixing matrix can be written as [9]

$$U' = U(1 + i\delta\theta),$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$+i \begin{pmatrix} U_{e2}\delta\theta^*_{12} + U_{e3}\delta\theta^*_{23,} & U_{e1}\delta\theta_{12} + U_{e3}\delta\theta^*_{23}, & U_{e1}\delta\theta_{13} + U_{e3}\delta\theta^*_{23} \\ U_{\mu 2}\delta\theta^*_{12} + U_{\mu 3}\delta\theta^*_{23,} & U_{\mu 1}\delta\theta_{12} + U_{\mu 3}\delta\theta^*_{23}, & U_{\mu 1}\delta\theta_{13} + U_{\mu 3}\delta\theta^*_{23} \\ U_{\tau 2}\delta\theta^*_{12} + U_{\tau 3}\delta\theta^*_{23}, & U_{\tau 1}\delta\theta_{12} + U_{\tau 3}\delta\theta^*_{23}, & U_{\tau 1}\delta\theta_{13} + U_{\tau 3}\delta\theta^*_{23} \end{pmatrix}. \quad (10)$$

Where $\delta\theta$ is a hermition matrix that is first order in μ[9,10]. The first order mass square difference $\Delta M^2_{ij} = M^2_i - M^2_j$,get modified [9,10] as

$$\Delta_{ij}^{'} = \Delta_{ij} + 2(M_i Re(m_{ii}) - M_j Re(m_{jj}), \tag{11}$$

and

$$m = \mu U^t \lambda U, \tag{12}$$

where

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} eV.$$

The change in the elements of the mixing matrix, which we parametrized by $\delta\theta$[9], is given by

$$\delta\theta_{ij} = \frac{iRe(m_{jj})(M_i + M_j) - Im(m_{jj})(M_i - M_j)}{\Delta M_{ij}^{'2}}. \tag{13}$$

The above equation determine only the off diagonal elements of matrix $\delta\theta_{ij}$. The diagonal element of $\delta\theta_{ij}$ can be set to zero by phase invariance.

Using Eq(10), we can calculate neutrino mixing angle due to Planck scale effects,

$$\frac{|U_{e2}^{'}|}{|U_{e1}^{'}|} = tan\theta_{12}^{'}, \tag{14}$$

$$\frac{|U_{\mu 3}^{'}|}{|U_{\tau 3}^{'}|} = tan\theta_{23}^{'}, \tag{15}$$

$$|U_{e3}^{'}| = sin\theta_{13}^{'} \tag{16}$$

For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$. Thus, from the above set of equations, we see that $U_{e1}^{'}$ and

U'_{e2} are much larger than U'_{e3}, $U'_{\mu 3}$ and $U'_{\tau 3}$. Hence we can expect much larger change in θ_{12} compared to θ_{13} and θ_{23} [10]. The rest of this paper provides detail on large scale changes due to quantum gravity effects on neutrino oscillations.

3 Neutrino Oscillation Parameter due to Quantum Gravity Effects

The quantum mechnical phenemenon of neutrino oscilation in vacuum is a direct consequence of non zero neutrino masses and mixings. The flavor eigenstate of the neutrino is related to the inertial mass eigenstates via linear combination

$$|\nu_\alpha >= \sum_{j=1,2,3} U_{\alpha i}|\nu_i >,$$

where α is the neutrino weak-flavor eigenstate index , α=$e, \mu, \tau, \rightarrowtail$index i relates the inertial mass eigenstates $|\nu_1 >, |\nu_2 >, |\nu_3 >$with masses $m_1, m_2$and $m_3$respectively and $U_{\alpha i}$is the 3×3standard unitary neutrino mixing matrix [11]. The standard formula for the transition probability between two flavor states α and β has the following form

$$P(\nu_e \to \nu_\mu) = \frac{1}{2} sin^2 2\theta_{12} \left(1 - cos\left(\frac{\Delta_{21} L}{2E}\right)\right), \tag{17}$$

where $\Delta_{21} = m_2^2 - m_1^2 > 0$ is the mass square differences and L is oscillation lentgh. There are three flavors of neutrinos and they mix to form three mass eigensates. This mixing is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{18}$$

where mixing matrix U parametrized as

$$U = R(\theta_{23})\Pi R(\theta_{13})\Pi^{*} R(\theta_{12}). \qquad (19)$$

In the above mixing matrix, Π is a diagonal matrix containing the CP violating phase δ and $R(\theta_{ij})$is is the form of rotation matrices. The mass eigenstates ν_i have eigenvalues m_i . Neutrino oscillation probabilities depend on the two mass squared differences $\Delta_{21} = m_2^2 - m_1^2$,$\Delta_{31} = m_3^2 - m_1^2$, the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and the CP violating phase δ. Using Eq(10), we can calculate neutrino mixing angle due to quantum gravity effects, For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$. Thus, from the above set of equations, we see that U'_{e1} and U'_{e2} are much larger than U'_{e3}, $U'_{\mu 3}$ and $U'_{\tau 3}$.

4 Numerical Results.

From Eq.(25) that the correction term of oscillation parameter depends on the type of neutrino mass spectrum. For hierarchical or invertes hierarchical spectrum the correction of oscillation parameter is negligible. We consider a degenerate neutrino Mass spectrum and take the common neutrino mass to be 2 eV, which is upper limit of tritium beta decay spectrum [13]. From definition of the matrix m in Eq(12), we find

$$m_{11} = \mu e^{i2a_1}(U_{e1} + U_{\mu 1} + U_{\tau 1})^2$$

$$m_{22} = \mu e^{i2a_2}(U_{e2} + U_{\mu 2} + U_{\tau 2})^2$$

The contribution of the term in the Planck scale correction, $\epsilon = 2(M_i Re(m_{11}) - M_j Re(m_{22})$, can be additive or subtractive depending on the values of the phase

a_1, a_2 and phase f_i. In our calculation, we used mixing angle $\theta_{13} = 10^o, \theta_{23} = \frac{\pi}{4}$, $tan\theta_{12}^2 = \frac{1-3sin\theta_{13}^2}{2}, \theta_{12} = 34^o$ and Dirac phase $\delta = \pm\frac{\pi}{2}$.We have taken $\Delta_{31} = 0.002eV^2$[14] and $\Delta_{21} = 0.00008eV^2$[15]. For simplicity, we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$and $\delta = \pm\frac{\pi}{2}$ .Number of experiments constrained it from above $sin^2 2\overline{\theta_{13}}. \bar{<} 0.17$ limit was obtained by CHOOZ experiment [16] and similar limit was given by Palo verde [17] (both reactor experiment). $sin^2 2\theta_{13} < 0.26$ limit was given by K2K [18] and the following limit $sin^2 2\theta_{13} < 0.15$ was given by MINOS [19] both being accelerator experiments. The T2K experiment [20] in a five year ν_μ run at the full J-PARC beam intensity, will be of the order of $sin^2 2\theta_{13} < 0.006$(90 %CL). The Day Bay project in china could reach a $sin^2 2\theta_{13}$ sensitivity below 0.01,while the RENO experiment in Korea should reach a sensitivity arround 0.02.In table (1) ,table(2).We list the modified neutrino oscillation parameter for some sample value of a_1 and a_2.

a_1	a_2	θ_{12}in degrees	θ_{23}in degrees	θ_{13}in degrees
0^o	0^o	37.15	45.00	10.00
0^o	45^o	36.46	44.96	9.94
0^o	90^o	33.99	44.87	9.96
0^o	135^o	34.80	44.91	10.02
0^o	180^o	37.15	45.00	10.00
45^o	0^o	36.34	45.04	9.87
45^o	45^o	35.59	45.00	9.81
45^o	90^o	33.19	44.91	9.84
45^o	135^o	34.07	44.95	9.90
45^o	180^o	36.34	45.04	9.87
90^o	0^o	34.00	45.12	9.93
90^o	45^o	33.31	45.08	9.87
90^o	90^o	31.07	45.00	9.89
90^o	135^o	31.86	45.03	9.95
90^o	180^o	34.00	45.12	9.93
135^o	0^o	34.66	45.08	10.05
135^o	45^o	34.03	45.05	9.99
135^o	90^o	31.72	44.96	10.02
135^o	135^o	32.45	45.00	10.08
135^o	180^o	34.66	45.08	10.05
180^o	0^o	37.15	45.00	10.00
180^o	45^o	36.46	44.96	9.94
180^o	90^o	33.99	44.87	9.96
180^o	135^o	34.80	44.91	10.02
180^o	180^o	37.15	45.00	10.00

Table 1: The modified mixing angels term for various value of phases. Input value are $\Delta_{31} = 2.0 \times 10^{-3} eV^2$, $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, Mixing angle $\theta_{13} = 10^o, \theta_{23} = \frac{\pi}{4}$, $tan\theta_{12}^2 = \frac{1-3sin\theta_{13}^2}{2}, \theta_{12} = 34^o$ and Dirac phase $\delta = \frac{\pi}{2}$.

a_1	a_2	θ_{12}in degrees	θ_{23}in degrees	θ_{13}in degrees
0^o	0^o	37.15	45.00	10.00
0^o	45^o	34.77	44.91	10.02
0^o	90^o	34.05	44.87	9.98
0^o	135^o	36.52	44.96	9.94
0^o	180^o	37.15	45.00	10.00
45^o	0^o	34.66	45.08	10.05
45^o	45^o	32.42	45.00	10.08
45^o	90^o	31.78	44.96	10.02
45^o	135^o	34.09	45.05	9.99
45^o	180^o	34.66	45.08	10.05
90^o	0^o	34.00	45.12	9.93
90^o	45^o	31.83	45.03	9.95
90^o	90^o	31.12	45.00	9.89
90^o	135^o	33.38	45.08	9.87
90^o	180^o	34.00	45.12	9.93
135^o	0^o	36.34	45.04	9.87
135^o	45^o	34.04	44.95	9.90
135^o	90^o	33.25	44.91	9.84
135^o	135^o	36.66	45.00	9.81
135^o	180^o	37.15	45.00	10.00
180^o	0^o	37.15	45.00	10.00
180^o	45^o	34.77	44.91	10.02
180^o	90^o	34.05	44.87	9.96
180^o	135^o	36.52	44.96	9.94
180^o	180^o	37.15	45.00	10.00

Table 2: The modified mixing angels term for various value of phase. Input value are $\Delta_{31} = 2.0 \times 10^{-3} eV^2$, $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, Mixing angle $\theta_{13} = 10^o, \theta_{23} = \frac{\pi}{4}$, $tan\theta_{12}^2 = \frac{1-3sin\theta_{13}^2}{2}, \theta_{12} = 34^o$ and Dirac phase$\delta = -\frac{\pi}{2}$.

5 Conclusions

We have considered the effects of an effective dimension-5 operator due to gravity on neutrino mixings in the Co-bimaximal scheme. On electroweak symmetry breaking this operators leads to additional mass terms.This model predict a value for the modified neutrino mixing angle $\theta_{12}^{planck}{=}\theta_{12}{\pm}3.15^o$,$\theta_{13}^{planck}{=}\theta_{13}\pm 0.08^o$ and $\theta_{23}^{planck} = \theta_{23} + 0.12^o$ which is correspondence to Planck scale $M_{pl} \approx 2.0 \times 10^{19} GeV$. This occurs,of course for degenerate neutrino mass with a common mass of about 2eV.Hence large correction to the Solar mixing angles are possible and has to be taken into account,while doing global analysis with co-bimaximal scheme.

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