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Energy Spectrum of Elementary Excitations of Electron Liquid of Conductors in Magnetic Field (II).

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Abstract

This article is a continuation of the article Ref.[1]. In the article Ref.[1] was introduced an effective mass of the elementary excitation in the plane, which is perpendicular to the magnetic field, but the dependence of it on the parameters of the task was not be determined. In this work we make a reasonable assumption about this dependence, and the energy spectrum is investigated on this base. A dependence the density of states on the energy is obtained. It has form of a staircase, parameters of which depend from the magnetic field. The Fermi energy also depends on the magnetic field. The dependence of energy of the elementary excitation on the spin orientation that stipulates the Pauli paramagnetism is taken into account.

Keywords: Electron liquid; magnetic field; elementary excitation; effective mass;energy spectrum; Fermi energy; Pauli paramagnetism.

1. Introduction

As was shown in Ref. [1] the conventional theory of electron liquid in a magnetic field is incorrect. This paper also showed that the elementary excitations of an electron liquid in a magnetic field are in the field of a quasi-electric potential

$$w(\rho) = -\left[\mathbf{I}_1(\Lambda R)\Lambda R\right]^{-1}\frac{eB^2R^2}{4m}\left[\mathbf{I}_0(\Lambda\rho)-1\right], \tag{1}$$

which exists only inside the volume under consideration. This volume is cylinder with radius R and height L. The magnetic field with induction B is directed parallel to the cylinder axis. It is introduced the cylindrical coordinates (ρ,φ,z). The charge of electron is $(-e)$, and the mass of electron is m. $\mathbf{I}_k(t)$ is modified Bessel function. The parameter Λ^{-1} is the characteristic length of a shielding that was determined in Ref. [1]:

$$\Lambda^2 = \frac{5e^2\sigma_0}{4\zeta_0\varepsilon_0} = 10^{20}\,\mathrm{m}^{-2}. \tag{2}$$

Here $\sigma_0 = 4 \cdot 10^{29}\,\text{m}^{-3}$ is the density of electrons (four electrons per unit cell), $\zeta_0 = \left(\hbar^2/2m\right)\left(3\pi^2\sigma_0\right)^{2/3} = 2{,}9 \cdot 10^{-18}\,\text{J}$ is the Fermi energy of free electron gas, and ε_0 is the electrical constant. From axial symmetry and the form of the potential $w(\rho)$ it follows that the radial Schrödinger equation for elementary excitation has the form:

$$\left\{-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \frac{l^2}{\rho^2}\right) + (-e)\,w_0(\rho) - E\right\}\Psi_l(\rho) = 0 \;. \tag{3}$$

Here the spin energy in a magnetic field does not taken into account, and we will assume that all states are doubly degenerate. An effective mass of the elementary excitation in the plane that is perpendicular to the magnetic field is denoted as m^*, and l is a quantum number of the angular momentum. The boundary condition for this equation is $\Psi_l(R) = 0$. Then from formulas (1) and (2) it follows that we can replace the potential by its asymptotic form:

$$w(\rho) = \frac{(-e)B^2R^2}{4m}\exp\left(\Lambda\rho - \Lambda R\right)\sqrt{\frac{R}{\rho}} \;. \tag{4}$$

This equation was investigated in Ref. [1] by a change of the variable $u = \mathbf{exp}\left(\Lambda\rho/2\right)$, $\rho = \left(2/\Lambda\right)\mathbf{ln}(u)$. In the neighborhood of the bound $\mathbf{ln}\,u = \Lambda\rho/2 >> 1$, and we can suppose $\mathbf{ln}\,u \approx \Lambda R/2 >> 1$.Then this equation asymptotically has the form:

$$\left[\frac{\partial^2}{\partial^2 u} + \frac{1}{u}\frac{\partial}{\partial u} + \frac{4\left(\tilde{E} - l^2/R^2\right)}{\Lambda^2 u^2} - \frac{2m^* e^2 B^2 R}{\hbar^2 m \Lambda^3\left[\mathbf{exp}\left(\Lambda R\right)\right]}\right]\Psi_l(u) = 0, \text{ where } \tilde{E} = \frac{2m^*}{\hbar^2}E \;. \tag{5}$$

The only solution to this equation that has zeros is

$$\Psi_l(u) = \mathbf{K}_{i\tau}(\gamma u), \quad \gamma = \frac{eB}{\Lambda\hbar}\sqrt{\frac{2Rm^*}{m\Lambda}}\,\mathbf{exp}\left(-\frac{\Lambda R}{2}\right), \quad \tau = \frac{2}{\Lambda}\sqrt{\tilde{E} - \frac{l^2}{R^2}} \;. \tag{6}$$

Here $\mathbf{K}_{i\tau}(t)$ $(t = \gamma u)$ is the Macdonald function with imaginary index. The boundary value of the argument, when $\rho = R$ is $t_b = Be\sqrt{2Rm^*/m\Lambda^3\hbar^2} \approx 0{,}86\,B\left(m^*/m\right)^{1/2}$. (For numerical estimates we take $R = 0.16$ m). We proved in the mathematical supplement to the Ref. [1] that when $\mathbf{K}_{i\tau}(t_b) = 0$ the relation between the argument and values of index modulus is:

$$\left[-\sqrt{\theta_n^2 - 1} + \theta_n \ln\left(\theta_n + \sqrt{\theta_n^2 - 1}\right)\right] = \frac{\pi}{t_b}\left(n + \frac{1}{2}\right), \quad \tau_n = \theta_n t_b, \quad \theta_n = \frac{\tau_n}{t_b} > 1 \text{ for all } n, \tag{7}$$

where n is arbitrary integer number or zero. This equation determines the infinite increasing sequence $\{\tau_n\}$ of the index values. The eigenvalues of elementary excitations energy depend on three quantum numbers: radial - n, angular momentum - l, and free motion that is parallel to the magnetic field - k :

$$E(n,l,k) = \frac{\hbar^2\Lambda^2}{8m^*}\tau_n^2 + \frac{\hbar^2 l^2}{2m^* R^2} + \frac{\hbar^2 k^2}{2mL^2} \;. \tag{8}$$

Each value n determines the beginning of the energy spectrum of two-dimensional particles with a mass m in the area $S = LR\sqrt{m^*/m}$. The number of states that are having energies smaller than $E > E_n$ in this spectrum with taking into account the spin degeneration is equal to

$$N_n(E) = \frac{S}{2\pi}\frac{2m}{\hbar^2}(E - E_n) = \frac{\sqrt{mm^*}LR}{\pi\hbar^2}(E - E_n). \tag{9}$$

That can be presented as the area of the rectangle with base $E - E_n$ and height $LR\sqrt{mm^*}/\pi\hbar^2$. If we place these rectangles one above the other in descending order of area, then they form a staircase above the energy axis. The sum of the areas of rectangles with $E_n < E$ is the total number of states of elementary excitations with energy less than E. Then the contour of the staircase is a graph of dependence of the state density in this volume $\mathfrak{N}_V(E)$ from the energy.

The dependence of the effective mass on the parameters of the task does not be determined in the Ref. [1]. Therefore the subsequent investigation of the spectrum in this article is incompletely. In this work we make a reasonable assumption of this dependence, and the spectrum is investigated on this base. In the third section the dependence of energy of the elementary excitation on the spin orientation is taken into account. Two gases that are differing by the spin orientation have the common Fermi energy. Therefore they must have different densities. The sum of their densities is equal to the electron density σ_0.

2. The density of states and the Fermi energy of the elementary excitations of the electron liquid in magnetic field

From formula (8) it can be seen that the effective mass m^* must be very large in order for $E(0,0,0)$ to have an acceptable value. When $B \to 0$ must be $m^* \to m$. Than we can suppose that $m^* = m(1+P)$, where P is a parameter that is proportional to B and large when $B \approx 1\ \mathrm{T}$. In this magnetic field we can take $m^* = mP$. The number of states $N_n(E)$ in formula (9) must be proportional to the cylinder volume $\pi R^2 L$. Then it is necessary that P be proportional to R^2. The small length in the theory of an electron in a magnetic field is $\lambda = \sqrt{\hbar/eB} = B^{-1/2}\cdot 2{,}57\cdot 10^{-8}\,\mathrm{m}$. Therefore the formula

$$m^* = m\left(1 + \frac{R^2}{\lambda^2}\right) \approx m\frac{eBR^2}{\hbar} \tag{10}$$

Meets all requirements. Substituting that formula into the formula (9) we can determine the height of stair steps in the state density graphic $LR\sqrt{mm^*}/\pi\hbar^2 = mV/\pi^2\hbar^2\lambda$. Then we can determine the density of states in a unit of volume $\mathfrak{N}(E)$, which does not depend on the shape of the volume.

The boundary value of the argument of the Macdonald function

$$t_b = Be\sqrt{\frac{2Rm^*}{\Lambda^3\hbar^2 m}} = \frac{1}{2\lambda^3}\left(\frac{2R}{\Lambda}\right)^{3/2} \approx B^{3/2}\cdot 5{,}8\cdot 10^6. \tag{11}$$

In Ref. [1] two approximate solutions of the equation(7) were obtained. When this equation can be approximate presented as $\theta_n \ln(2\theta_n/\mathbf{e}) + O(\theta_n^{-2}) = \pi(n+1/2)t_b^{-1}$ the approximate solution can be presented as $\theta_n = \pi(n+1/2)t_b^{-1}\left[\ln(2\pi/\mathbf{e}) + \ln(n+1/2) - \ln(t_b)\right]^{-1}$. Here $\mathbf{e}$ is the Napierian base. It is evident that must be $2\pi(n+1/2)/\mathbf{e} > t_b$. For $n < t_b$ we present $\theta_n^2 = 1 + \gamma_n^2$, $\gamma_n < 1$. Than the equation (7) is approximated as $\gamma_n^3 = 3\pi(n+1/2)t_b^{-1}$, and we obtain $\theta_n^2 = 1 + \left[3\pi(n+1/2)t_b^{-1}\right]^{2/3} = 1 + \frac{\lambda^2\Lambda}{2R}\left[6\pi(n+1/2)\right]^{2/3}$. Suppose that $1 << n_F < t_b$. Then the spectrum of values $E(n,0,0)$ in the area $n \le n_F$ has the form

$$E(n,0,0) = \frac{\hbar^2\Lambda^2\lambda^2}{8mR^2}t_b^2\theta_n^2 = \frac{e^2B^2R}{4m\Lambda}\left[1 + \frac{\lambda^2\Lambda}{2R}\left[6\pi(n+1/2)\right]^{2/3}\right] = \frac{e^2B^2R}{4m\Lambda} + \frac{\hbar eB}{8m}\left[6\pi(n+1/2)\right]^{2/3}. \tag{12}$$

The interval $[E(n_F);E(n_F+1)]$, in which the Fermi energy is located, can be obtained as solution of the inequality

$$\frac{m}{\pi^2\hbar^2\lambda}\sum_{n=0}^{f}\{E(f)-E_n\}=\frac{(6\pi)^{2/3}}{8\pi^2\lambda^3}\left[(f+1)(f+1/2)^{2/3}-\sum_{n=0}^{f}(n+1/2)^{2/3}\right]\Rightarrow\begin{cases}>\sigma_0, & \text{if } f=n_F+1\\ <\sigma_0, & \text{if } f=n_F\end{cases}. \quad (13)$$

For determining n_F we can replace inequality (13) by the equation and omit the negligibly small terms. We shall obtain:

$$\frac{(6\pi)^{2/3}}{8\pi^2\lambda^3}\left\{n_F^{5/3}-\sum_{n=0}^{n_F}n^{2/3}\right\}\approx\frac{(6\pi)^{2/3}}{8\pi^2\lambda^3}\left\{n_F^{5/3}-\frac{3}{5}n_F^{5/3}\right\}=\sigma_0,\ n_F=\frac{(20\pi^2)^{3/5}(\lambda^3\sigma_0)^{3/5}}{(6\pi)^{2/5}}=B^{-0,9}\cdot 9,25\cdot 10^4 \quad (14)$$

If the magnetic field is sufficiently large, the assumption that $1<<n_F<t_b$ is self-consistent. The Fermi energy equal:

$$E_F=\frac{e^2B^2R}{4m\Lambda}+(6\pi)^{2/3}\frac{\hbar eB}{8m}n_F^{2/3}=\left(1,12\cdot10^{-19}\cdot B^2+3,33\cdot10^{-20}\cdot B^{0,4}\right)\text{ J}. \quad (15)$$

The whole spectrum shifts from the minimum of potential energy $-ew(\rho)$ (see formula (1)) by the value $-ew(R)\approx e^2B^2R/4m\Lambda$. The coordinates of the bounds of steps on the staircase on the axis of energy are proportional to $\hbar eB/m=\hbar\omega$, but the distances between them decrease in proportion to $n^{-1/3}$. From formulas (13) and (14) it follows that the second term in the Fermi energy is proportional to $B^{2/5}$. Therefore, when the magnetic field changes, the Fermi energy will move relative to the spectrum. When in the course of this moving the Fermi energy coincides with the energy coordinate of the boundary of the step at the staircase, features appear in the properties of the electron liquid.

3. The energy spectrum with taking into account the interaction of electron spins with the magnetic field

It may seem that the conservation of angular momentum makes it impossible to polarize spins in a magnetic field. But it is not. The Hamiltonian of an electron in the homogeneous magnetic field may be presented as

$$\hat{H}=\frac{1}{2m}\widehat{\mathbf{p}}^2+\frac{eB}{2m}\left(-y\hat{p}_x+x\hat{p}_y+2\hat{s}_z\right)+\frac{e^2B^2}{8m}r^2+U=\frac{1}{2m}\widehat{\mathbf{p}}^2+\frac{eB}{2m}\left(\hat{L}_z+2\hat{s}_z\right)+\frac{e^2B^2}{8m}r^2+U=$$
$$\frac{1}{2m}\widehat{\mathbf{p}}^2+\frac{eB}{2m}\hat{J}_z+\frac{e^2B^2}{8m}r^2+U+\frac{eB}{2m}\hat{s}_z.$$

Here $\hat{L}_z$ is the *z*- component of the orbital angular momentum operator, $\hat{s}_z$ is the spin operator and $\hat{J}_z=\hat{L}_z+\hat{s}_z$ is the *z*-component of the operator of total angular momentum. As was shown in Ref. [2], when the total angular momentum of an electron liquid retains a zero value, the term, which is proportional to $\hat{J}_z$ is excluded from the effective Hamiltonian. Then we obtain the spectrum that is consist from two spectrum (12) that are shifted on $\pm\hbar eB/4m$. That can be considered as two gases, which must have equal Fermi energies, because the spin can change their direction. Therefore the densities of these gases must be $(\sigma_0/2)\pm\delta\sigma$, where $\delta\sigma$ is determined by the equation $E_{F+}=E_{F-}$. The Fermi energies of these gases may be obtained in the same way as in the previous section. In the first approximation over $2\delta\sigma/\sigma_0$ we obtain:

$$\delta\sigma\approx 0,37\left(\frac{eB\sigma_0}{\pi^2\hbar}\right)^{3/5}=1,2\cdot10^8\left(B\sigma_0\right)^{3/5}\ \text{m}^{-3}$$

References

[1] Dubrovskyi, I (2018). Inhomogeneous density and energy spectrum of elementary excitations of electron liquid of conductors in magnetic field.*BJMP*, **4**(1), 313-322.

[2] Dubrovskyi,I (2011). Statistical mechanics that takes into account angular momentum conservation law – Theory and application. In JC Moreno-Pirajan (Ed.), *Thermodynamics – Interaction Studies – Solids, Liquids and Gases*, Ch.17**, (**pp. 445-468). ISBN: 978-953-307-563-1, InTech. doi:org/10.5772/19645 .