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Unknown classical electrodynamics

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Abstract

In this paper, we analyze some little-known inconsistencies and inaccuracies in classical electrodynamics and suggest ways to eliminate them.

Key words: Classical electrodynamics, special theory of relativity, Marinov Motor.

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I. INTRODUCTION

As early as the beginning of the 20th century, difficulties and problems appeared in classical electrodynamics. They are known. In connection with the rapid development of quantum electrodynamics and in connection with the progress in this field, it seemed that this theory would help eliminate problems in classical electrodynamics. However, with the passage of time, these hopes gradually faded, until they turned into illusions. Quantum electrodynamics itself is faced with problems. But most of these problems have classical roots.

In this article we have collected materials concerning mathematical ambiguities and misconceptions in classical electrodynamics. Not all ambiguities and misconceptions are considered. But the most important of them are included in the article. Where it was possible, we corrected ambiguities and gave new explanations for the problems. In the article we do not put forward hypotheses. We are just clearing the field for further research. The task is extremely necessary, but ungrateful. Not everyone is ready to part with their delusions. We tried to write concisely, but give detailed, evidentiary explanations.

II.THE GENERALIZED POYNTING'S LAW OF ENERGY-MOMENTUM CONSERVATION

II.1.The derivation of the Poynting's energy conservation law and the problem "4/3"

The proof, proposed by Poynting, is very simple. We briefly reproduce it for the convenience of the subsequent discussion. We write the Maxwell equations

$$\mathrm{rot}\,\mathbf{H} = \varepsilon\frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}\,, \qquad \text{(II.1.1)}$$

$$\text{rot}\,\mathbf{E} = -\mu\frac{\partial\mathbf{H}}{\partial t}\,, \tag{II.1.2}$$

$$\text{div}\mathbf{E} = \frac{\varrho}{\varepsilon}, \tag{II.1.3}$$

$$\text{div}\mathbf{H} = 0\,. \tag{II.1.4}$$

Multiplying expression (II.1.1) by $\mathbf{E}$, and expression (II.1.2) by $\mathbf{H}$ and adding up the results, after simple transformations we obtain

$$\text{div}\mathbf{S} + \frac{\partial w}{\partial t} + p = 0\,, \tag{II.1.5}$$

where $\mathbf{S} = [\mathbf{E}\times\mathbf{H}]$ is the energy flux density of an electromagnetic wave, $w = \varepsilon\mathbf{E}^2/\,2 + \mu\,\mathbf{H}^2/\,2$ is the electromagnetic wave energy density,$p = \mathbf{jE}$is the power density of external forces. The following circumstance is striking: two equations (II.1.3) and (II.1.4) were not used in the derivation of the Poynting's conservation law (II.1.5).We are talking about this, because the Poynting vector often gives a "misfire". For example, for more than 100 years there is a known problem of "4/3".

In accordance with the well-known formula $E = mc^2$, the electromagnetic mass of a resting particle can be determined in a dual way: either through the square of the electric charge field, or through the space charge density and its potential

$$m_e = \int \frac{\varepsilon(\text{grad}\,\varphi)^2}{2c^2}dV = \int \frac{\varrho\varphi}{2c^2}dV, \tag{II.1.6}$$

where ϱ and φ are, respectively, the space charge density and the potential of this charge.

In classical mechanics, the inertial mass of a particle m is related to its momentum $\mathbf{P}$ by the relation $\mathbf{P} = m\mathbf{v}$. The same relation holds (Umov's law) for the energy density of a particle or a material medium w with an energy flux density $\mathbf{S}$: $\mathbf{S} = w\mathbf{v}$. It can be assumed (Thomson, 1881) that the same properties should have the density of the electromagnetic energy of the charge field

$$\mathbf{S}_e = w_e\mathbf{v}\,, \tag{II.1.7}$$

where $w_e = \varepsilon(\text{grad}\varphi)^2/2$isthe energy density of the electromagnetic mass.

The problem of the electromagnetic mass arose after unsuccessful attempts to connect the electromagnetic mass of a charged particle with its electromagnetic momentum and kinetic energy, just as it is done in classical mechanics. Establishing such a connection could confirm the electromagnetic nature of matter.

Indeed, the electromagnetic momentum of the charge field $\mathbf{P}_e$ can be calculated by relying on the Poynting vector $\mathbf{S}$, and the kinetic energy of the field K_e can logically be related to the energy of the magnetic field, since the magnetic field is absent in a stationary charge. The magnetic field of the charge arises when the charge moves. It would seem that each element of a moving charge, having a velocity $\mathbf{v}$, must have an electromagnetic momentum directed along the velocity vector. However, the researchers on this path encountered difficulties, which at that time could not be solved. Calculations for a charged particle with a uniform distribution of the space charge over the surface led to the following relationships that are not characteristic of mechanics:

$$\mathbf{P}_e = \int \frac{[\mathbf{E}\times\mathbf{H}]}{c^2}dV = \frac{4}{3}m_e\mathbf{v}\,;\quad K_e = \int \frac{\mu\mathbf{H}^2}{2c^2}dV = \frac{4}{3}m_e\frac{\mathbf{v}^2}{2}. \tag{II.1.8}$$

As we see, a strange "4/3" coefficient appeared in the formulas instead of one. For this reason, the problem of electromagnetic mass was called "problem 4/3".

Formulas (II.1.8) give integral relations for a charge model in which the entire charge is distributed in a thin surface layer. The coefficient 4/3 in expressions (II.1.8) was obtained precisely for such a model. With a uniform charge distribution in a spherical volume, we will have a different factor. In order to "correct" this factor and connect electrodynamics with mechanics, a hypothesis was proposed about the existence of a non-electromagnetic mass charge in a charge. This mass should be responsible for the stability of the charge "torn" by the Coulomb repulsion forces. As a result, the sum of the electromagnetic and non-electromagnetic "masses" should give the classical inertial mass of the particle.

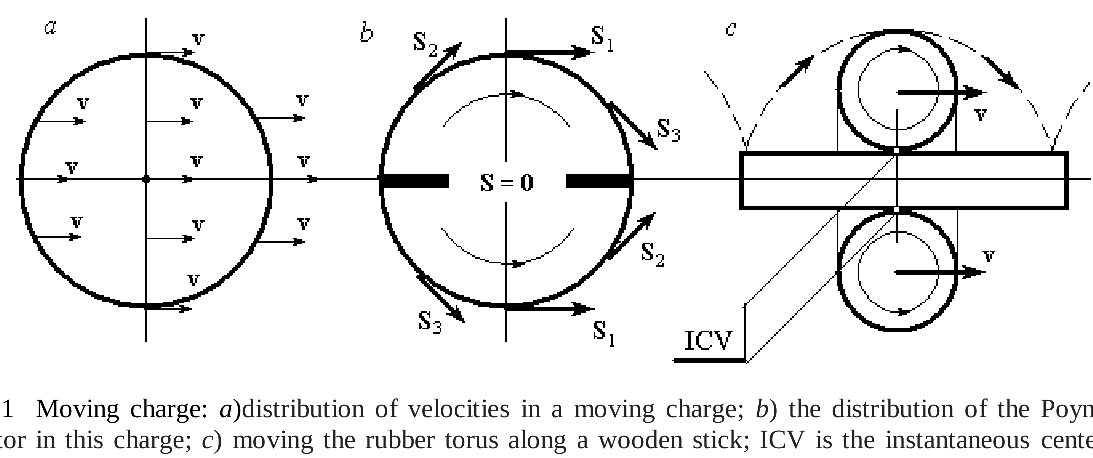
However, this was not a solution to the problem, since the "plus-defectiveness" of the electromagnetic mass is compensated by the "minus-defectiveness" of the non-electromagnetic mass.

II.2. Confusion with Poynting vector

Not knowing how to solve the problem, sometimes substitute for a verbal "surrogate" solution, creating the appearance of a solution. Let us analyze a detailed picture of the particle flux density with a uniform charge distribution, relying on the Poynting vector.

Consider a charge moving with a constant velocity v along the z axis. This means that any element of the charge has the same velocity **v** (see Fig. 1a). However, as shown in the same figure (see Fig. 1b), for local charge points the local Poynting vectors $\boldsymbol{S}$ have different values and directions. At points furthest from the z axis, the density of the vector $\boldsymbol{S}$ is maximal, and on the center line it is zero, since there is no magnetic field here.The direction of the Poynting vector resembles the displacement of a rubber torus worn on a stick. The inner layers of the torus do not move due to friction about the stick, as shown in Fig. 1c. Therefore, to move the torus it is necessary to "twist" the upper layers of the torus. In this case, the cross-sectional layers of the torus (having the form of a circle, as shown in Fig. 1c) are rolled along a stick. Their instantaneous center of velocities (ICV) is located on the surface of the stick. The instantaneous center of velocities for a moving charge is the segment (see 1b), where the Poynting vector is zero ($\boldsymbol{S} = 0$).

That is where the questions arise. Why does the direction of the Poynting vector not coincide with the velocity vector of the motion of the charge parts? Why is there no circular motion of the Poynting vector in the reference system, where the charge is *stationary*, and in the moving system there is a circular flow of the electromagnetic pulse (in accordance with the Poynting vector)? Why are the different charge points having the same velocity vector and the same density giving a different contribution to the total electromagnetic charge momentum?



Fig,1 Moving charge: *a*)distribution of velocities in a moving charge; *b*) the distribution of the Poynting vector in this charge; *c*) moving the rubber torus along a wooden stick; ICV is the instantaneous center of velocities.

The absurdity of the picture is confirmed by the theorem (L. D. Landau) [1], according to which the motion of the body can always be represented as the sum of two independent motions: *translational and rotational*. Consequently, if there is a rotational motion in one inertial frame of reference, then it must exist in any other inertial system. If there is no rotational motion, then it should not be in other inertial systems. Here it is a clear discrepancy (disagreement) between mechanics and electrodynamics. The situation is aggravated by the following circumstance. We assumed that the charge is spherically symmetric. But this is a hypothesis. The shape of the charge is unknown to us. With the same success, we can assume that the charge has the shape of an ellipsoid with a uniform charge distribution. In this case, we have a paradoxical result. The scalar electromagnetic mass assumes a "tensor" character

$$P_i = m_{ik} v_k \,, \qquad \text{(II.2.1)}$$

where P_i is the electromagnetic charge pulse, m_{ik} is the electromagnetic mass tensor; v_k is the velocity vector.This is really embarrassing! For sure, many have come across this result. So what is next? Further on, they took for granted erroneous positions of modern electrodynamics.

As we have seen, the Poynting vector can not be used to describe the electromagnetic mass. It *can be assumed that some kind of incorrectness* (error) was made in the derivation of the Poynting conservation law. Indeed, the proof proposed by Poynting is not the only one. The Poynting conservation law can be obtained in other ways. Let us try one of them.

II.3. The density of the Lagrange function for an electromagnetic field

It is strange, why does not the mathematical formalism well developed by mechanics and mathematicians be used to prove the Poynting theorem? For example, in the textbook [1] (§32. Energy-momentum tensor), a technique is given for obtaining the energy-momentum tensor, from which the conservation laws in a generalized form easily follow. To do this, it suffices to correctly write down the Lagrange function of the system.

As is known, the Lagrange function is not single-valued. But it must have a form that is invariant under the Galileo transformation (classical theory) or Lorentz (relativistic version). In [1] (§33) the following expression is given for the density of the Lagrange function

$$\Lambda = -\frac{1}{16\pi} F_{kl} F^{kl} = -\frac{1}{16\pi}\left(\frac{\partial A_i}{\partial x_k} - \frac{\partial A_k}{\partial x_i}\right)^2 . \qquad \text{(II.3.1)}$$

However, it is inconvenient for us to use this kind of density of the Lagrange function.

In the textbook [1], the construction of the theoretical fundamentals of electrodynamics proceeds from the Lagrange function for the charge. Then the electromagnetic field tensor F_{kl} is obtained. From it go to the energy-momentum tensor of the electromagnetic field, to the Maxwell equations and the Poynting theorem.

We will carry out the analysis in the reverse order and start with the density of the Lagrange function for the electromagnetic field of the wave, moving to the charge fields. We write this expression for the density of the Lagrange function

$$\Lambda = \frac{1}{\mu}\left[-\frac{(F_{ik})^{\mathbf{2}}}{4} + \mu j_i A_i\right] = -\frac{1}{4\mu}\left[\left(\frac{\partial A_k}{\partial x_i}\right)^2 - 2\frac{\partial A_i}{\partial x_k}\frac{\partial A_k}{\partial x_i} + \left(\frac{\partial A_i}{\partial x_k}\right)^2\right] + \frac{j_i A_i}{4\mu}. \qquad \text{(II.3.2)}$$

Since the Lagrange function is not uniquely determined, we transform expression (II.3.2) and give it another form of the Lagrange function, using the action integral

$$S = \int \Lambda\, d\Omega = \int \frac{1}{\mu}\left[-\frac{(F_{ik})^{2}}{4} + \mu j_i A_i\right] d\Omega\,, \qquad \text{(II.3.3)}$$

where $d\Omega = dx_1 dx_2 dx_3 dx_4$; $j_i = c\varrho u_i$ is 4-vector of current density; $u_i = dx_i/ds$ is 4-vector of speed; ϱ is space charge density.

We expand the integrand, transform and integrate by parts

$$\begin{aligned} S &= \int \frac{1}{\mu}\left[-\frac{1}{2}\left(\frac{\partial A_i}{\partial x_k}\right)^2 + \frac{1}{2}\frac{\partial}{\partial x_k}\left(A_i \frac{\partial A_k}{\partial x_i}\right) + \mu j_i A_i\right] d\Omega \\ &= \int \frac{1}{\mu}\left[-\frac{1}{2}\left(\frac{\partial A_i}{\partial x_k}\right)^2 + +\mu j_i A_i\right] d\Omega + \int \frac{1}{2\mu} A_i \frac{\partial A_k}{\partial x_i} dS_k\,. \end{aligned} \qquad \text{(II.3.4)}$$

In the second integral of the final expression (II.3.4), the limits of integration are infinity, where the field disappears when integrating over the coordinates. When integrating with respect to time, the initial and final points of variation are fixed, and there the variation of the integral is zero. Consequently, the last integral in expression (II.3.4) vanishes. Thus, we obtain a new very simple expression for the density of the Lagrange function

$$\Lambda = -\frac{1}{2\mu}\left(\frac{\partial A_i}{\partial x_k}\right)^2 + j_i A_i\,. \qquad \text{(II.3.5)}$$

This kind of density of the Lagrange function can be found in textbooks on quantum electrodynamics [2].The expression (II.3.5) is completely equivalent to the expression (II.3.1).

II.4 Maxwell's equations in the Lorentz gauge

Now we can get the "equations of motion", i.e. equations for finding the potentials of the electromagnetic field generated by the 4-vector of the current j_k. To this end, we write the functional (the action integral), which we will vary.

$$\delta S = \int \left[-\frac{1}{\mu}\frac{\partial A_i}{\partial x_k}\frac{\partial \delta A_i}{\partial x_k} + j_i \delta A_i\right] d\Omega\,. \qquad \text{(II.4.1)}$$

Integrating by parts, we obtain

$$\delta S = -\frac{1}{\mu}\int \left(\frac{\partial A_i}{\partial x_k}\delta A_i\right) dS_k + \int \frac{1}{\mu}\left[\frac{\partial^2 A_i}{\partial x_k^2} + \mu j_i\right] \delta A_i\, d\Omega = 0. \qquad \text{(II.4.2)}$$

The first integral over the hypersurface S_k vanishes for the same reasons as the last integral in (VI.4). Thus, we obtain the final system of equations for the 4-potential A_i

$$\frac{\partial^2 A_i}{\partial x_k^2} = -\mu j_i\,, \qquad \text{(II.4.3)}$$

to which the continuity equations should be added:

$$\frac{\partial A_i}{\partial x_i} = 0 \quad \text{and} \frac{\partial j_i}{\partial x_i} = 0. \tag{II.4.4}$$

The system of equations (II. 4.3)-(II. 4.4) represents the Maxwell equations in the Lorentz gauge:

$$\begin{gathered} \frac{\partial^2 \mathbf{A}}{\partial x^2} + \frac{\partial^2 \mathbf{A}}{\partial y^2} + \frac{\partial^2 \mathbf{A}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{j} \\ \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{\varrho}{\varepsilon} \\ \mathrm{div}\mathbf{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0 \\ \mathrm{div}\mathbf{j} + \frac{\partial \varrho}{\partial t} = 0\,. \end{gathered} \tag{II.4.5}$$

Thus, a new expression for the Lagrangian density leads to the *correct* equations of electrodynamics (the Maxwell equations in the Lorentz gauge). It remains unclear why this simple and mathematically correct approach is not used to describe the equations of electrodynamics?

II.5 Energy-momentum tensor and conservation laws

Analytical mechanics provides a way to construct the energy-momentum tensor from a given Lagrange function. The method is described in [1]. The energy-momentum tensor is

$$T_{ik} = \delta_{ik}\Lambda - \sum_l \frac{\partial A_l}{\partial x_i}\frac{\partial \Lambda}{\partial\left(\frac{\partial A_l}{\partial x_k}\right)}, \tag{II.5.1}$$

where

$$\Lambda = -\frac{1}{2\mu}\left(\frac{\partial A_i}{\partial x_k}\right)^2. \tag{II.5.2}$$

The calculations yield the following result

$$T_{ik} = \frac{1}{\mu}\frac{\partial A_l}{\partial x_i}\frac{\partial A_l}{\partial x_k} - \frac{1}{2\mu}\delta_{ik}\left(\frac{\partial A_l}{\partial x_i}\right)^2. \tag{II.5.3}$$

It is not difficult to see that the energy-momentum tensor is symmetric: $T_{ik} = T_{ki}$. It is known that the 4-divergence of this tensor for free space (when the fields are described outside sources) is zero, i.e. $\partial T_{ik}/\partial x_k = 0$.

From this expression follows the laws of conservation of energy and momentum of the wave. We will write down the results for a *space free from the sources of the fields*.

The law of conservation of the energy flux density $\mathbf{S}$ of the electromagnetic wave field

$$\frac{\partial \mathbf{S}}{\partial t}+\frac{1}{c^2}\operatorname{grad} w=0\,. \qquad \text{(II.5.4)}$$

The law of conservation of the energy density w of the electromagnetic wave field

$$\operatorname{div}\mathbf{S}+\frac{\partial w}{\partial t}=0, \qquad \text{(II.5.5)}$$

where

$$\mathbf{S}=-\frac{1}{\mu}\frac{\partial \mathbf{A}}{\partial t}\operatorname{div}\mathbf{A}-\frac{1}{\mu}\frac{\partial \mathbf{A}}{\partial t}\times\operatorname{rot}\mathbf{A}+\varepsilon\left(\operatorname{grad}\varphi\frac{\partial \varphi}{\partial t}\right); \qquad \text{(II.5.6)}$$

$$w=\frac{1}{2\mu}\left[(\operatorname{div}\mathbf{A})^2+(\operatorname{rot}\mathbf{A})^2+\left(\frac{\partial \mathbf{A}}{\partial t}\right)^2\right]-\frac{\varepsilon}{2}\left[(\operatorname{grad}\varphi)^2+\left(\frac{\partial \varphi}{\partial ct}\right)^2\right]. \qquad \text{(II.5.7)}$$

We analytically have obtained *generalized* Poynting conservation laws that describe *not only* the law of conservation of the energy *density* of an electromagnetic wave, *but also* the law of conservation of the energy *flux* density.

Let us now represent the vector potential $\mathbf{A}$ in the form of a sum of a rotational $\mathbf{A}_1$ and an irrotational $\mathbf{A}_2$ potential. $\mathbf{A}=\mathbf{A}_1+\mathbf{A}_2$.

From the obtained relations, very interesting conclusions follow:

1) In the general case, the Maxwell equations in the Lorentz gauge describe *three different types* of energy flows:

The first energy flow is a known flux of *transverse* electromagnetic waves, described by the Poynting vector. Its density is

$$\mathbf{S}_1=-\frac{1}{\mu}\frac{\partial \mathbf{A}_1}{\partial t}\times\operatorname{rot}\mathbf{A}_1=\mathbf{E}\times\mathbf{H}\,, \qquad \text{(II.5.8)}$$

where $\mathbf{E}$ and $\mathbf{H}$ are curl components of electromagnetic fields.

The second energy flow is a stream of *longitudinal* electric waves of the vector potential $\mathbf{A}_2$. Its density is

$$\mathbf{S}_2=-\frac{1}{\mu}\frac{\partial \mathbf{A}_2}{\partial t}\operatorname{div}\mathbf{A}_2\,. \qquad \text{(II.5.9)}$$

The third energy flow is stream of *longitudinal* waves formed by a scalar potential φ. Its density is

$$\mathbf{S}_3 = \frac{\partial \varphi}{\partial t} \operatorname{grad} \varphi. \qquad (\mathrm{II}.5.10)$$

2)The energy density and flux densities$\mathbf{S}_1$ and $\mathbf{S}_2$ formed by the vector potential $\mathbf{A}$ are *positive*, and the energy density and the flux density $\mathbf{S}_3$ created by the scalar potential φ are *negative*. This is not a new fact. This is known to some experts on quantum field theory. But this fact, as usual, is little known to physicists who specialize in other areas.

3)A new interesting consequence follows from the expressions (II. 5.4) and (II. 5.5). In the free space, the flux density and energy density must satisfy the *wave* equation, i.e. the flux density and energy density are also *retarded*, similar to the potentials of the fields of an electromagnetic wave:

$$\Delta \mathbf{S} - \frac{1}{c^2}\frac{\partial^2 \mathbf{S}}{\partial t^2} = 0; \qquad \Delta w - \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} = 0\,. \qquad (\mathrm{II}.5.11)$$

This means that the solution of some problems, for example, by diffraction of waves associated with the solution of vector wave equations, can be reduced to the same problems, but described by the wave equation for the scalar energy density w.In other words, in principle, sometimes it is possible to reduce the cumbersome calculations when solving similar problems.

4) The results obtained can easily be extended to any wave processes described by the wave equation. We have obtained conservation laws for electromagnetic waves in free space. The Poynting energy conservation law can be generalized by including the power density of the field sources. The results are listed in Table 1.

5) The limiting transition from wave phenomena to quasi-static phenomena is *impossible* in principle because of the *negative* energy of the field of the scalar potential. At the same time, it is *impossible* to solve *the electromagnetic mass* problem within the framework of retarded potentials. These fundamentally new results change a lot in understanding the phenomena of electrodynamics and allow you to get rid of misconceptions and prejudices.

Table 1

Transverse waves of the vector potential

$\mathbf{S}_1 = -\frac{1}{\mu}\frac{\partial \mathbf{A}_1}{\partial t} \times \operatorname{rot} \mathbf{A}_1$	$w_1 = \frac{1}{2\mu}\left[(\operatorname{rot}\mathbf{A}_1)^2 + \left(\frac{\partial \mathbf{A}_1}{c\partial t}\right)^2\right]$	$p_1 = -\mathbf{j}_1 \frac{\partial \mathbf{A}_1}{\partial t}$

Longitudinal waves of the vector potential

$\mathbf{S}_2 = -\frac{1}{\mu}\frac{\partial \mathbf{A}_2}{\partial t} \operatorname{div}\mathbf{A}_2$	$w_2 = \frac{1}{2\mu}\left[(\operatorname{div}\mathbf{A}_2)^2 + \left(\frac{\partial \mathbf{A}_2}{c\partial t}\right)^2\right]$	$p_2 = -\mathbf{j}_2 \frac{\partial \mathbf{A}_2}{\partial t}$

Longitudinal waves of the scalar potential

$\mathbf{S}_3 = \varepsilon \frac{\partial \varphi}{\partial t} \operatorname{grad}\varphi$	$w_3 = -\frac{\varepsilon}{2}\left[(\operatorname{grad}\varphi)^2 + \left(\frac{\partial \varphi}{\partial ct}\right)^2\right]$	$p_3 = \varrho \frac{\partial \varphi}{\partial t}$

II.6 The condition for the absence of longitudinal waves

It is obvious that longitudinal waves will not exist if there are no sources that excite these waves. To solve this problem, we need to consider the right-hand side of the Maxwell equations in the Lorentz gauge for the potentials $\mathbf{A}_2$ and φ creating longitudinal waves. We write down the necessary equations for analysis.

$$\Delta\mathbf{A}_2 - \frac{1}{c^2}\frac{\partial^2\mathbf{A}_2}{\partial t^2} = -\mu\mathbf{j}_2;\ \ \mathrm{rot}\mathbf{A}_2 = 0; \mathrm{rot}\mathbf{j}_2 = 0; \qquad (\mathrm{II}.6.1)$$

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{\varrho}{\varepsilon}; \qquad (\mathrm{II}.6.2)$$

$$\mathrm{div}\mathbf{A}_2 + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0\,. \qquad (\mathrm{II}.6.3)$$

Let us use here the idea of L. D. Landau. [1] on the possibility of eliminating one of the four equations (see Chapter 3, paragraph 18, "Gradient Invariance"). For example, we can eliminate the equation for the scalar potential in order to bring two wave equations (II. 4.5) to one vector equation.For this purpose, in (II. 6.1) we differentiate the equation for $\mathbf{A}_2$ in time, and in (II. 6.2) we apply the gradient operator to all the terms. And then we summarize the results. We obtained the wave equation for the longitudinal electric field $\mathbf{E}_L$

$$\begin{gathered}\Delta\mathbf{E}_L - \frac{1}{c^2}\frac{\partial^2\mathbf{E}_L}{\partial t^2} = \Delta\left(-\frac{\partial\mathbf{A}_2}{\partial t} - \mathrm{grad}\,\varphi\right)\\ -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(-\frac{\partial\mathbf{A}_2}{\partial t} - \mathrm{grad}\,\varphi\right) = \mu\frac{\partial\mathbf{j}_2}{\partial t} + \frac{1}{\varepsilon}\mathrm{grad}\,\varrho\,.\end{gathered} \qquad (\mathrm{II}.6.4)$$

Thus, the electric field that determines the longitudinal waves of the vector $\mathbf{E}_L$ is described by the expression (II. 6.4). On the right side there are *sources* of longitudinal electric field.

In order to the field $\mathbf{E}_L = 0$, it is necessary that the sources of this field are absent, i.e. it is necessary that

$$\mu\frac{\partial\mathbf{j}_2}{\partial t} + \frac{1}{\varepsilon}\mathrm{grad}\,\varrho = 0. \qquad (\mathrm{II}.6.5)$$

Besides, we can use the continuity equation for the irrotational current component

$$\mathrm{div}\,\mathbf{j}_2 + \frac{\partial\varrho}{\partial t} = 0\,. \qquad (\mathrm{II}.6.6)$$

Both conditions (II. 6.5) and (II. 6.6) lead to the following final wave equations

$$\Delta\mathbf{j}_2 - \frac{1}{c^2}\frac{\partial^2\mathbf{j}_2}{\partial t^2} = 0 \quad \text{and} \quad \Delta\varrho - \frac{1}{c^2}\frac{\partial^2\varrho}{\partial t^2} = 0. \qquad \text{(II.6.7)}$$

We found an interesting fact: *the absence of longitudinal waves* will be if and only if the charge density and the density of the irrotational current component satisfy the wave equation, i.e. are "late" or "outstripping"!

II.7 Discussion of the results of Chapter II

The conclusions that we have obtained have a rigorous mathematical confirmation. We have to comprehend them. This is important, since standard textbooks on field theory (on electrodynamics) give *completely different results*. Let us start in order.

1. We have established that the Maxwell equations in the Lorentz gauge can describe three types of flows: $\mathbf{S}_1$ is the standard flux density of a transverse electromagnetic wave; $\mathbf{S}_2$ is the flux density of the longitudinal wave of the vector potential; $\mathbf{S}_3$ is the flux density of the longitudinal wave of the scalar potential.

2. There are two problems. The first problem: how does the flow of longitudinal waves correlate with the Maxwell equations in the Coulomb gauge, where there exist *only transverse waves* (flux density $\mathbf{S}_1$)? The second problem is the problem of matching experiments. Until now, in nature, no longitudinal waves have been observed experimentally, although in order of magnitude these waves should have the same order of magnitude as the longitudinal waves of the vector potential. All this casts doubt on "gauge invariance".

3. The condition for mutual cancellation of longitudinal waves of the scalar and vector potential gives hope that gauge invariance should occur under certain conditions. As we have seen, this is not always possible.

4. The condition for the absence of longitudinal waves imposes stringent conditions on the densities of external currents and charges. They should be functions of advanced and retarded potentials. For this reason, we would call them "non-inertial". As a consequence, the fields of inertial charges should be described (strange as it may seem) by *other equations* within the framework of Maxwell's equations. Recall that we *do not change* the mathematical formalism of classical electrodynamics. We rethink it and give a new interpretation, eliminating mistakes and prejudices.

5. We have established that the Poynting vector has applicability limits. *It can not be used to describe charge fields*. For example, the electromagnetic charge mass is negative, since the field energy of the scalar potential is negative.

We will continue to discuss all these questions in other Chapters. Perhaps the results presented above were received by someone earlier. But we have not found them in the scientific and technical literature.

III. THEFORGOTTEN CONSERVATION LAW OF UMOV

III.1 The Umov Vector

In the previous chapter, we obtained results that should be stated simultaneously with the new ones, since they are connected by a single logic. We list the outlined problems.

1. The problem of the electromagnetic mass of an inertial charge.

2. Conditions for the validity of gauge invariance.

3. Inertial and non-inertial charges and currents.

Obviously, the solutions that we find will generate new questions that require an answer. In this chapter, we will mainly deal with mathematical problems. Questions of interpretation, questions related to the Lorentz transformation, we will consider later. Now we will continue to seek a solution to the problem of the electromagnetic mass of *inertial charges*, as well as to analyze the mathematical features of the solutions of Maxwell's equations.

Let us prove the Umov conservation law for inertial charges. We write the Maxwell equations in the Lorentz gauge

$$\frac{\partial^2 A_i}{\partial x_l^2} = -\mu j_i \,, \qquad \text{(III.1.1)}$$

$$\frac{\partial A_i}{\partial x_i} = 0 \quad \text{and} \, \frac{\partial j_i}{\partial x_i} = 0. \qquad \text{(III.1.2)}$$

where$u_i = dx_i/ds$, $j_i = c\varrho u_i$, $A_i = \varphi u_i/c$, the values of ϱ and φ are taken in the reference frame associated with the charge (**v**= 0).

We show that the law of conservation of Umovimplies from Eq. (III.1.1).To prove this, we multiply the right and left parts of the equation (III.1.1) by $-c/2\mu \cdot \partial A_k/\partial x_i$ and transform the result obtained.

The right part will be

$$\frac{c}{2} j_i \frac{\partial A_k}{\partial x_i} = \frac{1}{2} c^2 \varrho u_i \frac{\partial A_k}{\partial x_i} = \frac{c^2 \varrho}{2} \frac{\partial \varphi u_k}{\partial x_i} u_i = \frac{c^2}{2} \varrho \varphi \frac{du_k}{ds} = 0. \qquad \text{(III.1.3)}$$

So, the right-hand side vanishes, since the potential φ is taken in its own frame of reference, where it does not depend on time, and the external forces do not act on the charge, and it does not experience acceleration$(du_k/ds = 0)$.

The left part will be

$$-\frac{c}{2\mu} \frac{\partial A_k}{\partial x_i} \frac{\partial^2 A_i}{\partial x_l^2} = -\frac{c}{2\mu} \frac{\partial}{\partial x_i} \left(A_k \frac{\partial^2 A_i}{\partial x_l^2} \right) = \frac{c}{2} \frac{\partial}{\partial x_i} (A_k j_i) = c \frac{\partial}{\partial x_i} \left(\frac{\varrho \varphi}{2} u_k u_i \right) = 0. \qquad \text{(III.1.4)}$$

Thus, we obtained on the left-hand side an expression for the divergence of *the energy-flux density tensor* for the charge field. If the components of this tensor are divided by the square of the speed of light and are integrated over the spatial volume, we obtain an expression for *the energy-momentum tensor*T_{ik} of a relativistic particle with the electromagnetic mass m_e. 4-divergence of the tensor T_{ik} is determined by the expression:

$$\frac{\partial}{\partial x_i} (T_{ik}) = \frac{\partial}{\partial x_i} (m_e c u_i u_k) = 0. \qquad \text{(III.1.5)}$$

It follows from the expression obtained that *the relativistic momentum of the electromagnetic mass is constant*. This is obvious, since the forces do not act on the charge, and the charge moves at a constant speed ($\partial \mathbf{P}_e / \partial t = 0$).

From (III.1.4) follows the Umov law of conservation of energy, which has the standard form:

$$\mathrm{div}\mathbf{S}_u + \frac{\partial w}{\partial t} = 0, \qquad \text{(III.1.6)}$$

where $\mathbf{S}_u = \frac{w\mathbf{v}}{\sqrt{1-(v/c)^2}}$, $w = \frac{\varrho\varphi}{\sqrt{1-(v/c)^2}}$is the flux density and the energy density of the charge field.

It is not difficult to see that the expression obtained corresponds to the classical expression up to a relativistic factor. And why do Maxwell's equations correspond to *two different* energy-momentum conservation laws? The answer is simple: each law corresponds to a specific *functional* solution (one law for retarded potentials, the other for instantaneous potentials within the Maxwell equations).

Umov's law describes the conservation of the energy of *instantaneously acting potentials*, and *Poynting's*law of conservation of the energy is applicable only for *retarded potentials*! This circumstance is key to understanding the phenomena of electrodynamics. Instantaneous potentials are "hidden" in the equation $A_i = \varphi u_i / c$ and condition (III.1.2).

III.2Some remarks about instantaneous action at a distance

Thus, in Maxwell's equations (even in the Lorentz gauge), the wave (retarded and advanced) potentials and the instantaneously acting potentials realizing *actio in distans* (action-at-the-distance) "get along".

O.D. Khvolson writes in his "Course of Physics" [3] (§ 4. Actio in distans): *"The term "actio in distans", i.e. "Action at a distance" denotes one of the most harmful teachings that have ever dominated in physics and hindered its development: it is a doctrine that allowed the direct action of something (A) to something else (B), which is from him on a certain and so a great distance, that there cannot be any contact between A and B ....*

... The disciple of Newton, Cotes, in the preface to the second edition of Principia, which Newton did not read before it was printed, first clearly expressed the idea of "actio in distans", that the bodies are directly attracted. On the one hand, the certainty that the view expressed in the preface to his book is approved by Newton, on the other - the grandiose development of celestial mechanics based entirely on the law of universal gravitation, as a fact, and not requiring any of its explanations, made scientists forget about the pure descriptive character of this law and see in it the complete expression of a really occurring physical phenomenon ...

*... The idea of action in the distant past, which prevailed in the last century, received new food, was further strengthened when, at the end of the century, it turned out from Coulomb's experiments that both magnetic and electrical interactions can be reduced to interactions of special hypothetical substances (two electricity and two magnetisms), which occurs directly in the distance and according to laws, quite analogous to Newton's law. ... In the first half of this century (*XIX century - our note*), actio in distans dominated science in full. ....*

... At the present time, it has become common to believe that actio in distans should not be allowed into any area of physical phenomena. But how to drive it out of the doctrine of universal gravitation?"

History really develops in a spiral. What was previously perceived as a natural explanation later turned into an absurdity. Now we are back to *instantaneous action at a distance*. And this is natural. Physicists *have not analyzed to the end* possible solutions to the Maxwell equations. It seemed to them (the intuition had let slip!) that the retarded potentials exhausted all sorts of interactions. Now we have to take a "step back" to go further.

In Chapter II, we obtained a result that actually denies the possibility of electrostatic (magnetostatic) interaction of inertial charges due to the *negative* magnitude of the electromagnetic mass that results from retarded potentials. The point is not even in quantitative relationships, but in a *qualitative difference*: the interaction energy has a *negative* sign! This leads to the fact that the charges of the same sign should be attracted, and the opposite signs must be repelled! This is absurd.

Above we strictly deduced the Umov's law of conservation, of which is associated with instantaneous action at a distance! The circle of the spiral ends.

III. 3Some remarks about causality and the concept of "interaction"

Now we have to describe the model of instantaneous interaction at a distance and connect this interaction with the principle of causality.

Let us quote GSE [4] (namely, the article "Interaction in physics"): "**INTERACTION** in *physics*, the impact of bodies or particles on each other, leading to a change in the state of their motion. In the mechanics of Newton, the mutual action of bodies on each other is quantitatively characterized by force. A more general characteristic of **I**. is the potential energy. Initially, in physics, the idea was established that I. between bodies can be carried out directly through an empty space, which does not take any part in the transfer of **I**.; while **I**. moves instantly.... This was the so-called. the concept of *long-range action* ...It has been proved that **I**. electrically charged bodies *is not instantaneously* realized and the displacement of one charged particle leads to a change in the forces acting on other particles, not at the same instant, but only after a finite time. ... Accordingly, there is a "*mediator*" that carries out the **I**. between the charged particles. This mediator was called an electromagnetic field. .... There was a new concept **-** the concept of short-range action, which was then extended to any other **I**."

Let us give our interpretation of this concept. "INTERACTION" is a *philosophicalcategory* that reflects the spatially-temporal *process* of mutual influence of the material objects under consideration. So, the *elementary* interaction is a *process* (!), which proceeds in an *elementary* volume for a *small* time interval. The total interaction consists of the sum of the elementary ones.

We want to emphasize once again that interaction is not a field, a substance or a material object. Any local interaction is a *process* that can be characterized by *intensity* in the chosen local volume, but *not by the velocity of displacement of an elementary volume* in space. The volume itself can move relative to the observer, but this displacement is not the "*speed of propagation of interaction*"!Interaction is impossible without mutual contact between objects. "Mutual contact" at a given point in space belongs to two objects at the same time. Therefore, the concept of "the speed of propagation of interactions" loses its physical meaning.

So, the process is not a material object. Consequently, the concept of "*the speed of propagation of interactions*" has no basis. This is an empty, meaningless concept (a concept that has no physical meaning). To illustrate, let us consider an example. Let two charges be at a great distance from each other. Coulomb instantaneous forces we neglect. We will consider the interaction with the help of retarded potentials. Let us assume that the first charge was acted upon by an object that changed the position of the first charge in space. The first charge emits a perturbation wave, which after a while will reach the second charge and will affect it.

Question: where can this "*speed of spreading of interactions*" be “inserted” here? Which part of the described process is the answer to this term? There is no such speed in nature! The described process of interaction consists of three parts:

1. *The interaction* of the first charge with an object and the appearance of a perturbed wave propagating from the charge.

2. *The propagation* of an electromagnetic wave from the first charge to the second. The usual process of wave propagation is flowing, and there are no interactions!

3. Then *the interaction* of the second charge with the disturbed wave begins.

The question of cause-effect relations in classical electrodynamics was considering detail by O. D. Jefimenko in [5]. Note that there are many different paradoxes, somehow related to cause-effect relations in classical electrodynamics (see, for example, [6]).

Now it is useful to consider the physical model of interaction at a distance. Imagine that the platform descends from the hill, and after striking it strikes the other, standing in its way. Such a collision refers to a "point" contact type. We put an elastic spring between the carts. If the spring has a mass, then when the moving cart is struck along the spring along the spring, a compression wave will propagate. The speed of this wave will depend on the rigidity and mass of the spring.

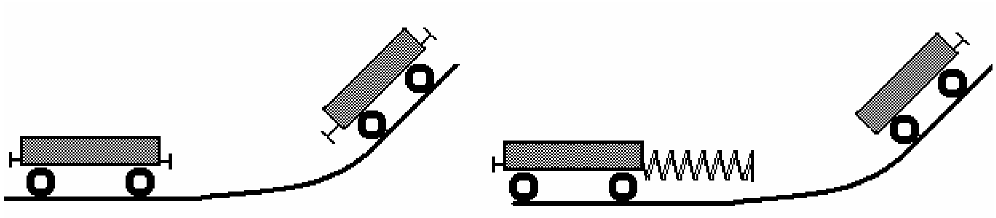


Fig. 2 Collision of carts.

Suppose now that the mass of the spring is zero. In the limit, the velocity of propagation of a wave from a moving cart to a stationary one and back will be infinite. The collision of the carts will no longer be a "point", since the carts are separated by a spring. However, the interaction will retain its contact character. Such an interaction was called a contact interaction of a point type.

Now we can consider the case of interaction of electric or gravitational charges. There are two possible explanations. The electromagnetic mass of a resting charge is determined by the formula

$$m = \int \frac{\varrho\varphi}{2c^2} dV. \qquad \text{(III.3.1)}$$

According to this approach, the inertial charge mass is concentrated in the charge itself. Consequently, the electric field surrounding the charge has no inertial properties. It is like an inertial spring, considered earlier. An analog of this field is the lines of force, which have elastic properties. They determine the contact nature of the interaction. Thus, the instantaneous action at a distance does not contradict the principle of causality and has its analogue interaction of a contact type.

III.3a Special results

Below we present the results of the research by Dr. M.V. Korneva (Physical faculty, department of electronics, Voronezh State University, Russia). She allowed us to use them in our article.

Speed of propagation of interactions. We have already written that interaction is a process. An obligatory condition for interaction is the presence of contact between objects. If there is direct contact, then interaction is possible. If there is no contact, then there is no interaction. An example of the lack of interaction in the presence of a contact is the intersection of two waves that go in different directions. The interaction of waves is absent due to the principle of superposition in classical electrodynamics. Direct contact is an area common to two objects. Therefore, the term "propagation speed of interactions" should belong to two objects at once or not. This is the main reason for the lack of a strict definition of the concept of "the speed of the spread of interactions." Generally speaking, *this term has no physical meaning*.

Class of transformations. However, the problem of speed limits is preserved by the relativistic factor $1/\sqrt{1-V^2/c^2}$. It turns out that the Lorentz transformation is not the only transformation preserving the invariance of the Maxwell equations. We will show this. We shall seek a class of transformations of 4-coordinates for which the wave equations retain their shape in accordance with the Galileo-Poincaré principle.

Consider two inertial reference frames K and K_0, which move relative to each other with velocity V along the x-axis. The space-time coordinates of the system $K(x, y, z, ct)$must be related to the corresponding coordinates$K_0(x_0, y_0, z_0, ct_0)$using the transformation matrix$[\boldsymbol{T}(V/c)]$.

$$[\boldsymbol{X}_0] = [\boldsymbol{T}(V/c)][\boldsymbol{X}], \qquad \text{(III.3a.1)}$$

where $[\boldsymbol{X}]$and $[\boldsymbol{X}_0]$are vector columns of 4-coordinates, $[\boldsymbol{T}(V/c)]$ is the matrix of transformation, depending only on the speed of the relative motion of the compared inertial systems.

The following requirements are imposed on the matrix $[\boldsymbol{T}]$:

1)the determinant of the matrix must be equal to one $\det[\boldsymbol{T}] = 1$;

2) there must exist an inverse transformation matrix from K_0 to K, i.e. the matrix $[\boldsymbol{T}(V/c)]^{-1}$;

3) the matrix of the inverse transformation must be obtained by replacing V by $-V$. This follows from the equality of inertial reference systems$[\boldsymbol{T}(V/c)]^{-1} = [\boldsymbol{T}(-V/c)]$. The product$[\boldsymbol{T}(V/c)] \cdot [\boldsymbol{T}(-V/c)] = [\boldsymbol{E}]$, where $[\boldsymbol{E}]$ is the unit diagonal matrix.

From these conditions it is possible to determine the general form of the matrix of transformations of coordinates and time preserving the invariant form of the wave equations. Equations corresponding to (III.3a.1) can be written in the following form:

$$x_0 = x\sqrt{1+f^2(V/c)} - Vt; y_0 = y;\ z_0 = z;\ \ ct_0 = ct\sqrt{1+f^2(V/c)} - x\,V/c, \qquad \text{(III.3a.2)}$$

where $f(V/c)$ is an odd function relative to V/c, V is the relative velocity of the inertial systems.The conditions listed above *are not sufficient*, unfortunately, to determine the explicit form of the function $f(V/c)$. It can be V/c, or $\sin(V/c)$, or $\sinh(V/c)$, etc.At low velocities, when the function $f(V/c) = V/c$ is much less than one, we have the general result:

$$x_0 = x\left[1 + \frac{(V/c)^2}{2}\right] - Vt; y_0 = y;\ z_0 = z;\ ct_0 = ct\left[1 + \frac{(V/c)^2}{2}\right] - x\,V/c. \quad \text{(III. 3a. 3)}$$

We give some special cases:

A) If $f(V/c) = v/\sqrt{v^2 - c^2}$ then we have the Lorentz transformation

$$x_0 = \frac{x - vt}{\sqrt{1 - (v/c)^2}}; y_0 = y;\ z_0 = z;\ ct_0 = \frac{ct - x\,v/c}{\sqrt{1 - (v/c)^2}}, \quad \text{(III. 3a. A)}$$

where v is the velocity entering into the Lorentz transformation.

B)If $f(V/c) = V/c$then we have a modified transformation

$$x_0 = x\sqrt{1 + (V/c)^2} - Vt; y_0 = y;\ z_0 = z;\ ct_0 = ct\sqrt{1 + (V/c)^2} - x\,V/c. \quad \text{(III. 3a. B)}$$

We see that the speed V in the modified transformation is related to the speed v in the Lorentz transformation by the relation$V/c = v/\sqrt{v^2 - c^2}$.

C)If $f(V/c) = \sinh(V/c)$then we have

$$x_0 = x \cdot \cosh(V/c) - ct\sinh(V/c);$$
$$y_0 = y;\ z_0 = z; \quad \text{(III. 3a. C)}$$
$$ct_0 = ct \cdot \cosh(V/c) - x \cdot \sinh(V/c).$$

We believe that *direct* measurements are necessary to determine the form of the function $f(V/c)$.

Group transformation properties. To each function $f(V/c)$in expression (III.3a.2),there corresponds a transformation that forms a group. Such a group is *non-commutative*. This property creates insurmountable difficulties in describing phenomena. We illustrate this with elementary examples.

Let the light source S move along the *x*-axis with the velocity **V**, as shown in Fig. 3.We need to find a reference system where this source is at rest.

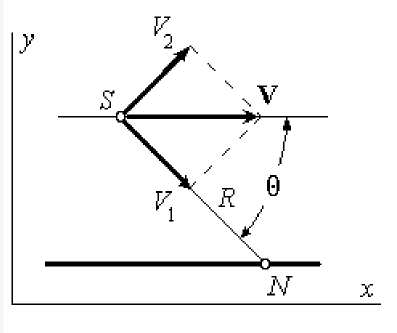


Fig. 3

The observer N sees this source at an angle θ. We can go to the correct system of the report in several ways. For example, we can expand the vector **V** by the sum of two orthogonal components. One component$V_1 = V\cos\theta$ is directed towards the observer N; the other component $V_2 = V\sin\theta$ has an orthogonal direction.

We can, for example, first use the transformation$[\mathbf{T}(V\cos\theta/c)]$, and then apply the transformation $[\mathbf{T}(V\sin\theta/c)]$. The general transformation then has the form:

$$\left[\boldsymbol{T}\left(\frac{V}{c}\right)\right] = \left[\boldsymbol{T}\left(\frac{V\cos\theta}{c}\right)\right]\cdot\left[\boldsymbol{T}\left(\frac{V\sin\theta}{c}\right)\right]. \qquad \text{(III.3a.4)}$$

But we can swap the transformation matrices:

$$\left[\widetilde{\boldsymbol{T}}\left(\frac{V}{c}\right)\right] = \left[\boldsymbol{T}\left(\frac{V\sin\theta}{c}\right)\right]\cdot\left[\boldsymbol{T}\left(\frac{V\cos\theta}{c}\right)\right]. \qquad \text{(III.3a.5)}$$

Using the expression (III.3a.4) or (III.3a.5), we find an inertial system in which the light source S is at rest:

$$[\boldsymbol{X}_0] = [\boldsymbol{T}(V/c)][\boldsymbol{X}]\text{and}\left[\widetilde{\boldsymbol{X}}_0\right] = \left[\widetilde{\boldsymbol{T}}(V/c)\right][\boldsymbol{X}], \qquad \text{(III.3a.6)}$$

where $[\boldsymbol{X}]$ is a 4-matrix column (x, y, z, ict).Obviously, we will get two different results, that is, we will find two different frames of reference$[\boldsymbol{X}_0] \neq \left[\widetilde{\boldsymbol{X}}_0\right]$. This is the *first* difficulty caused by the noncommutativity of the group.

Let us now consider the *second* difficulty. We will try, with the help of the inverse transformation, to return the particle back to our frame of reference. It is possible to quickly find the matrix of the inverse transformation if in the matrix of the direct transformation we replace the sign of the velocity V by the opposite one:

$$\left[\boldsymbol{T}\left(\frac{V}{c}\right)\right]^{-1} = \left[\boldsymbol{T}\left(\frac{-V}{c}\right)\right]. \qquad \text{(III.3a.7)}$$

However, because of noncommutativity, we cannot return the source to our frame of reference to the same point.

$$\left[\boldsymbol{T}\left(\frac{V}{c}\right)\right]\times\left[\boldsymbol{T}\left(\frac{V}{c}\right)\right]^{-1}=\left[\boldsymbol{T}\left(\frac{V}{c}\right)\right]\times\left[\boldsymbol{T}\left(\frac{-V}{c}\right)\right]$$
$$=\left[\boldsymbol{T}\left(\frac{V\cos\theta}{c}\right)\right]\cdot\left[\boldsymbol{T}\left(\frac{V\sin\theta}{c}\right)\right]\times\left[\boldsymbol{T}\left(\frac{-V\cos\theta}{c}\right)\right]\cdot\left[\boldsymbol{T}\left(\frac{-V\sin\theta}{c}\right)\right]\neq[\mathbf{E}]\,, \tag{III.3a.8}$$

where $[\boldsymbol{E}]$ is the unit diagonal matrix.

Now we can state a hypothesis and answer the question: Why did not Poincaré defend his priority in creating the STR? We assume that Poincaré saw the non-commutative character of the Lorentz group. He understood that the direction he had previously had no prospects. Poincaré lost interest in this idea and did not defend his priority. Perhaps he was looking for a new solution, but premature death violated plans.

We are sincerely grateful to Dr. M.V. Korneva for providing interesting materials and for the possibility of placing them in our article.

III.4 Instant interaction and the Lorentz transformation

In the first section of this chapter, we found that Maxwell’s equations describe instantaneously acting potentials in addition to retarded potentials. Further we showed that the instantaneous action at a distance does not contradict the causality principle and is based on the “contact type” interaction model.

Now we need to understand the nature of “duality” of potentials in Maxwell's equations using the example of a scalar potential. We want to answer the questions: why can the wave equations (the Lorentz gauge) give solutions in the form of potentials of an instantaneous nature and do not such potentials conflict with the Lorentz transformation? We will conditionally call a charge whose potential is a retarded, *virtual charge*. “Non-inertial charges” is a special case of virtual charges. A virtual charge can have any speed, and the velocity of a non-inertial charge is fixed and equal to the speed of light. However, both types of charges create a *retarded* potential.

Let the virtual charge be a sphere on whose surface the charge with a surface density $\sigma = q/4\pi a^2$, where a is the radius of the sphere, is uniformly distributed. The charge is stationary. The equation for the potential of the virtual charge field has the form:

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{q}{4\pi\varepsilon a^2}\delta(r-a). \tag{III.4.1}$$

The potential at $r = 0$ must be limited. Assume that the virtual charge is born at the initial time ($t = 0$). To solve the wave equation, we must give the initial conditions. We choose the initial conditions to be zero.Here there are two *prejudices* that we need to show. First, the conviction that Maxwell's equations are not capable of describing the "birth" of charges is sufficiently firmly entrenched in the minds of those who professionally deal with problems of electrodynamics. However, the presence of "initial conditions" refutes this fact. The wave equation describes potentials starting from the instant $t = 0$. The right-hand side of the wave equation (by virtue of this) is identically zero for $t < 0$.All processes up to the time $t = 0$ are *“compressed and captured” precisely in the initial conditions*. Thus, the process of "appearance" ("birth") of a charge does not contradict the mathematical description. Immediately note that this process responds to virtual charges and currents.

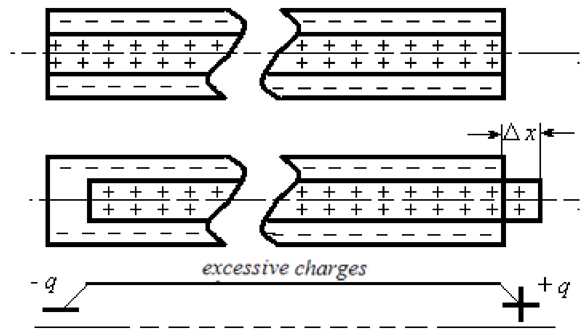


Fig. 4

In addition, the process of "birth" of a single charge contradicts the law of conservation of charges. We show that the linearity of the Maxwell equations allows us to bypass this fact. Consider two thin coaxial charged cylinders inserted into each other, as shown in Fig. 4. Due to the fact that charges uniformly distributed over the surface have opposite signs and are equal in magnitude, the field outside these cylinders (excluding edge effects) will not be observed. Now we move one cylinder along the common axis by a very small distance Δx. Then an "excessive" charge of the negative sign will appear to the left of the system edge, and to the right – a positive charge equal in magnitude to negative. Thus, in accordance with the law of conservation of charge, we obtained at a great distance from each other two dissimilar charges.

The Maxwell equations in the Lorentz gauge are linear differential equations. For this reason, we can use the superposition principle to describe the appearance of potentials of charge fields. In other words, we can give a separate description of the "birth" of each of the charges and a description of the potential of each of these two charges. Below we do this for a positive charge. Potentials of a negative charge can be described in a similar way.

We will not describe the decision procedure. The potential described by Eq. (III.4.1) is equal to the sum of two potentials (Fig. 4), one of which moves from a to infinity along the radius, and the second to the center and, reflecting from the origin with phase loss by π (hard "core"), moves from the center, subtracting from the first for $r > a$ (Fig. 5). The potential φ for $r > a$ is *retarded*.

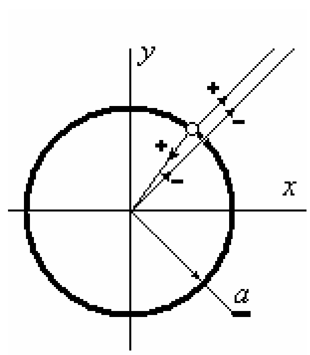


Fig.5

For a point virtual charge (for$a \to 0$), the potential has the form ($r > 0$):

$$\varphi = \frac{q}{4\pi\varepsilon r}\eta(ct - r); \ \text{where}\eta(\xi) = \begin{vmatrix} 1 & \xi \geq 0 \\ 0 & \xi < 0 \end{vmatrix}. \qquad \text{(III.4.2)}$$

Now we can relate the moment of "birth" of a charge at an infinitely remote time. The charge potential in magnitude will be constant, not depending on time. This does not mean that the potential is "static". In each consecutive infinitesimal intervals of time, thin layers of potential "bud off" from the charge and are carried one after the other to infinity, decreasing inversely proportional to the distance from the charge r^{-1}.Due to the fact that the resultant charge potential is the difference between the retarding potential that is direct and reflected from the "core", it is pointless to talk about the presence of a Poynting flow associated with the motion of potentials.

Let us return to the wave equation (III.4.1). In this equation is “inserted” the Poisson equation of the form

$$\Delta\varphi = -\frac{q}{4\pi\varepsilon a^2}\delta(r-a)\,. \tag{III.4.3}$$

It describes the instantaneously acting potential, “generated” by an inertial charge. We do not change the designation of the charge (q), although we will now consider the potential of a not virtual but an *inertial* charge. We hope that this will not lead anyone astray. It seems that equations (III.4.1) and (III.4.3) formally coincide if we assume that the charge is at rest and exists for an infinitely long time. However, the solution of equation (III.4.3), which is equal to

$$\varphi = \frac{q}{4\pi\varepsilon r} \tag{III.4.4}$$

differs from the solution (III. 4.2) by a factor which for $t \to \infty$ tends to 1.But this is not the same, since the potential boundary (III.4.2) and, consequently, the potential of the virtual charge themselves exist only in the part of space for$r < ct$ (the factor $\eta(ct-r)$), while the potential (III.4.4) of the inertial charge realizes instantaneous action at a distance and exists arbitrarily long in *the entire free space*.

The Maxwell equations in the form (III.4.3) do not describe the process of "birth" of instantaneous acting potentials when an *inertial* charge occurs. Such a potential and its source exist arbitrarily long ago and are connected by an instantaneous connection in all points of space. Recall that the signs of the energies of the potentials (III.4.2) and (III.4.4) are opposite!

It seems that there is a limiting transition from retarded potentials (III.4.1) to instantaneous potentials (III.4.3). To do this, it is sufficient to set the speed of light c to infinity in equation (III.4.1). But this is an illusion. In the wave equation, the quantity c is the parameter of the equation having the dimension of the square of the velocity!

To see this, we write down the potentials of a moving virtual charge using the Lorentz transformation. Using the Lorentz transformation or solving the equation "in the forehead," it is not difficult to show that the retarded potential φ^* of a moving virtual charge is

$$\varphi^* = \frac{q}{4\pi\varepsilon R_0\sqrt{1-(v/c)^2}}\eta(ct-R_0), \tag{III.4.5}$$

where

$$R_0 = \frac{\sqrt{(x-vt)^2+(1-v^2/c^2)(y^2+z^2)}}{\sqrt{1-(v/c)^2}}.$$

We did not abuse the strokes over independent variables. The coordinates and time refer to the observation point separated from the radiation point by the distance R_0. We pay attention to the factor η, which cannot be omitted even for a very long lifetime of the charge! This error is typical for all textbooks, without exception. The authors carefully copy each other, not bothering to check themselves and ignoring the physical content of the formulas.

Usually, the Galilean transformation is applied to the equation (III.4.5), which is not entirely correct. If we are guided by the invariance of the *wave* equation with respect to the observer's transition from one inertial frame of reference to another, then simultaneously we transform not only the wave equation, but also the Poisson equation "hidden" in it.

Uniform motion of the inertial charge creates a potential φ_0. Under the transformation, the coordinates change, and the form of the Poisson equation for this potential also changes. In the new frame of reference, it acquires the form

$$\Delta\varphi_0 - \frac{v^2\partial^2\varphi_0}{c^2\partial x^2} = -\frac{q}{4\pi\varepsilon a^2\sqrt{1-(v/c)^2}}\delta(x-vt,y,z)$$

or (III.4.6)

$$\frac{\partial^2\varphi_0}{\partial x^2}\left(1-\frac{v^2}{c^2}\right) + \frac{\partial^2\varphi_0}{\partial y^2} + \frac{\partial^2\varphi_0}{\partial z^2} = -\frac{q}{4\pi\varepsilon a^2\sqrt{1-(v/c)^2}}\,.$$

The solution of Eq. (III.4.6) is well known:

$$\varphi_0 = \frac{q}{4\pi\varepsilon R_0\sqrt{1-(v/c)^2}} = \frac{q}{4\pi\varepsilon\sqrt{(x-vt)^2+(1-v^2/c^2)(y^2+z^2)}}\,. \qquad \text{(III.4.7)}$$

A similar result will be obtained if the Lorentz transformation to expression (III.4.4) is applicable. The whole problem is that until now all textbooks state that expression (III.4.7) is a *retarded* (!) *potential*! In fact, it *resembles a retarded shape*, but is instantaneously acting at a distance potential. Once again I want to repeat: "*The authors of the textbooks diligently copy each other, not bothering to check themselves, and do not delve into the physical content of the formulas!*"

Let us now consider the influence of the Lorentz gauge on the solution of the wave equation. It can be established that the potential φ0 is instantaneous in another way. The scalar potential φ_0 of a moving charge satisfies equation

$$\Delta\varphi_0 - \frac{\partial^2\varphi_0}{\partial t^2} = -\frac{q}{4\pi\varepsilon a^2\sqrt{1-(v/c)^2}}\delta(x-vt,y,z). \qquad \text{(III.4.8)}$$

In turn, the vector potential of a *uniformly* moving charge is related to the scalar potential by a relation that can be written in the form

$$\mathbf{A}_0 = \varphi_0\mathbf{v}/c^{\mathbf{2}}. \qquad \text{(III.4.9)}$$

There is, in addition, the Lorentz gauge condition

$$\operatorname{div}\mathbf{A}_0 + \frac{1}{c^2}\frac{\partial\varphi_0}{\partial t}\,. \qquad \text{(III.4.10)}$$

If we substitute the expression for the vector potential in the Lorentz gauge condition, then these additional conditions jointly give the continuity equation for the scalar potential φ

$$\operatorname{div}\mathbf{v}\varphi_0 + \frac{\partial\varphi_0}{\partial t} = 0. \qquad \text{(III.4.11)}$$

It follows from (III.4.11) that the derivative of the potential in time (which we can regard as the initial condition for $t = 0$) cannot be given arbitrarily. For example, it cannot be zero, as in the solution of the wave equation (III.4.1). Moreover, we can, using (III.4.11), calculate the second derivative of the potential with respect to time and exclude it from the wave equation. When a point inertial charge moves along the x axis, the following expressions can be found:

$$\frac{\partial \varphi_0}{\partial t} = -\mathrm{div}\,\mathbf{v}\varphi_0 = -v\frac{\partial \varphi_0}{\partial x};$$

$$\frac{\partial^2 \varphi_0}{\partial t^2} = -\frac{\partial}{\partial t}\left(v\frac{\partial \varphi_0}{\partial x}\right) = -v\frac{\partial}{\partial x}\frac{\partial \varphi_0}{\partial t} - \frac{\partial \varphi_0}{\partial x}\frac{\partial v}{\partial t} = v^2\frac{\partial^2 \varphi_0}{\partial x^2} - \frac{\partial \varphi_0}{\partial x}\frac{\partial v}{\partial t}. \qquad \text{(III. 4.12)}$$

If the motion is uniform, then expression (III.4.12) is simplified:

$$\frac{\partial^2 \varphi_0}{\partial t^2} = -\frac{\partial}{\partial t}\left(v\frac{\partial \varphi_0}{\partial x}\right) = v^2\frac{\partial^2 \varphi_0}{\partial x^2}. \qquad \text{(III. 4.13)}$$

Taking (III.4.13) into account, it is easy to reduce the wave equation to a Poisson equation (to an equation of elliptic type):

$$\frac{\partial^2 \varphi_0}{\partial x^2}\left(1 - \frac{v^2}{c^2}\right) + \frac{\partial^2 \varphi_0}{\partial y^2} + \frac{\partial^2 \varphi_0}{\partial z^2} = -\frac{q}{4\pi\varepsilon a^2\sqrt{1-(v/c)^2}}\delta(x - vt, y, z). \qquad \text{(III. 4.14)}$$

Pay attention to the following fact. Now we *do not need to specify the initial conditions*, since the time derivative of the potential φ_0 in equation (III.4.14) is absent!Next, we make the change

$$\xi = \frac{x}{\sqrt{1-(v/c)^2}}$$

and turn expression (III.4.14) into the Poisson equation

$$\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = -\frac{q}{\varepsilon}\delta\left(\xi\sqrt{1-(v/c)^2} - vt, y, z\right). \qquad \text{(III. 4.15)}$$

The solution of this equation is the Lorentz potential, which is long-range (Poisson's equation!)

$$\varphi_L = \frac{q}{4\pi\varepsilon\sqrt{(x-vt)^2 + (1 - v^2/c^2)(y^2+z^2)}}. \qquad \text{(III. 4.16)}$$

The potential (III.4.16) satisfies the equations (III.4.1) and (III.4.14) simultaneously. As already mentioned, the potential (III.4.16) is not retarded. He describes the instantaneous action at a distance, because it is the solution of equation (III.4.15).

From this follows the possibility of removing another prejudice: the Lorentz-covariance of the equations of physics *is not a guarantee* of the absence of instantaneous action at a distance! The Lorentz transformation is an ordinary algebraic transformation. It does not turn the retard potentials into instantaneous ones and back!

IV. ANALYSIS OF THE RELATIVISTIC INTERACTION OF CHARGES

IV.1 The action integral for two charges

In the previous chapter, we showed that the mass of a charge is of an electromagnetic nature (Umov's law). This allows us to state a consistent theory of the interaction of electric charges. As is known, the action integral for a free relativistic particle is:

$$S = \int_{s_1}^{s_2} (-mc)\, ds. \qquad \text{(IV. 1.1)}$$

Substituting the expression (III.1.6) into the expression (IV.1.1), we obtain

$$S=\int_{s_1}^{s_2}\left[\iint\left(-\frac{\varrho\varphi}{2c}\right)dV\right]ds=\int_{s_1}^{s_2}\left[\iint\frac{\varrho\varphi}{2c}u_i^2\,dV\right]ds=\frac{1}{2c}\int_{s_1}^{s_2}\left(\int j_iA_idV\right)ds\,, \qquad \text{(IV.1.2)}$$

where$j_i=c\varrho u_i$ is 4-vector of current density, $A_i=\varphi u_i/c$ is 4-potential of the charge. The potential$A_i=\varphi u_i/c$ is instantaneous. The expression (IV.1.2) uses the identity $u_i^2=-1$.

Let our charge be divided into two charges ($e=e_1+e_2$). Let us investigate the case when the distance between charges is much larger than the dimensions of the charges. As an aid, we write the electric dipole moment of the charge. The integrand of expression (IV.1.2) is

$$j_iA_idsdV=A_ic\varrho u_idsdV=A_ic\varrho dx_idV=A_idP_i^edV\,. \qquad \text{(IV.1.3)}$$

The value $dP_i^e=c\varrho dx_i$is the differential of the electric dipole moment of the charge. Let us assign a numerical index for potentials of charges, for charge fields, etc. Parameters of the first charge have the index 1 or (1), the parameters of the second charge have the index 2 or (2). We now consider the expression (IV.1.3) for two charges:

$$S=\frac{1}{2c}\int_{s_1}^{s_2}\left(\int j_iA_i\,dV\right)ds=\int\frac{1}{2c}\int_{s_1}^{s_2}\left(dP_i^{e(2)}+dP_i^{e(1)}\right)\left(A_i^{(1)}+A_i^{(2)}\right)dV. \quad \text{(IV.1.4)}$$

4-potentials have the following physical meaning. $A_i^{(2)}$ is the vector potential acting on the charge e_1. This potential is generated by the charge e_2 in a place where there is a charge e_1. The potential $A_i^{(1)}$ is determined in the same way.

$$S=\int\frac{1}{2c}\int_{s_1}^{s_2}\left(dP_i^{e(1)}+dP_i^{e(2)}\right)\left(A_i^{(1)}+A_i^{(2)}\right)dV$$
$$=\int\frac{1}{2c}\int_{s_1}^{s_2}\left[-\varrho_1\varphi_1ds-\varrho_2\varphi_2ds+c\varrho_1u_i^{(1)}A_i^{(2)}dx_i^{(1)}+c\varrho_2u_i^{(2)}A_i^{(1)}dx_i^{(2)}\right]dV. \qquad \text{(IV.1.5)}$$

Now we integrate each term of the integrand over an infinite volume V, taking into account the expressions for the electromagnetic masses of the charges:

$$\int\frac{\varrho_1\varphi_1}{2c^2}dV=m_1\text{and}\int\frac{\varrho_2\varphi_2}{2c^2}dV=m_2\,. \qquad \text{(IV.1.5a)}$$

Now consider the third term in expression (IV.1.5). The dimensions of the charges are much smaller than the distance between the charges. This condition allows us to consider the potential of the first charge$A_i^{(2)}$ as a constant.

$$\frac{1}{2c}\int j_i^{(1)}A_i^{(2)}\,dV=\frac{1}{2c}\varphi_2u_i^{(1)}u_i^{(2)}\int\varrho_1\,dV=\frac{e_1}{2c}\varphi_2u_i^{(1)}u_i^{(2)}=\frac{e_1}{2}u_i^{(1)}A_i^{(2)}, \quad \text{(IV.1.6)}$$

where

$$e_1=\int\varrho_1dV\text{ and }\quad A_i^{(2)}=\frac{\varphi_2u_i^{(2)}}{c}\,. \qquad \text{(IV.1.6a)}$$

Similarly, the fourth term of expression (IV.1.5) can be integrated. So, we get the following result:

$$S = \int_{s_1}^{s_2} \left[-m_1 c ds + \frac{1}{2} e_1 u_i^{(1)} A_i^{(2)} ds + \frac{1}{2} e_2 u_i^{(2)} A_i^{(1)} ds - m_2 c ds \right]. \qquad \text{(IV. 1.7)}$$

Now we use the following expressions:$u_i^{(1)} ds = dx_i^{(1)}$ and$u_i^{(2)} ds = dx_i^{(2)}$.

$$S = \int_{s_1}^{s_2} \left[-m_1 c ds + \frac{1}{2} e_1 A_i^{(2)} dx_i^{(1)} + \frac{1}{2} e_2 A_i^{(1)} dx_i^{(2)} - m_2 c ds \right]. \qquad \text{(IV. 1.8)}$$

We can give other forms to the expression (IV.1.8), if we use identities

$$-ds = \left(u_i^{(1)}\right)^2 ds = u_i^{(1)} dx_i^{(1)} = u_i^{(2)} dx_i^{(2)} \text{and} \quad e_1 \varphi_2 = e_2 \varphi_1 . \qquad \text{(IV. 1.8a)}$$

These action integrals are convenient for obtaining equations of charge motion.

$$S = \int_{s_1}^{s_2} \left[-m_1 c ds + e_1 A_i^{(2)} dx_i^{(1)} - m_2 c ds \right], \qquad \text{(IV. 1.9)}$$

$$S = \int_{s_1}^{s_2} \left[-m_1 c ds + e_2 A_i^{(1)} dx_i^{(2)} - m_2 c ds \right]. \qquad \text{(IV. 1.10)}$$

IV.2 Equation of motion of one charge in the field of another charge

Now, from equation (IV.1.9), we can derive the equation of motion for the first charge e_1, provided that the position and velocity of the charge e_2 are "frozen" ($\delta x_i^{(2)} = 0,\ \delta u_i^{(2)} = 0$). We change the coordinates of the first charge (see [2]):

$$\delta S = \int_{s_1}^{s_2} \left[-m_1 c \delta ds + e_1 A_i^{(2)} \delta dx_i^{(1)} + e_1 \delta A_i^{(2)} dx_i^{(1)} \right]. \qquad \text{(IV. 2.1)}$$

Using [2], we can write the final expression:

$$\delta S = \int_{s_1}^{s_2} \left[-m_1 c \frac{du_i^{(1)}}{ds} + e_1 \left(\frac{\partial A_k^{(2)}}{\partial x_i^{(1)}} - \frac{\partial A_i^{(2)}}{\partial x_k^{(1)}} \right) u_k^{(1)} \right] \delta x_i^{(1)} ds = 0. \qquad \text{(IV. 2.2)}$$

From (IV.2.2), due to the arbitrariness of the variation$\delta x_i^{(1)}$, we obtain the equation of motion of the first charge:

$$m_1 c \frac{du_i^{(1)}}{ds} = e_1 \left(\frac{\partial A_k^{(2)}}{\partial x_i^{(1)}} - \frac{\partial A_i^{(2)}}{\partial x_k^{(1)}} \right) u_k^{(1)}. \qquad \text{(IV. 2.3)}$$

If we remove indices 1 and 2, we obtain the standard equation of motion, which is given in all textbooks on electrodynamics. Similarly, it is easy to obtain the equation of motion for the second particle using expression (IV.1.10). We can also obtain the equation of motion of the second particle by replacing the indices 1 by 2 and 2 by 1 in the expression (IV.2.3).

Here we see an amazing fact. Physicists use instantaneous potentials, not fully understanding this!

It is easy and convenient to show the physical meaning of the results obtained using examples of the classical interaction of charges, when v is much smaller than c.

V. THE AMBIGUITIES IN DESCRIBING THE INTERACTION OF INERTIAL CHARGES

V.1 Preliminary remarks

Attention is drawn to the *similarity* of the laws of quasi-static electrodynamics for inertial charges, obeying the Coulomb law, and the law of universal gravitation of Newton. Considering in this Chapter *electrodynamics errors*, we will keep in mind this *analogy*. Classical mechanics is built on the principle of relativity of Galileo, who argues that the fundamental laws of physics are the same in all inertial systems. In classical mechanics, the realization of this principle is obvious (for example, the law of universal gravitation, Coulomb's law, etc.). In the above laws, the interaction is determined by the relative distance between the two bodies $\mathbf{R}_{12} = \mathbf{R}_1 - \mathbf{R}_2$. The observer's transition to a new inertial system preserves the relative distance between the two bodies.

In standard textbooks, the nonrelativistic expression for charge interactions *is derived carelessly* (for example, [1], [9], etc.). The Lagrange function responsible for the interaction is written as follows [1]:

$$L_{\text{int}} = ceu_i A_i \approx -e\varphi + e\mathbf{vA}. \qquad \text{(V.I.1)}$$

This is an *incorrect* (erroneous) result. It can be *assumed* that this "inference" is, in fact, an elementary fit to obtain the Lorentz formula for two *nonrelativistic* interacting charges. To show the error, we will write the expression (V.I.1) in detail

$$L_{12} = \frac{ce_1 u_{i1} e_2 u_{i2}}{4\pi\varepsilon r_{12}} = \frac{ce_1 e_2 u_{12}}{4\pi\varepsilon r_{12}} \ , \qquad \text{(V.I.2)}$$

where$u_{12} = u_{i1}u_{i2}$ is the true scalar invariant under the Lorentz transformation:

$$u_{12} \approx -\left[1 + \frac{(\mathbf{v}_1 - \mathbf{v}_2)^2}{2c^2}\right] = -\left[1 - \frac{\mathbf{v}_1\mathbf{v}_2}{c^2} + \frac{v_1^2 + v_2^2}{2c^2}\right]. \qquad \text{(V.I.3)}$$

The terms in the brackets (right-hand side (V.I.3)), satisfy condition

$$\frac{e_1 e_2 (v_1^2 + v_2^2)}{8\pi\varepsilon c^2 r_{12}} \ll \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \ . \qquad \text{(V.I.4)}$$

It seems that we can, based on (V.I.4), neglect members in (V.I.2) to obtain (V.I.1). But this is a *mistake*. It had a negative impact on the development of physics. In addition to the incorrect mathematical result, a *epistemological* error entered the physics simultaneously. Thanks to it, work and power ceased to be invariant quantities, not only with respect to the Lorentz transformation, but also with respect to the Galilean transformation.

V.2 Interaction of two charges

We will correct errors in describing the interaction of inertial charges. Recall that we must reject the requirement of the Lorentz-covariance of the equations. Now we are not limited by this requirement in the choice of the interaction Lagrangian. The only condition is its invariance under the Galileo transformation. Such an interaction Lagrangian should depend on the *relative distance* between the charges and the *relative velocity* of their motion.

Let us write down the general form of the charge interaction Lagrangian:

$$L_{\text{int}} = -e_1 e_2 \frac{f\left(\frac{v_{12}^2}{c^2}\right)}{4\pi\varepsilon r_{12}} \ , \qquad (\text{V.2.}L)$$

where $f\left(\frac{v_{12}^2}{c^2}\right)$ can be $\sqrt{1+\left(\frac{v_{12}}{c}\right)^2}$, or $\operatorname{ch}\left(\frac{v_{12}}{c}\right)$, or $\frac{1}{\sqrt{1-\left(\frac{v_{12}}{c}\right)^2}}$ etc.

The relative velocity is $v_{12}=|\mathbf{v}_1-\mathbf{v}_2|$. The modern technique allows experimentally to establish the form of the function f for both small and large velocities.

We write the action integral:

$$S=\int\left[\frac{m_1v_1^2}{2}-\frac{e_1e_2}{4\pi\varepsilon r_{12}}f\left(\frac{v_{12}^2}{c^2}\right)+\frac{m_2v_2^2}{2}\right]dt\,. \qquad (V.2.1)$$

For the convenience of analysis and the simplicity of the calculations, we confine ourselves to small relative velocities. The proliferation of conclusions for high velocities does not present fundamental difficulties. We write down the action integral S for $v\ll c$:

$$S\approx\int\left\{\frac{m_1v_1^2}{2}-\frac{e_1e_2}{4\pi\varepsilon r_{12}}\left[1+\frac{(\mathbf{v}_1-\mathbf{v}_2)^2}{2c^2}\right]+\frac{m_2v_2^2}{2}\right\}dt\,. \qquad (V.2.2)$$

As we see, this expression corresponds to the formula (V.I.3). We can give the expression (V.2.2) the standard form:

$$S\approx\int\left(\frac{m_1v_1^2}{2}-e_1\varphi_2-\frac{1}{2}e_1\mathbf{v}_{12}\mathbf{A}_2+\frac{m_2v_2^2}{2}\right)dt\,, \qquad (V.2.3)$$

where $\mathbf{v}_{12}=\mathbf{v}_1-\mathbf{v}_2$,$\mathbf{A}_2=\varphi_2\mathbf{v}_{12}/c^2$ is the vector potential acting on the charge e_1, which is created by the charge $e_2$moving relative to the charge e_1.From the action integral (V.2.3) follows the equation of motion for the first charge, provided that the charge e_2 is “frozen” ($\mathbf{R}_2$ and $\mathbf{v}_2$ are constant):

$$m_1\frac{d\mathbf{v}_1}{dt}=-e_1\operatorname{grad}\varphi_2-e_1\frac{d\mathbf{A}_2}{2dt}-\frac{1}{2}e_1(\mathbf{v}_1-\mathbf{v}_2)\times\operatorname{rot}\mathbf{A}_2\,. \qquad (V.2.4)$$

The equation of motion of the second charge can be obtained in the same way:

$$m_2\frac{d\mathbf{v}_2}{dt}=-e_2\operatorname{grad}\varphi_1-e_2\frac{d\mathbf{A}_1}{2dt}-\frac{1}{2}e_2(\mathbf{v}_2-\mathbf{v}_1)\times\operatorname{rot}\mathbf{A}_1\,, \qquad (V.2.5)$$

where $\mathbf{A}_1=\varphi_1(\mathbf{v}_2-\mathbf{v}_1)/c^2$ .It seems that the factor 1/2 in expression (V.2.5) contradicts the modern point of view. However, no direct experiments were performed to directly verify the interaction of two charges at low velocities $v\ll c$. We consider below the interaction of a charge with a current and show the correctness of our calculations.

Expressions (V.2.4) and (V.2.5) are invariant under the Galileo transformation. The interaction of charges does not depend on the choice of the inertial system by the observer. The third Newton principle (the action is equal to the counteraction) is always fulfilled: $\mathbf{F}_{12}=-\mathbf{F}_{21}$.

V.3 Interaction of a charge and a conductor with a current

We will assume that the individual positive and negative charges of the conductor are much larger than the free charge q, and we will not take into account its effect on the conductor. At the point where the charge q moves, the positive charge of the conductor creates a potential φ_1, and the negative potential φ_2 as shown in Fig. 6. The conductor is quasineutral, i.e. the total scalar potential outside the conductor is equal to zero $\varphi_1+\varphi_2=0$.

We write the Lagrange function, taking into account that it is equal to the sum of Lagrangians of charge interaction with positive and negative charges of the conductor.

$$L = \frac{mv^2}{2} - q\varphi_1\left[1 + \frac{(\mathbf{v}_1 - \mathbf{v}\ \ )^2}{2c^2}\right] - q\varphi_2\left[1 + \frac{(\mathbf{v}_2 - \mathbf{v}\ \ )^2}{2c^2}\right]. \qquad (V.3.1)$$

Taking into account the quasineutrality of the conductor, we can write the Lagrange function in the following form:

$$L = \frac{mv^2}{2} - q\varphi_1\frac{(\mathbf{v}_1 - \mathbf{v}_2)\left(\mathbf{v} - \frac{(\mathbf{v}_1 + \mathbf{v}_2)}{2}\right)}{c^2}\ . \qquad (V.3.2)$$

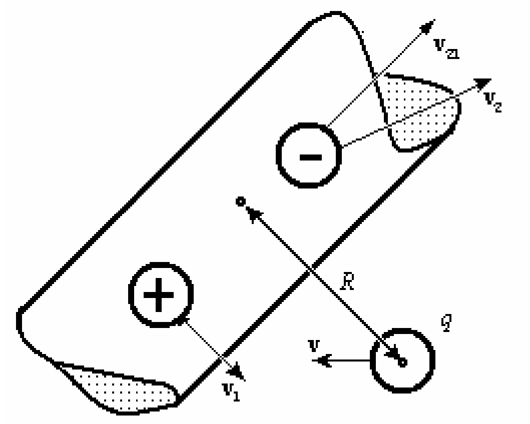


Fig. 6Notation: $\mathbf{v}_1$ is speed of positive charges of a conductor (ions); $\mathbf{v}_2$ is the velocity of conduction electrons of the conductor; v is the free charge, $\mathbf{v}_{21} = \mathbf{v}_2 - \mathbf{v}_1$ is the average velocity of the conduction electrons in the conductor.

The expression (V.3.2) can be given the standard form after the introduction of the following notation:

- $\mathbf{A} \ = \varphi_1(\mathbf{v}_1 - \mathbf{v}_2)/c^2$ is a vector potential of the conductor at the charge point q,
- $\mathbf{v}_0 = (\mathbf{v}_1 + \mathbf{v}_2)/2$ is a base frame velocity,
- $\mathbf{v}_r = \mathbf{v} \ \ - \mathbf{v}_0$ is a velocity of charge q in the base system.

The basic frame of reference is an inertial system in which the positive charges (ions) of the conductor and the conduction electrons *move at opposite speeds in equal directions*. This is a kind of "center of inertia" of conductor charges. The expression (V.3.2) can now be written in "standard" form:

$$L = \frac{mv^2}{2} + q\mathbf{v}_r\mathbf{A}\,. \qquad (V.3.2a)$$

It is known that the average velocity of conduction electrons in a conductor $(\mathbf{v}_1 - \mathbf{v}_2)$ is very small. Therefore, the potential φ_1 is practically a function $(\mathbf{R} - \mathbf{v}_0 t) \approx (\mathbf{R} - \mathbf{v}_1 t)$. In other words, we can assume that the reference frame of the conductor is connected with the conductor itself. Taking this into account, we can write the equation of charge motion, on the right-hand side of which there is a Lorentz force

$$m\ \frac{d\mathbf{v}}{dt} = -q\frac{\partial\mathbf{A}}{\partial t} + q\mathbf{v}_r \times \mathrm{rot}\mathbf{A}\ \ . \qquad (V.3.3)$$

Expression (V.3.3) can be written in another form:

$$m\ \frac{d\mathbf{v}}{dt}=q\mathbf{E}^{'}-q\frac{\partial\mathbf{A}}{\partial t}+q\mathbf{v}\times\mathrm{rot}\mathbf{A}\ \ , \qquad (\mathrm{V}.3.3\mathrm{a})$$

where$\mathbf{E}^{'}=-\mathbf{v}_0\times\mathrm{rot}\mathbf{A}\ \ =-\mathbf{v}_0\times\mathbf{B}$.

This is a known result of the conversion of a magnetic field using the Lorentz transformation applied to a charge moving in the reference frame. In the base system, as it was said, the positive and negative charges of the conductor have the same speed, but move in opposite directions. The charge reference system moves relative to the base system with a velocity $\mathbf{v}_0$ [7], [9].

Let us return to the expression (V.3.3) and replace in it the vector potential by the scalar

$$m\ \frac{d\mathbf{v}}{dt}=q\ \mathrm{grad}\ \varphi_1\frac{(\mathbf{v}_1-\mathbf{v}_2)\mathbf{v}_r}{c^2}\ . \qquad (\mathrm{V}.3.4)$$

It follows from (V.3.4) that the charges do not interact with the conductor if:

1) $\mathbf{v}_1-\mathbf{v}_2=0$: trivial case of absence of current in the conductor;

2) $\mathbf{v}\ \ -\mathbf{v}_0=0$: the charge rests in the base reference system of the conductor;

3) $(\mathbf{v}_1-\mathbf{v}_2)\mathbf{v}_r=0$, but $\mathbf{v}_1-\mathbf{v}_2\neq 0$ and $\mathbf{v}_r\neq 0$,The point charge moves in the base system perpendicular to the conductor. If we were to consider a spherical extended charge, then with this motion, it would begin to rotate.

Usually the average velocity of conduction electrons is small. Therefore, it is possible to assume approximately that the basic reference frame of a conductor is connected with a conductor. If the moving charge crosses the lines of force of the magnetic field, then the force acts on the charge. If the charge is at rest in the base system, then there are no magnetic forces. We can extend the concept of the "reference frame" to closed circuits with current, electromagnetic and magnets. A magnet, an electromagnet, etc. there are basic reference frames.

V.4 Work and force. Errors of interpretation

Let us now find out the content of the concepts “force” and “work”. Incorrect interpretation of these concepts gives rise to paradoxes and errors in explaining the phenomena of electrodynamics. Especially many such errors in the explanation of magnetic phenomena. We will discuss them in the next chapter (Chapter VI).

The concept of “force” can be given in classical mechanics the following definition: *Force* is the *property* of a material object (the source of a given property), which manifests itself in the interaction of material objects and leads to a change in the state of the interacting objects (momentum, trajectory, etc.)

Note that the *force* is the *property* of the object, and not a certain material object. “Naked” force, i.e. forces without a source (as properties without an object) do not happen. Power always has its source. The sources of forces can be a variety of material objects: a charge with its own field, an electromagnetic wave that carries its own property - a force characteristic, i.e. tension of its field, etc. Force manifests itself only in *interaction*, i.e. in mutual action. The mutuality of action in classical mechanics is reflected by Newton's third principle. For the manifestation of strength, at least two objects are needed that must interact.

Work is the second side (energy characteristic) of interaction. We give the following definition: Work is an objective quantitative characteristic of a qualitative change in the motion of matter that characterizes the energy side of interaction. *Work* is an objective concept. The work is determined in mechanics by the *relative* motion of material objects and does not depend on the position of the observer. This property determines the *invariance of the work* with respect to the Galilean transformation; independence of work from volitional choice by the observer of the inertial frame of reference. Below we will consider examples to explain the characteristic errors of interpretation.

Example 1. Consider two interacting bodies. The equations of motion of these bodies have the form:

$$m_1 \frac{d\mathbf{v}_1}{dt} = -\mathbf{F}_{12} \text{and} m_2 \frac{d\mathbf{v}_2}{dt} = -\mathbf{F}_{21} . \quad (V.4.1)$$

Let us calculate the differential of the work.

$$dA = \mathbf{F}_{12}(\mathbf{v}_1 - \mathbf{v}_2)dt = \mathbf{F}_{12}d\mathbf{R}_{12} = \mathbf{F}_{21}d\mathbf{R}_{21} . \quad (V.4.2)$$

The work done by each particle is

$$dA_1 = \frac{m_2}{m_1 + m_2}\mathbf{F}_{12}d\mathbf{R}_{12} \text{and} dA_2 = \frac{m_1}{m_1 + m_2}\mathbf{F}_{12}d\mathbf{R}_{12}. \quad (V.4.3)$$

Expressions (V.4.3) do not depend on the choice of the inertial frame of reference. Often in textbooks you can find the following expression for the work done by bodies:

$$d\widetilde{A}_1 = \mathbf{F}_{12}\mathbf{v}_1 dt \text{ and } d\widetilde{A}_1 = \mathbf{F}_{12}\mathbf{v}_1 dt . \quad (V.4.4)$$

Expression (V.4.4) can be regarded as a standard *epistemological* error. Power is always a property of the interacting body. This property is mistakenly detached from the particle and transformed into some kind of independent substance that rests in the observer's reference frame. As a result of this approach, “*work*” appears, which depends on the subjective choice of the inertial frame of reference (virtual work) by the observer. It can not be regarded as a *real* work [7].

Example 2. In textbooks, one can read that

$$dA = d\frac{m^2}{2} = e\mathbf{vE}dt . \quad (V.4.5)$$

We must limit the use of this expression to the following condition. Expression (V.4.5) is valid only if the source of the field **E***is at rest* in the observer's reference frame. In the general case, expression (V.4.5) is incorrect, since it does not take into account the motion of the source of the electric field.

Example 3. Another example of epistemological error. A description of the so-called "convection" potential is given in the textbook [7]. Let the charges rest. The Coulomb forces of two fixed charges are balanced by mechanical forces. An observer who moves past charges with a constant speed, it will seem that the moment of forces acts on the charges.

Briefly expound the explanation given in [7] (pp. 348-349):

Two electrons moving parallel to each other at the same speed **u** interact with each other. The strength of the interaction is determined by the expression for the Lorentz force

$$\mathbf{F} = e(\mathbf{E} + \mathbf{u} \times \mathbf{B}), \quad (V.4.6)$$

which, by expansion of the vector product, becomes

$$\mathbf{F} = -\frac{e^2}{4\pi\varepsilon}\nabla\left(\frac{1 - u^2/c^2}{s}\right) = -\nabla\psi, \quad (V.4.7)$$

where ψ is called the convection potential.We draw your attention to the fact that the convective potential is instantaneous, not delayed. Further we quote [7] (p. 349): “… we would conclude that the force $\mathbf{F}_2$exerted by the electron e_1 at (x_1, y_1, z_1) on the electron e_2 at (x_2, y_2, z_2) is perpendicular to the ellipsoid

$$s = \sqrt{(x_1 - x_2)^2 + (1 - u^2/c^2)[(y_1 - y_2)^2 + (z_1 - z_2)^2]} = \text{constant}, \quad (V.4.8)$$

as shown in Fig. 8 (Fig. 19-3 in [7]).” However, we will continue quoting [7]: “On the other hand, the reaction force$\mathbf{F}_1$on the electron e_1 is perpendicular to the corresponding ellipsoid (shown by the dashed line in the figure) referred to the co-moving electron e_2. Hence, except when the line between the electrons is parallel or perpendicular to the direction of motion, the forces of action and reaction do not appear be collinear. Therefore if the two electrons were connected by a rigid bar there would be a couple acting about an axis perpendicular to the plane of the line joining the electrons and the direction of motion. This will be recognized as the torque also predicted by Ampère’s law when current elements are substituted for the moving charges, and which Trouton and Noble attempted to measure. The paradox produced by the observation of a null effect indicates the difficulties in interpreting the velocity of moving charges in pre-relativistic electrodynamics.

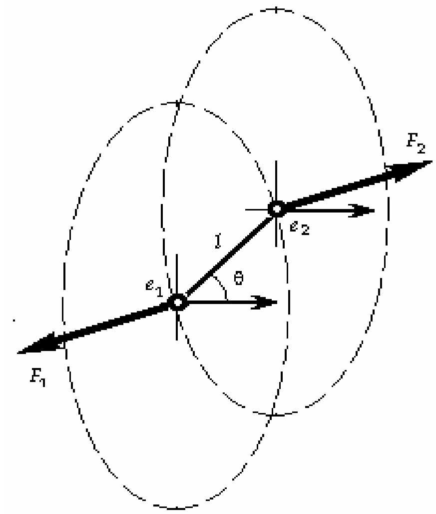


Fig. 7

The torque predicted here is real enough to an observer moving with a velocity **u** relative to the two charges, and should in that case be measurable if there were no mechanical considerations involved. We have already noted that the assumption of a “rigid” bar is not consistent with the theory of relativity… The problem as a whole is similar to that in which the torque is balanced by the gain in angular momentum: in every case equilibrium is a property invariant under a Lorentz transformation.”

It is clear that here we are not dealing with an *explanation* of the physical phenomenon predicted by SRT, but with a *declaration* about the “invariance” of equilibrium in any inertial frame of reference (“in every case equilibrium is a property invariant under a Lorentz transformation”[7])

Let us, however, try to find out the cause of the paradox. We will not now consider the formalism of relativistic formulas. We will be interested only in the epistemological (epistemological) error of explanation. The error is due to the existing definition of the field strength.

In numerous textbooks one can read (see, for example, [11]):“The electric field intensity is defined as the force per unit positive charge that would be experienced by a stationary point charge, or *test charge*, at a given location in the field:$\mathbf{E} = \mathbf{F}/q_t$.”A similar definition was adopted in [12]:“The electric field strength at a given point in space is numerically equal to the force acting on a single positive point charge (test charge)”.However, the definitions given are incomplete and, therefore, they are *erroneous*. The test charge must rest at the point $P(x, y, z)$, where the field (the observer's reference frame) is measured. *This is very important*! If the charge moves through the point $P(x, y, z)$ at a velocity$\mathbf{u}$, then another force $\mathbf{F}^{'} \neq \mathbf{F}$ will act on the charge.For this reason, the explanation in [7] contains an error. The author attributes the force to the *moving* charge, which would act on a *stationary* test charge. As a consequence, we calculate the torque that does not exist in reality. We recall that the force is invariant under the Galileo transformation. Let us give *our* definition of the electric field strength **E**, which is suitable for relativistic and classical variants.

Definition. *The electric field strength* (at a given point in space and at a given instant of time) is *the force characteristic* of this field, numerically equal to the force acting on a unit positive charge (ie, on a *test charge*), *at rest* at this point, and having a direction, coinciding with the direction of the force vector.

This definition is correct: First, the philosophical side of the definition of "force characteristics" allows us not to perceive tension as an independent kind of matter. It reflects one of the *properties* of a *phenomenon* such as an electromagnetic field. We note that the "*energy characteristic*" of the electric field is the potential (including convectional), since it is defined through the concept of "*work*". Force is one of the properties of a wave or material body. Without the introduction of such refinements, confusion is possible. For example, some researchers mistakenly try to consider force as an *independent* "material object", existing as it were, regardless of the source that creates this force. The charges interact, and the forces arising between them are the *properties* of the charges (the sources of these forces).Secondly, we want to draw attention to the appearance in the definition of the concept of "strength" of the word "*at rest*". The point is that at a given moment of time at a given point in space we can "place" a *moving* single charge in the field under investigation. Naturally, *another* force will act on it from the field side (*another* field strength will be measured), different from the one that would act on the charge *at rest*.

Let us give an example. Let us have a homogeneous magnetic field of a magnet, which *is at rest* in our reference frame. If the test charge is *at rest*, then the magnetic field will not act on it. The intensity of the electric field acting on the test unit charge is zero. But if the charge moves with a speed **v** relative to the magnet, then, according to the Lorentz formula, the force will act on it and, the electric field intensity proportional to it will exist

$$\mathbf{E} = \mathbf{F}/e = \mathbf{v} \times \mathbf{B}. \qquad (\mathrm{V}.4.9)$$

Let us now consider the case when this magnet with its field moves with a constant velocity **u** in our frame of reference. Sometimes it is possible to find assertions that in this case also the magnetic field will not influence the resting charge. At the same time supporters of this point of view "nod" to the above Lorentz formula. Indeed, if the velocity of the charge is zero, then the force (the strength of the electric field) must be zero. *But this is an erroneous point of view*. A moving magnetic field generates an electric field strength equal to

$$\mathbf{E}' = -\mathbf{u} \times \mathbf{B}\,. \qquad (\mathrm{V}.4.10)$$

This tension creates a force that will affect the test charge *at rest* in our inertial reference frame. Under its influence, the free charge starts to move at an accelerated rate, i.e. to do work. Now, relying on the definition of the electric field strength, we can give a consistent explanation for the "convection potential."

So, let us turn to Fig. 8 and consider the field strength created by the first charge e_1, which exists at the point in space where the moving charge e_2 is at the moment. For this purpose (in accordance with the definition of the concept of "electric field strength"), we place a *fixed* test charge at a given point in space at the time corresponding to the flight of the second charge.

Naturally, this *resting* charge will be acted upon by a force determined by the Lorentz formula. But will the same force act on a *moving* charge? The answer to this question should generally be *negative*. The moving charge will be acted upon by a *different* force than the one we measured with a stationary test charge.

But let us return to the paradox in question. What do we have? And we have *a substitution* of forces, if we speak from the point of view of physics. We *illegally substitute* the force that acts on the moving charge, the other by the force that acts on the charge that is *stationary* in our reference frame. If we calculated the forces calculated for a fixed charge by *real* forces, then we would not find any paradox related to the appearance of the torque.

Interaction in modern mechanics has an *objective* character, as is the case in Newtonian mechanics. It cannot depend on the choice of an inertial frame of reference by the observer.

V.5 Interaction of two conductors

Consider the interaction of two conductors with currents. We can represent the conductor in the form of an ionic lattice of positive charges and conduction electrons. Let the first conductor (i.e. its positive ion array) move with velocity$\mathbf{v}_1$, and the second conductor move with velocity $\mathbf{v}_3$, as shown in Fig. 8.

The Lagrange function is determined by the sum of the pair interactions of positive and negative charges of two conductors. We select the volume dV in the second conductor. In this conductor, ϱ_3 and ϱ_4 are the densities of positive and negative charges, respectively. Let in this volume positive charges of the first conductor create a potentialφ_1, and negative ones φ_2.

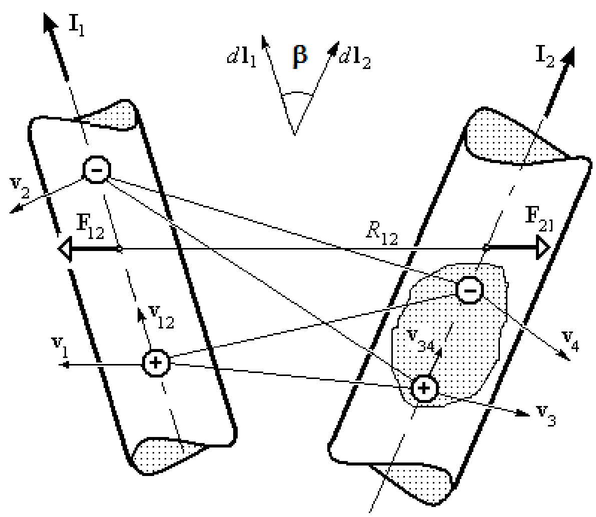


Fig. 8 The notation in the figure is as follows: $\mathbf{v}_1$ isthe velocity of positive charges of conductor 1; $\mathbf{v}_2$ is the average velocity of negative charges of conductor 1; $\mathbf{v}_3$ isthe velocity of positive charges of conductor 2; $\mathbf{v}_4$ is the average speed of negative charges of conductor 2; $\mathbf{v}_{21} = \mathbf{v}_2 - \mathbf{v}_1$ is the average velocity of negative charges in conductor 1 relative to positive ones; $\mathbf{v}_{43} = \mathbf{v}_4 - \mathbf{v}_3$ is the average velocity of negative charges in conductor 2 relative to positive.

We regard both conductors as quasineutral systems: $\varrho_3 + \varrho_4 = 0$; $\varphi_1 + \varphi_2 = 0$.The density of the interaction Lagrangian is

$$\Lambda = -\frac{\varphi_1\varrho_3}{c^2}\left(1+\frac{(\mathbf{v}_1-\mathbf{v}_3)^2}{2c^2}\right)-\frac{\varphi_1\varrho_4}{c^2}\left(1+\frac{(\mathbf{v}_1-\mathbf{v}_4)^2}{2c^2}\right)$$
$$-\frac{\varphi_2\varrho_3}{c^2}\left(1+\frac{(\mathbf{v}_2-\mathbf{v}_3)^2}{2c^2}\right)-\frac{\varphi_2\varrho_4}{c^2}\left(1+\frac{(\mathbf{v}_2-\mathbf{v}_4)^2}{2c^2}\right)=\frac{\varphi_1\mathbf{v}_{12}}{c^2}\varrho_3\mathbf{v}_{34}=\mathbf{jA}\,, \qquad (\mathrm{V}.5.1)$$

where $\mathbf{j}=\varrho_3\mathbf{v}_{34}$ is the current density in conductor 2,$\mathbf{A}=\varphi_1\mathbf{v}_{12}/c^2=\varphi_2\mathbf{v}_{21}/c^2$is the vector potential created by the conductor 1 in the volume dV of the conductor 2.We see that the density of the Lagrange function coincides with the known function. However, this is only an outward resemblance. The form of the function (V.5.1) is a consequence of the complete compensation of the Coulomb potentials in quasineutral systems. This is not a relativistic effect. The expression (V.5.1) is *invariant under the Galileo transformation*.

To obtain the Lagrange function, it is necessary (V.5.1) to integrate over the entire volume containing the conductors:

$$L=\int \mathbf{jA}\,dV. \qquad (\mathrm{V}.5.2)$$

Let the elements of the length of the conductors dl_1 and dl_2 and the dimensions of their cross sections s_1 and s_2 be small in comparison with the distance R_{12} between these conductors. Then we can write the vector potential of the first conductor in a known form:

$$\mathbf{A}=\frac{\mu}{4\pi}\frac{I_1 d\mathbf{l}_1}{R_{12}}, \qquad (\mathrm{V}.5.3)$$

whereI_1 is the current flowing through the cross section of the first conductor,$I_1=\int \varrho_2\mathbf{v}_{21}\,d\mathbf{s}_1$.We substitute the expression (V.5.3) into the formula (V.5.2).

$$L=\int \frac{\mu}{4\pi}\mathbf{j}\frac{I_1 d\mathbf{l}_1}{R_{12}}dV\,. \qquad (\mathrm{V}.5.4)$$

The volume $dV$is small. The vector potential $\mathbf{A}$ can be considered constant in this volume. Taking this into account, expression (V.5.4) takes the final form

$$L=\frac{\mu}{4\pi}\mathbf{j}\frac{I_1 d\mathbf{l}_1}{R_{12}}\int \mathbf{j}\,dV=\frac{\mu}{4\pi}\frac{(I_1 d\mathbf{l}_1\cdot I_2 d\mathbf{l}_2)}{R_{12}}\,, \qquad (\mathrm{V}.5.5)$$

where $I_2=\int \varrho_4\mathbf{v}_{43}\,d\mathbf{s}_2$.Note that expression (V.5.5) is invariant under the Galileo transformation. Now, based on (V.5.5), we can consider the interaction of two infinitesimal conductors with currents, i.e. interaction of two elementary currents.

In order to clarify the features of the interaction of elementary currents, we write down the action integral, relying on (V.5.5):

$$S = \int L\,dt = \int \frac{\mu}{4\pi}\frac{(I_1 d\mathbf{l}_1 \cdot I_2 d\mathbf{l}_2)}{R_{12}} dt \ . \qquad (V.5.6)$$

We can vary only two values: $\mathbf{R}_{12}$vector distance between the two conductors and $\boldsymbol{\varphi}(\mathbf{l}_1, \mathbf{l}_2)$ the angle of mutual orientation of the current element.We will vary $\mathbf{R}_{12}$ at a constant angle $\boldsymbol{\varphi}(\mathbf{l}_1, \mathbf{l}_2)$:

$$\begin{aligned} \delta S = \delta \int \frac{\mu}{4\pi}\frac{(I_1 d\mathbf{l}_1 \cdot I_2 d\mathbf{l}_2)}{R_{12}} dt = \frac{\mu}{4\pi}\int (I_1 d\mathbf{l}_1 \cdot I_2 d\mathbf{l}_2)\,\delta\left(\frac{1}{R_{12}}\right) dt \\ = \int \frac{\mu}{4\pi}\frac{(I_1 d\mathbf{l}_1 \cdot I_2 d\mathbf{l}_2)}{R_{12}^3}\mathbf{R}_{12}\delta\mathbf{R}_{12} dt = \int \mathbf{F}_{12}\,\delta\mathbf{R}_{12} dt = 0. \end{aligned} \qquad (V.5.6a)$$

It follows from this that

$$\mathbf{F}_{12} = \frac{\mu}{4\pi}\frac{(I_1 d\mathbf{l}_1 \cdot I_2 d\mathbf{l}_2)}{R_{12}^3}\mathbf{R}_{12} = -\,\mathbf{F}_{21}\ . \qquad (V.5.7)$$

As we see, Newton's third law is fulfilled.Now we will vary the angle of mutual orientation of the current elements $\boldsymbol{\beta}(\mathbf{l}_1, \mathbf{l}_2)$ with the unchanged distance $\mathbf{R}_{12}$:

$$\begin{aligned} \delta S = \int \delta \frac{\mu}{4\pi}\frac{(I_1 d\mathbf{l}_1 \cdot I_2 d\mathbf{l}_2)}{R_{12}} dt = \int \frac{\mu I_1 I_2}{4\pi R_{12}}([d\mathbf{l}_1 \times \delta\boldsymbol{\beta}(\mathbf{l}_1, \mathbf{l}_2)])dt \\ = -\int \frac{\mu I_1 I_2}{4\pi}\big([d\mathbf{l}_1 \times d\mathbf{l}_2]\delta\boldsymbol{\beta}(\mathbf{l}_1, \mathbf{l}_2)\big)dt \\ = \int \mathbf{M}_{21}\,\delta\boldsymbol{\beta}(\mathbf{l}_1, \mathbf{l}_2)dt = -\int \mathbf{M}_{12}\,\delta\boldsymbol{\beta}(\mathbf{l}_1, \mathbf{l}_2)dt = 0. \end{aligned} \qquad (V.5.6b)$$

It follows from this that

$$\mathbf{M}_{21} = \frac{\mu}{4\pi}\frac{(I_1 d\mathbf{l}_1 \times I_2 d\mathbf{l}_2)}{R_{12}} = -\mathbf{M}_{12}. \qquad (V.5.8)$$

The results (V.5.7) and (V.5.8) *completely describe the phenomena* associated with the interaction of two elementary currents. The third law of Newton is not violated. The correctness of the derivation can be confirmed by using the expression for the Lorentz force in the absence of electrostatic Coulomb forces (q are charges in the corresponding conductors in Fig. 9):

$$\mathbf{F}_{12} = -q_2\frac{\partial \mathbf{A}_1}{\partial t} + q_2\mathbf{v}_2 \times \mathrm{rot}\mathbf{A}_1\ . \qquad (V.5.9)$$

Let us calculate the values:

$$\mathbf{A}_1 = \frac{\varphi_1 \mathbf{v}_1}{c^2} = \mu \frac{q_1 \mathbf{v}_1}{4\pi R_{12}}; \quad q_2 \frac{\partial \mathbf{A}_1}{\partial t} = q_2 \frac{\mathbf{v}_1}{c^2} \frac{\partial \varphi_1}{\partial t} = -\frac{\mu}{4\pi R_{12}^3} q_1 \mathbf{v}_1 (q_2 \mathbf{v}_2 \mathbf{R}_{21});$$
$$q_2 \mathbf{v}_2 \times \mathrm{rot} \mathbf{A}_1 = -\frac{\mu}{4\pi R_{12}^3} q_2 \mathbf{v}_2 \times (\mathbf{R}_{21} \times q_1 \mathbf{v}_1). \quad (\mathrm{V.5.9a})$$

Substituting these expressions into Eq. (V.5.9), we obtain

$$\mathbf{F}_{12} = \frac{\mu \mathbf{R}_{21} (q_1 \mathbf{v}_1 \cdot q_2 \mathbf{v}_2)}{4\pi R_{12}^3} = -\mathbf{F}_{21}. \quad (\mathrm{V.5.10})$$

In its form, the expression obtained corresponds to the expression (V.5.7).Indeed, if $q_1 \mathbf{v}_1$ corresponds to $I_1 d\mathbf{l}_1$, and $q_2 \mathbf{v}_2$ corresponds to $I_2 d\mathbf{l}_2$, then we come to the expression (V.5.7), which was to be shown.

VI. PROBLEMS IN EXPLAINING MAGNETIC PHENOMENA

VI.1 Unipolar induction

Conventional classical electrodynamics could never give a correct explanation to the phenomenon of the unipolar induction (see, for example, [10]). The unipolar generator based on this phenomenon first was developed by Michael Faraday in 1831 as one of his explorations of the phenomena of induced EMF. Here we give a new explanation to this phenomenon in the framework of Newton's classical mechanics. A qualitative explanation does not present fundamental difficulties. However, a quantitative example, as a rule, involves cumbersome calculations, for which its clarity is lost. This is the first reason that made us look for the simplest models for analysis. The second reason was to find the most universal model on which we could explore different models of unipolar generators. The model of a unipolar generator is shown in Fig. 9. The device contains a current ring equivalent to a magnet, and a conductive disk with a sliding contact. The ring and disk can rotate independently of each other with different angular velocities. Such a device is universal and allows modeling unipolar generators of different types. For example, if the disk and the current ring rotate at the same angular velocity, we have a unipolar generator with a rotating magnet. If the current ring is stationary, but the disk rotates, then we are dealing with another type of unipolar generator.

Let us consider the operation of a unipolar generator in the general case. We assume that $h \ll a$ (see Fig. 10). In other words, the rotating disk, the current ring and the AVC chain lie in the same plane $z = 0$.

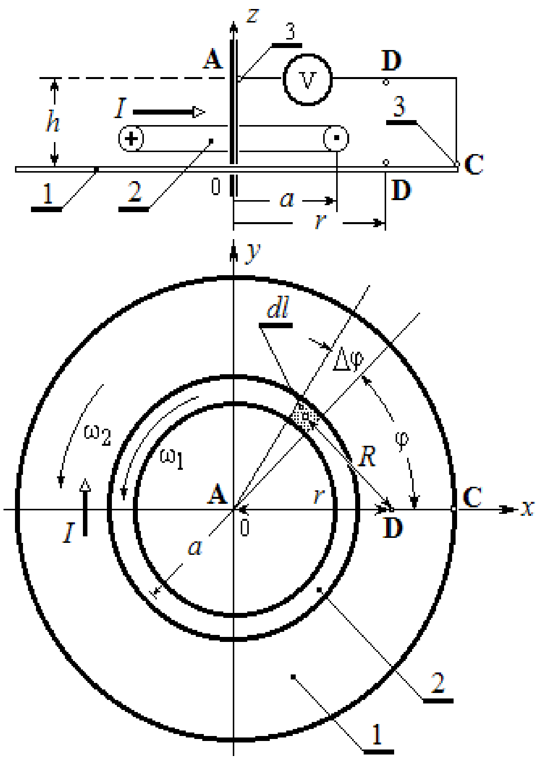


Fig. 9. 1 is conducting disk; 2 is a ring with a current; 3 is sliding contact.

Let us make some preliminary remarks. The EMF of the induction is generated by the current ring in two parts of the closed circuit AVCOA. In the first fixed part of the ABC circuit, the EMF of induction U_1 is excited. If the ring with the current is stationary, the EMF $U_1 = 0$. The second section, where the induction EMF occurs, is the OC section on the disk. Here, the EMF of U_2 is induced. The total EMF in the AVCOA circuit is

$$U = U_1 - U_2 \,. \tag{VI. 1.1}$$

When $\omega_1 = 0$, the whole chain of AVCOA is at rest and the total EMF is zero, $U = 0$.

The procedure for calculating EMF U is simple. We will calculate the total field strength at some point D on the x axis. The value of U is obtained as a result of integration of the total field strength. We single out the element dl on the ring with current. It can be regarded as an element of a current that moves with velocity $\mathbf{v}_0$.

1. Let the point D of the fixed chain AVC be located at a distance R from the z axis. It is easy to see that the field strength at the point D is[1]

$$d\mathbf{E}_1 = -grad(d\phi) = -\frac{(\mathbf{v}_{12}\mathbf{v}_0)}{c^2}\frac{dq_1}{d\varphi}\frac{\mathbf{R}}{4\pi\varepsilon R^3}d\varphi \,, \tag{VI. 1.2}$$

where q_1 is the total positive charge of the rotating ring with current, R is the distance between dl and the point D, $\mathbf{v}_0$ is the speed of the reference frame of the element with the current dl $(v_0, v_{12} \ll c$,

$$R = \sqrt{r^2 + a^2 - 2ar\cos\varphi} \,. \tag{VI. 1.3}$$

[1]In this chapter, we denote by the symbol φ the angle of the cylindrical coordinate system, and denote the scalar potential by the symbol ϕ.

2. We now consider the point D on a rotating disc. The velocity of the displacement of point D is:

$$v = \omega_2 r\,. \tag{VI. 1.4}$$

The field strength at this point D is

$$d\mathbf{E}_2 = -\frac{[\mathbf{v}_{12}(\mathbf{v}_0 - \mathbf{v})]}{c^2}\frac{dq_1}{d\varphi}\frac{\mathbf{R}}{4\pi\varepsilon R^3}d\varphi\,. \tag{VI. 1.5}$$

Let us consider the physical meaning of the equation (VI.1.5). Obviously, the field strength can be represented as the sum of the strengths:

$$d\mathbf{E}_2 = d\mathbf{E}_2^{'} + d\mathbf{E}_2^{''}, \tag{VI. 1.6}$$

where

$$d\mathbf{E}_2^{'} = -\frac{(\mathbf{v}_{12}\mathbf{v}_0)}{c^2}\frac{dq_1}{d\varphi}\frac{\mathbf{R}}{4\pi\varepsilon R^3}d\varphi \tag{VI. 1.6a}$$

is the field strength, which is excited under the condition that the ring with the current rotates, and the conducting disk is stationary, and

$$d\mathbf{E}_2^{''} = \frac{(\mathbf{v}_{12}\mathbf{v})}{c^2}\frac{dq_1}{d\varphi}\frac{\mathbf{R}}{4\pi\varepsilon R^3}d\varphi \tag{VI. 1.6b}$$

is the field strength, which is excited under the condition that the conducting disk rotates, and now the ring with the current is fixed.

3. The total field strength is equal to the difference in the field strengths:

$$d\mathbf{E}_2 = d\mathbf{E}_1 - d\mathbf{E}_2\,. \tag{VI. 1.7}$$

It is easy to see that the components dE_1 and $dE_2^{'}$ cancel each other, and we obtain the following components of the general field strength $d\mathbf{E}$:

$$dE_r = -\mu\frac{(\mathbf{v}_{12}\mathbf{v})}{c^2}\cos\varphi\frac{dq_1}{d\varphi}\frac{(a - r\cos\varphi)}{R^3}d\varphi\,, \tag{VI. 1.8}$$

$$dE_\varphi = -\mu\frac{(\mathbf{v}_{12}\mathbf{v})}{c^2}\cos\varphi\frac{dq_1}{d\varphi}\frac{a\sin\varphi}{R^3}d\varphi. \tag{VI. 1.9}$$

The total field strength created by the entire current ring is calculated by integrating these expressions in the range from 0 to 2π. It is obvious that in the total field strength $\mathbf{E}$ only the radial component is conserved, because of the parity of dE_r and the oddity of dE_φ.

$$dE_r = -\mu\frac{Iar\omega_2}{4\pi}\int_0^{2\pi}\frac{\cos\varphi\,(a - r\cos\varphi)}{R^3}d\varphi\,; \quad E_\varphi = 0, \tag{VI. 1.10}$$

where $I = v_{12}\dfrac{dq_1}{dl}\,, \qquad dl = ad\varphi\,.$

Now, integrating E_r over r, we calculate the EMF of induction U.

$$U = \int_0^C E_r dr = -\int_0^C \mu\frac{Iar\omega_2}{4\pi}\left[\int_0^{2\pi}\frac{\cos\varphi\,(a - r\cos\varphi)}{R^3}d\varphi\right]dr. \tag{VI. 1.11}$$

It can be seen from the formula that this EMF does not depend on the angular velocity ω_1.

4. Now we show that the EMF (VI.1.11) can be calculated in another way, for example, using the Faraday law.

$$U = -\frac{d\Phi}{dt}\,. \qquad \text{(VI. 1.12)}$$

Consider points C and C*, which are located, as shown in Fig. 10. Point C is located on the fixed sliding contact, and C* on the rotating disk.At the initial time t the coordinates of these points are equal. At the next instant $t + \Delta t$ the point C* will move and take the position C**. The total flow Φ that flows through the ACC** sector is equal to

$$\Phi = \int_0^{\varphi(t)} \left[\int_0^C rB(r)\,dr\right] d\varphi\ . \qquad \text{(VI. 1.13)}$$

This flux Φ does not depend on the angular velocity ω_1. Using the expression (VI.1.12), we find the EMF U:

$$U = -\frac{d\Phi}{dt} = -\omega_2 \int_0^C rB(r)\,dr\,, \qquad \text{(VI. 1.14)}$$

where$\omega_2 = d\varphi(t)/dt$.

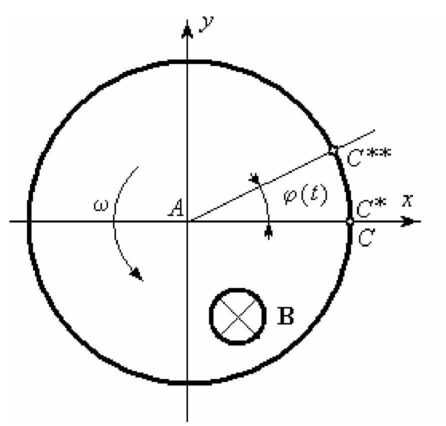


Fig. 10

Now, using the Bio-Savart law, we calculate the magnetic field induction $B(r)$.

$$B(r) = \int_0^{2\pi} \mu \frac{Ia}{4\pi} \frac{\cos\varphi\,(a - r\cos\varphi)}{R^3} d\varphi. \qquad \text{(VI. 1.15)}$$

If we compare equations (VI.1.14) and (VI.1.15) with the expression (VI.1.11), then it turns out that they are equivalent.

Thus, we carried out a detailed analysis of unipolar induction.

VI.2Marinov Motor

The principle of the action of the Marinov motor was described in [13], and experiments confirming the experiment of Marinov in [14], [15]. One of the realizations of this motor is shown in Fig. 11.

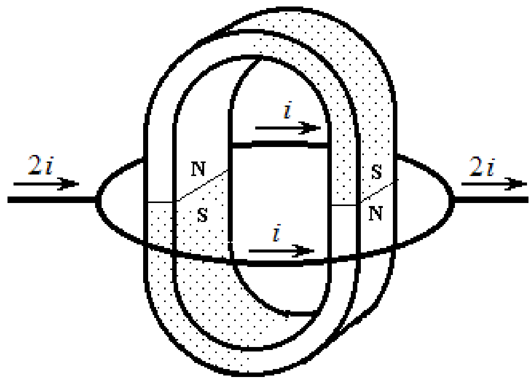


Fig. 11

Two horseshoe magnets are connected by opposite poles. In the plane that passes through the connection of the poles of the magnets, a circular conducting coil is located. As a turn, Marinov used mercury. After two sliding contacts to the mercury ring, a current source is connected, which creates a current i in each of the halves of the ring.

According to the Lorentz formula, the force acting on any element $d\mathbf{l}$ of this ring is:$d\mathbf{F}$=[$id\mathbf{l} \times$ $\mathbf{B}$, where i is the current; $\mathbf{B}$ is the magnetic field induction; $d\mathbf{l}$ is the element of the conducting ring.It must act perpendicular to the element $d\mathbf{l}$. Such a force can not create a torque acting on the ring. However, this rotation was not only observed experimentally, but the magnitude of the rotational moment was measured [14], [15]. An explanation of this phenomenon was proposed.

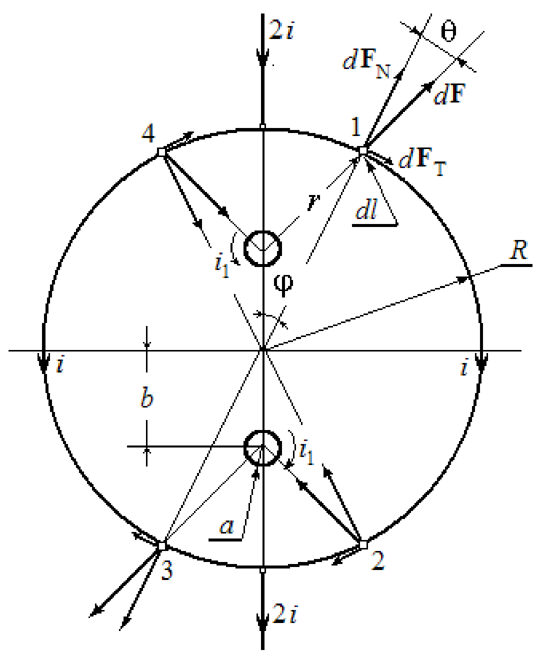


Fig. 12

We give an explanation based on the Newtonian theory. A simplified scheme of the Marinov motor is shown in Fig. 12. A permanent annular magnet, whose magnetic field lines are perpendicular to the plane of the figure, is represented by two closed ring currentsi_1. The outer conductor ring is provided with two sliding contacts connected to the DC source. All the elements mentioned are in the same plane.

Consider the current element $d\mathbf{l}$, located at position 1 in Fig. 12. The force $d\mathbf{F}$ $(a \ll r)$, which acts on this element of the current from the side of the ring current, is directed along the line connecting $d\mathbf{l}$ with the ring current in accordance with the principle of equality of action and counteraction. This force has two projections. The first projection of the $d\mathbf{F}_N$ is directed along the radius R. The second projection $d\mathbf{F}_T$ is directed along the tangent to the circle with the current i. This force creates the angular momentum$d\mathbf{M}$.

$$d\mathbf{M} = R d\mathbf{F}_T = R d\mathbf{F} \sin\theta. \qquad \text{(VI. 2.1)}$$

The other three current elements, arranged symmetrically, as shown in Fig. 13, create exactly the same torque moments. The total moment acting on the ring with current i is:

$$\mathbf{M} = 4R \int_0^{\pi} \frac{d\mathbf{F}}{d\varphi} \sin\theta d\varphi\,. \qquad \text{(VI. 2.2)}$$

At the same time, according to the Lorentz theory, the force acting on a conductor with a current is always perpendicular to the conductor with this current and the torque acting on the ring with current i must be zero. The explanation of magnetic phenomena from the position of Newtonian mechanics has no difficulties and allows us to obtain correct explanations for these phenomena.

VII. CONCLUSION

Modern classical theories (mechanics, electrodynamics) at the beginning of the 20th century were undeservedly considered “outdated theories” in relation, for example, to quantum theories. However, scientists have found that this view is erroneous. The development of new theories now depends on solving the problems of classical theories.

To date, many critical articles have been published and hypotheses have been proposed for eliminating problems in classical theories. Among the critics there are many well-known scientists [16].Unfortunately, scientists analyze special cases, considering electrodynamics to be a "complete theory". This view was held by R. Feynman. He wrote [11]: “*... I must say right away that all the rest of the physics is not as well tested as electrodynamics* …”

The aim of *our* study was, *just*, electrodynamics. We got interesting results that are unknown in the scientific world. So,

1. In ChapterII we analyzed the problem of the electromagnetic mass and showed that the solution to this problem does not exist within the framework of retarded potentials. Using the Maxwell equations in the Lorentz gauge, we have *rigorously* proved the generalized Poynting law. We found the condition for the absence of longitudinal waves in electrodynamics and showed that electromagnetic waves are radiated by charges that have no inertia.However,for the sake of objectivity, we must refer the reader to an article where it is shown that massless particlesdo not radiate [17].

2. In Chapter III we got an amazing result. It turned out that the second law of conservation of energy-momentum takes place in the framework of Maxwell’s equations in the Lorentz gauge. This is the law of conservation of Umov in the relativistic form. Umov’s law solves the problem of electromagnetic mass. In the same Chapter, we showed that instantaneous action at a distance can also take place within the framework of the Lorentz transformation.Moreover, Dr. M. Korneva discovered that the Maxwell equations are invariant with respect to a large class of transformations.

Some of these transformations do not prohibit superluminal velocities. We believe that an experimental verification of the Lorentz transformation in the future is necessary.

3. The obtained solution of the electromagnetic mass problem allowed to give a rigorous description of the relativistic interaction of electric charges (Chapter IV).

4. In Chapter V we found a standard transition to a description of the interaction of charges in the framework of classical representations. It turned out that the classical Lagrange function is invariant under the Galileo transformation. The interaction of a charge with a current and the interaction of two currents were described. Some errors in the interpretation of magnetic phenomena were corrected.

5. Chapter VI is devoted to explaining two phenomena where a stable point of view has not yet been formed. The results of the previous chapters allowed us to give a logically rigorous explanation of the phenomenon of unipolar induction of Faraday and describe the principle of the action of the Marinov motor.

We believe that interesting results have been obtained and not only theoretical but also experimental studies in these fields of science are needed in the future.

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