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How to minimize energy loss during an energy transfer between a source and a receiver

Patrick CORNILLE
Advanced Electromagnetic Systems
7 Rue de la Perdriere
59320 Ennetieres-en-Weppes,F rance
E-mail: patrick.cornille@libertysurf.fr

Abstract

In this paper, w e review the one capacitor problem with a new approach. We will study how to minimize the energy transfer between the power supply and the capacitor by choosing a voltage with a specific time behavior. We will introduce a new stepping method to charge a capacitor that almost cancel the resistive losses. We will for the first time explain the mathematical reasons why an irreversible process becomes reversible by such a method.

Keywords: Entropy; Capacitor problem; Energy transfer; Reversibility; Heat losses.

1 INTRODUCTION

There is a well-known problem that appears in the literature, the so-called: One or Two capacitor problem. The problem concerns an uncharged capacitor which is connected at time $t = 0$ to a battery or a charged capacitor. Simple calculations show that after the charging process is completed, half of the initial electrostatic energy disappears, apparently giving rise to a paradox. For several decades, this paradox has not received a satisfactory explanation even to day.

The search for answering this paradox allows to highlight many subtleties about energy transfer processes and about the role played by the dissipative phenomena during the transfer of energy from one system (the source) to another system (the receiver). In this regard, it is very useful comparing the two capacitors system with others similar systems showing the same energy behavior.

This is the case of two identical tanks *Singal* [1] connected by a hose at their bottoms and filled with water. We let the water flows from the filled tank into the empty tank and we measure the fluid level in the empty tank as a function of time. The water flows from where the level is higher to where it is lower. The system reaches equilibrium when the levels become equal. *Fuchs* [2,p.39] demonstrated that the time behavior of the tank problem is the same as in the RC model examined in this paper. *Bonanno* [3] in his paper quotes several other systems.

Different solutions are given in the literature for solving the capacitor problem. Some authors think that the missing energy is radiated away by the accelerated charges circulating in the connecting wires and the capacitor plates. To resolve this situation, *Boykin* [4] has adopted the magnetic dipole model and has shown that just the radiation effect due to the magnetic dipole model can explain the missing energy. Other authors as $Choy$ [5] considers that the electric dipole radiation from the capacitor part can also explain the missing energy. Some author as $Al-Jaber$ [6] calculates the energy balance based on the Poynting vector, other author as $Powell$ [7] adds a self-inductance in the circuit.

$Singal$ [1] in the section 4 of his paper argues that the missing energy as possible radiation losses calculated from Larmor's formula cannot resolve the paradox. In fact, the radiation losses, due to the acceleration of the charges will be extremely small and can be made arbitrarily small by making a quasi static charging process. We can also argue against the radiation solution in the RC problem by noting that in the tank problem, there is no radiation effect possible in that case. Finally, we can quote several papers [8-12] which are worth to be read concerning this problem.

2 THE LAW OF ENERGY CONSERVATION VS THE LAW OF BALANCE

A system or a body is usually defined as a set of particles in interaction with one another. This system can interact or not with another system. We say that a system is closed if the system has no exchange of mass and energy with its surroundings. However, this definition is too loose and we need an accurate mathematical definition to define what is a closed system. There is only one condition to define a closed system, namely the forces of interaction of the particles or the body must verify the Newton's third law as explained by $Cornille$ [13-14] and $Pinheiro$ [15]. The law of motion for a body of mass m has for expression:

$$m\frac{d\mathbf{U}}{dt} - \mathbf{F}_i = \mathbf{F}_e \tag{1}$$

where we have introduced a partition of forces between the internal force which follows the Newton's third law for that body and the external force which can be any force including a force deriving from a potential function but this force is applied to the center of mass of the system or the body.

The partition of forces between internal and external forces is a fundamental property of the physical world because this partition allows a system to evolve freely with a motion of its own, the external forces being present or not. If this partition did not exist, then we will not have, for example, the freedom to walk on earth while the earth is rushing through space with a speed of 200 kilometers per second around the Milky Way.

If we assume that the internal force derives from a potential function $E_P[\mathbf{R}(t)]$ which does not depend explicitly on time, then we can scalar multiply both sides of the previous equation by $\mathbf{U}$ and write the equation in a well known form:

$$\frac{dE_M}{dt} = \mathbf{U} \cdot \mathbf{F}_e \tag{2}$$

where the mechanical energy E_M has for definition:

$$E_M = \frac{1}{2}m\mathbf{U}^2 + E_P(\mathbf{R}) \tag{3}$$

Therefore, the system is closed if the mechanical energy is constant, that is to say the external force is zero. We have defined the classical conservation law of energy that some physicists consider as the most important concept in physics. This is not so for several reasons: the first one is the presence of an external force which can change the energy of the system, for example if the force is a dissipative force, then the total energy of the system will decrease or on the contrary the energy of the system may increase as in the case of the harmonic oscillator examined in the appendix where the gravitational force as an external force will increase the energy of the spring.

It is important to note that we classify the dissipative force *Guemez* [16] as an external force since the dissipation mechanism as the resistance of the wires connecting the battery to the capacitor in the RC model is really outside the capacitor.

The second reason is the existence of well known forces which do not satisfy the Newton's third law, in that case the mechanical energy is not conserved. We can quote two famous laws which violate this principle: the Lorentz force *Cornille* [17] and the Coriolis force *Cornille* [14]. Moreover, there is a third case when the potential function has an explicit time dependence. All what we have discussed until now may seem trivial to the reader, we will prove in this paper that it is not the case.

Equation (2) defines what we call an equation of energy transfer between two independent systems when the external force is present. The fact that we have an equation with a sign equal is considered by some physicists as a conservation law of energy. This is not the case since the equation is not related to the classical definition of the mechanical energy. Some authors such as *Fuchs* [2, p.20] defines rightfully this equation as a law of balance. This definition is well chosen since the equation of energy transfer depends on a variable which quantifies this unbalance as the voltage difference between two capacitors, the difference of level of the fluids in two tanks or the temperature difference between two heat baths.

All the previous discussion leads naturally to the different concepts of entropy, reversibility versus irreversibility, equilibrium states versus non equilibrium states which are very difficult subjects to apprehend. However, these concepts do not concern only thermodynamics but apply to all branches of physics, this is the reason why this subject is so important. We will take a simple example often given in the literature to make our point of view very clear.

Consider the case of an egg that we let drop on the ground from a height h, we know that the mechanical energy is conserved since we have for the initial state $E_K = 0$ and $E_P = mgh$ and for the final state $E_K = 0.5 * mU^2 = mgh$ and $E_P = 0$, the egg breaks irreversibly when it reaches the ground and the kinetic energy is transformed into heat which results in an increase of the entropy of the system. The egg with the gravitational potential function form a closed system and there is no ambiguity. The physical situation examined in this paper is quite different since it involves the transfer of energy between two independent systems: the source and the receiver. Indeed, this is what is done in thermodynamics when we consider the transfer of heat between two different heat baths.

The fact that we can minimize the heat losses during the transfer of energy between the source and receiver by processing the energy transfer by small steps is known for a long time but no clear understanding and mathematical demonstration of the phenomena have been given in the literature to date in spite of the fact that this principle, as the Newton's third law, is one of the most important principle in physics since many practical applications can use this principle, some of them will be examined in this paper.

Another explanation to the missing energy problem is given by $Al - Jaber$ [18], the author assumes that the missing energy is taken into account by the kinetic energy of the recoiling system during the process of energy transfer. We know that the recoil motion in a closed system is due to Newton's third law and that the conservation law of energy takes into account the recoil kinetic energy.

In the capacitor problem, this hypothesis would imply a motion of the capacitor during the charging process. Therefore, if we suspend a capacitor by a wire to the ceiling of the laboratory, we can expect to observe a motion as in the famous Trouton-Noble experiment which was replicated recently by $Gabillard$ [19] where a torque motion is indeed observed when the capacitor is charged with a high voltage. But this motion is not due to the Newton's third law. On the contrary, this motion is the consequence of the violation of the Newton's third law by the magnetic part of the Lorentz force $Cornille$ [14].

3 THE ONE CAPACITOR SOLUTION

The magnitude of the voltage V across a capacitor with capacitance C charged via a resistance R by a power supply with a voltage U is solution of the first order differential equation:

$$RC\,\frac{dV}{dt} + V = U \tag{4}$$

The general solution of the above equation is given by the formula:

$$V(t) = \alpha\, e^{-\alpha\, t} \int_0^t e^{\alpha\, s}\, U(s) ds + U_0\, e^{-\alpha\, t} \tag{5}$$

with the definition $\alpha = 1/RC$ and the initial condition $V(0) = U_0$.

Let us multiply by $I(t) = C * dV/dt$ the equation (4) and integrate each term to obtain the equation:

$$\int_0^T RI^2(t)dt + \int_0^T V(t)I(t)dt = \int_0^T U(t)I(t)dt \tag{6}$$

The energy equation above can be written in the symbolic form:

$$E_J(T) + E_C(T) = E_S(T) \tag{7}$$

Let assume that the voltage of the power supply is written in the form $U(t) = U_S\, f(t)$ where U_S is a constant. The first case is the classical RC problem of charge of a capacitor by a battery:

$$f_1(t) = 1 \qquad V(t) = U_S(1 - e^{-\alpha\, t}) \tag{8}$$

$$E_J(T) = E_0(1 - e^{-2\alpha\, T}) \qquad E_C(T) = E_0(1 - e^{-\alpha\, T})^2 \tag{9}$$

$$E_S(T) = 2E_0(1 - e^{-\alpha\, T}) \tag{10}$$

with the definition $E_0 = CU_S^2/2$.

$Mita$ [20] made a wrong assumption in their equation (17) namely that the voltage across the capacitor is the same that the voltage of the power supply which cannot be true since the voltage across the capacitor is a solution of the differential equation (4). However, we can choose the voltage of the power supply as the same as the voltage across the capacitor which is our second case:

$$f_2(t) = 1 - e^{-\alpha t} \qquad V(t) = U_S(1 - e^{-\alpha t} - \alpha t e^{-\alpha t}) \tag{11}$$

$$E_J(T) = \frac{1}{2} E_0[1 - (1 + 2\alpha T + 2\alpha^2 T^2)e^{-2\alpha T}] \tag{12}$$

$$E_C(T) = E_0(1 - e^{-\alpha T} - \alpha T e^{-\alpha T})^2 \tag{13}$$

$$E_S(T) = \frac{1}{2} E_0[3 - 4(1 + \alpha T)e^{-\alpha T} + (1 + 2\alpha T)e^{-2\alpha T}] \tag{14}$$

The third case is:

$$f_3(t) = \frac{t}{T} \qquad V(t) = U_S(e^{-\alpha t} + \alpha t - 1)/(\alpha T) \tag{15}$$

$$E_J(T) = E_0[2\alpha T + 4e^{-\alpha T} - e^{-2\alpha T} - 3]/(\alpha T)^2 \tag{16}$$

$$E_C(T) = E_0[e^{-\alpha T} + \alpha T - 1]^2/(\alpha T)^2 \tag{17}$$

$$E_S(T) = E_0[(\alpha T)^2 + 2e^{-\alpha T} + \alpha T e^{-2\alpha T} - 2]/(\alpha T)^2 \tag{18}$$

The above RC model shows that there is no missing energy for finites values of R and T since the equation (7) is rigorously verified, therefore, there is no paradox. If we assume the presence of non-zero resistance R connecting the capacitor to the power supply, then the amount of energy loss in the resistance is exactly the same as the missing energy. We cannot set the resistance to zero in equation (4) because the equation $V = U$ would imply an instantaneous transfer of energy from the source to the capacitor which is physically impossible. Therefore, a resistance is needed in any electrical circuit to account for a finite time for the transfer of energy between the source and the receiver.

Actually $R \to 0$ is a mathematical idealization which may not hold good when we go below certain very low resistance values. Indeed, one can imagine using superconducting wires for the connections, but even in that case, a critical field limit must exist in the superconducting state. If we lower its temperature, the resistance of the conductor will reduce steadily up to a certain point below which it may suddenly become zero as the material turns into a superconductor. At this turnover point, there is a discontinuity in the resistance and one does not have $R \to 0$ as a limit.

Some authors state that the energy loss is independent of the resistance R. This conclusion arises because the quantity T in the preceding equations is taken to infinity. In experiments, we always deal with a finite value of the quantity T. We can also see by comparing the three cases that the amount of dissipation energy is not always half of the supplied energy when $T \to \infty$.

Therefore, the fundamental question we should ask is how to minimize the energy loss during the transfer of energy between the source and the receiver. This question is not often investigated in the literature except in the recent french thesis *Cornogolub* [21]. It suffices to look for an extremum of the dissipative power function $P(t) = RI^2(t)$, namely:

$$\frac{dP}{dt} = 2RI\frac{dI}{dt} = 0 \Rightarrow RI = RC\frac{dV}{dt} = Ct \tag{19}$$

The above constraint gives the following definition for $f_4(t)$ which is very similar to $f_3(t)$:

$$f_4(t) = (t + RC)/T \qquad V(t) = U_S\, t/T \tag{20}$$

$$E_J(T) = 2E_0/(\alpha T) \qquad E_S(T) = 2E_0(1 + 2/\alpha T) \tag{21}$$

By comparing the definitions of $f_4(t)$ and $f_1(t)$, we see that the constant RC/T in $f_4(t)$ cancels the exponential term in the voltage expression across the capacitor in equation (15). We note that indeed, the energy loss $E_J(T)$ can be a small quantity for $T >> RC$.

We can give a numerical example by taking $C = 10\,\mu F, R = 100\,\Omega, U_S = 100\,V$, we get $E_0 = 0.05\ J$ and plot versus time in the figures 1 and 2 the voltage $V(t)$ and the current $I(t)$ to see the difference in behavior for the four cases.

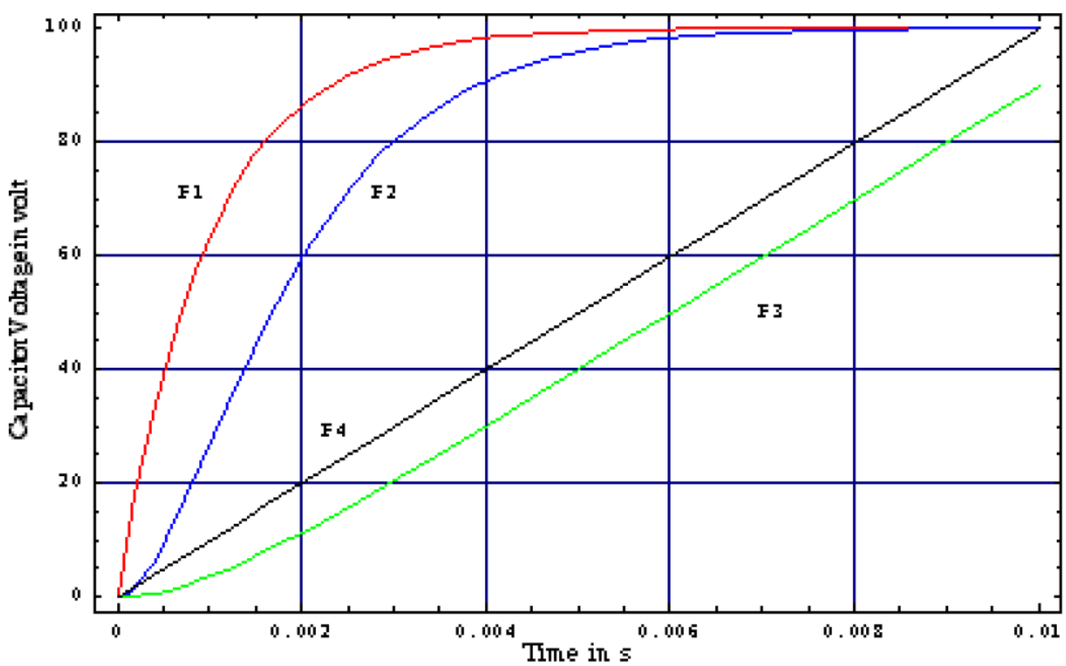

Figure 1. Capacitor voltage versus time

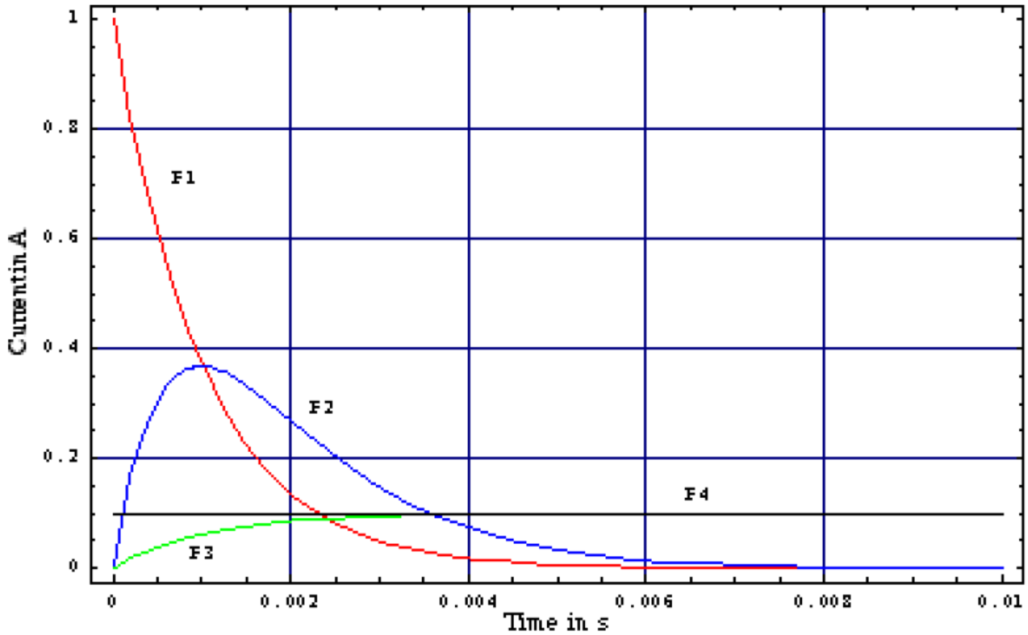

Figure 2. Capacitor current versus time

Until now, we have considered the case where the capacitor was not initially charged, let us assumed now that the capacitor is charged at time $t = 0$ with a voltage such that $\beta = U_0/U_S = 1/10$. The solution is given by the equations:

$$f_5(t) = 1 \qquad V(t) = U_S[1 + (\beta - 1)e^{-\alpha t}] \tag{22}$$

$$E_J(T) = E_0(1-\beta)^2(1 - e^{-2\alpha T}) \quad E_S(T) = 2E_0(1-\beta)(1 - e^{-\alpha T}) \tag{23}$$

$$E_C(T) = E_0[(1 + (\beta - 1)e^{-\alpha T})^2 - \beta^2] \tag{24}$$

Table 1. Capacitor energies versus the source function

Function	$E_J(T)$	$E_C(T)$	$E_S(T)$	$E_C(T)/E_S(T)$
f1(T)	50.00	49.99	99.99	0.499
f2(T)	25.00	49.95	74.95	0.666
f3(T)	8.50	40.50	50.00	0.810
f4(T)	10.00	50.00	60.00	0.833
f5(T)	40.05	49.49	89.99	0.549

In table 1, the different energies are given in mJ. We note that the ratio $E_C(T)/E_S(T)$ is better with an initial condition than without, even if we take into account the initial energy of the charged capacitor since we have $E_C(T)/(E_S(T) + E_C(0)) = 0.547$.

This can be explained by the fact that the initially charged capacitor sent a negative current towards the source which partially compensates the positive current making the total current more constant. We proved previously that the energy loss is decreased in this case. This is the key of understanding why a charging process by steps where the initial voltage of the capacitor is also increased by step will almost cancel the energy losses by Joule heating as we will show hereafter.

The stepping process used in the literature is directly applied to the source or to the receiver, for example: for the capacitor, it is the voltage of the power supply which is divided into N steps *Heinrich* [22] each of them with a voltage increase $U = U_S/N$, this is a staircase voltage which in the limit $N \to \infty$ gives the continuous functions $f3$ or $f4$. For the harmonic oscillator *Fundaun* [23], we add N masses successively, each of them being $m = M/N$. These stepping processes are not practical from an experimental point of view and do not allow to get a physical insight in the phenomena.

4 APPLICATION OF THE STEPPING PROCESS

We will now present a different stepping process which consists to switch the energy transfer between the source and the receiver. The two stepping processes are quite different since the former gives a capacitor voltage curve below the classical charging curve as for the cases $f_3(t)$ and $f_4(t)$ while for the later, the curve is above the classicl voltage curve as shown in figure 3.

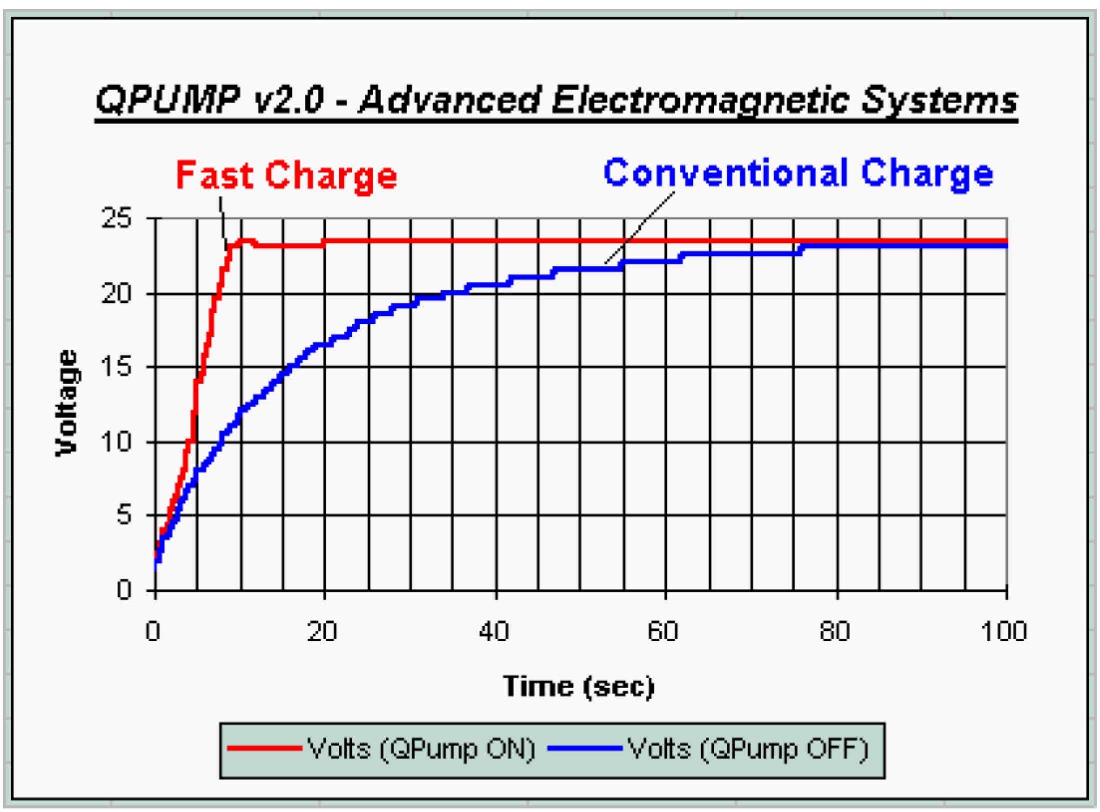

Figure 3. Capacitor voltage versus time with and without stepping

With the development of the electric vehicle market, the need for charging rapidly and efficiently batteries and super capacitors becomes a major concern. The curve 3 shows the voltage behavior during the charging process of a super capacitor (1 farad) with and without a stepping method. The measured efficiency is above 90%.

Common alkaline batteries cannot be recharged with an ordinary battery charger because the resistive losses heat the batteries which can explode. However, a stepping battery charger can be used to recharge alkaline batteries more than 10 times since the Joule heating is almost cancel. This special battery charger has been on sale in Europe with many counterfeiters in Asia. A US patent, *Cornille* [24] was recently granted to the author for that technology.

We will now generalize the study of section 2 to a more complicated but very interesting case. An equation of motion describes the behavior of a physical system of mass m in terms of its motion as a function of time. More specifically, the equation of motion is now given by the formulation

$$\frac{d\mathbf{P}}{dt} = \mathbf{F} \tag{25}$$

where the momentum has for definition $\mathbf{P} = m(t)\mathbf{U}$. The velocity $\mathbf{U}$ of the system is defined with respect to an inertial reference frame.

if we assume that the force $\mathbf{F}$ applied to the system derives from a potential function $E_P(\mathbf{R}, t)$, then the equation above can be rewritten in the form:

$$m\,\frac{d\mathbf{U}}{dt} + \frac{dm}{dt}\,\mathbf{U} = -\nabla E_P \tag{26}$$

If the preceding equation is scalarly multiplied by $\mathbf{U}$, then we obtain

$$\frac{m}{2}\,\frac{d\mathbf{U}^2}{dt} + \frac{dE_P}{dt} = \frac{\partial E_P}{\partial t} - \frac{dm}{dt}\,\mathbf{U}^2 \tag{27}$$

where we have used the definition:

$$\frac{dE_P}{dt} = \frac{\partial E_P}{\partial t} + \mathbf{U}\cdot\nabla\, E_P \tag{28}$$

The second term in the right hand side of equation (27) is a dissipative term if we verify the condition $dm/dt > 0$.

We can integrate equation (27):

$$\frac{1}{2}\int_0^T m\,\frac{d\mathbf{U}^2}{dt}\,dt + \int_0^T \frac{dE_P}{dt}\,dt = \int_0^T \frac{\partial E_P}{\partial t}\,dt - \int_0^T \frac{dm}{dt}\,\mathbf{U}^2\,dt \tag{29}$$

We have the identity

$$\frac{1}{2}\int_0^T m\,\frac{d\mathbf{U}^2}{dt}\,dt = \left[\frac{1}{2}\,m\mathbf{U}^2\right]_0^T - \frac{1}{2}\int_0^T \frac{dm}{dt}\,\mathbf{U}^2\,dt \tag{30}$$

If we substitute the preceding equation in equation (29), we obtain a final equation:

$$\left[\frac{1}{2}\,m\mathbf{U}^2 + E_P\right]_0^T = \int_0^T \frac{\partial E_P}{\partial t}\,dt - \frac{1}{2}\int_0^T \frac{dm}{dt}\,\mathbf{U}^2\,dt \tag{31}$$

The term in the left hand side of the equation is the mechanical energy of the system while in the right hand side, we have two terms: the first one is the contribution of the explicit time variation of the potential and the second term is the dissipative term.

Now we can give an explanation concerning the nature of the system described by the above equation. Let us consider the case of an electrostatic linear accelerator which is a huge capacitor charged by a power supply. It is either a Cockcroft-Walton accelerator, which uses a diode capacitor voltage multiplier to produce high voltage or a Van de Graaff accelerator which uses a moving fabric belt to carry charges to the plates of the capacitor. An electrostatic accelerator used a static high voltage to accelerate charged particles in an evacuated tube with an electrode at either end which are the plates of the capacitor. Since the charged particle passed only once through the potential difference, the output energy is limited to the accelerating voltage of the machine. This method is still used today, with the electrostatic accelerators greatly out-numbering any other type, they are more suited to lower energy studies owing to the practical voltage limit of about $1MV$ for air insulated machines.

Einstein predicted in his theory of relativity that no particle having mass can travel as fast as the speed of light. No matter how much energy one adds to an object with mass, its speed cannot reach that limit. This limit results from the fact that mass and energy are tied together by the formula $E = m_0\gamma\, c^2$. This equation predicts that nothing with mass can move as fast as light.

In modern accelerators, particles are sped up to very nearly the speed of light. For example, the main injector at Fermi National Accelerator Laboratory accelerates protons to 0.99997 times the speed of light. As the speed of a charged particle gets closer and closer to the speed of light, an accelerator would require an infinite amount of energy to increase the kinetic energy of the charged particle and in the process the mass of this particle would become infinite.

The speed limit is the result of the braking force which is included in the dissipative term in the right hand side of equation (31). This force is dissipative if the condition $dm/dt > 0$ is verified. We know that this force depends on the square of the velocity and is therefore a magnetic force. *Cornille* [14,25] demonstrated how the Ampere force which has a longitudinal component along the direction of motion of the electron can explain the nature of the braking force. Now we can apply the above analysis of the capacitor problem to our capacitor accelerator with a charged particle $-q$ in motion between two plates separated with a distance D charged with the charges $-Q(t)$ and $+Q(t)$. We can define two potential functions where their sum is:

$$E_P[\mathbf{R}(t), t] = qQ(t)\left[\frac{1}{R} + \frac{1}{D-R}\right] \quad \Rightarrow \quad \frac{\partial E_P}{\partial t} = qC\,\frac{dV}{dt}\left[\frac{1}{R} + \frac{1}{D-R}\right] \tag{32}$$

We can verify that these potentials have the correct form and sign by calculating the forces which must have the same direction towards the + Q(t) for an electron but not the same magnitude. A more exact formulation of the potential function can be obtained by doing a numerical integration of the interaction potentials between the electron and the charges on each plate of the capacitor as done by *Cornille* [25].

If the charging process of the capacitor C is divided into N steps which results in a voltage increase such that the voltage V of the power supply verifies the condition $dV/dt > 0$, then we should obtain two advantages: first, it should decrease by two the energy necessary to charge the capacitor, secondly, the two terms in the right hand side of equation (40) should compensate one another or even cancel opening the possibility to break the light speed limit. Only an experiment can prove or disprove the validity of this hypothesis.

The last term in equation (31) is a velocity dependent damping force with a non constant coefficient which includes an acceleration dependent term since for $m(t) = F[U(t)]$, we have $dm/dt = dF/dU * dU/dt$. Since the radiation effect involves the acceleration of the charge, one can expect a damping force depending on both U and dU/dt. It is well known that the radiation effect can be defined as a resistive term *Cornille* [14, p. 386]. Therefore, the dissipation energy term incorporates an Ohmic like resistive force and a radiative resistive force. One may ask the question whether or not a neutral particle has a light speed limit.

5 CONCLUSION

In the present paper, we have proved that there is no missing energy in the RC problem and therefore no paradox occurs provided we keep with finite values of the parameters R and T. We clarified the concept of energy transfer when there is a dissipation effect by showing the importance of the time behavior of the source voltage. We have also demonstrated the fundamental role of the initial conditions to explain why we can decrease the heat losses at will with a new stepping method. Finally, we have examined several experimental applications where we can minimize the energy transfer between the source and the receiver.

APPENDIX: THE HARMONIC OSCILLATOR MODEL

Let us now consider the case of a harmonic oscillator where a frictional force is present. If an external force $\mathbf{F}_e$ is applied to an object of mass m in a harmonic potential the equation of motion can be written as follows

$$m\,\frac{d^2\mathbf{R}}{dt^2} + r\,\frac{d\mathbf{R}}{dt} + k\mathbf{R} = \mathbf{F}_e \tag{33}$$

where r and k are respectively the friction constant and the spring constant. If the preceding equation is scalarly multiplied by $\mathbf{U} = d\mathbf{R}/dt$, then we can rewrite the above equation as follows:

$$\frac{dE_M}{dt} = \mathbf{F}_e \cdot \mathbf{U} - r\,\mathbf{U}^2 \tag{34}$$

where the mechanical energy has for definition:

$$E_M = \frac{1}{2}\,m\mathbf{U}^2 + \frac{1}{2}\,k\mathbf{R}^2 \tag{35}$$

Let us assume that the mass m is at rest at time $t = 0$ when the spring stretches to balance the constant gravitational force $\mathbf{F}_e = m\mathbf{g}$. The mass will come to a complete stop at the new equilibrium position, therefore the initial and final velocity are both zero, which means that the variation of the kinetic energy becomes zero. Let us consider the case where the linear spring is loaded by adding a mass m at each step *Gupta* [26], then the variation of the elastic energy stored in the spring due to loading is given by the relation:

$$\Delta E_P = \int_0^{\mathbf{R}(T)} \mathbf{F}_P(\mathbf{R}) \cdot d\mathbf{R} = \sum_{n=1}^{N} \int_{R_{n-1}}^{R_n} \mathbf{F}_P(\mathbf{R}) \cdot d\mathbf{R} \tag{36}$$

knowing that the spring force is $\mathbf{F}_P(\mathbf{R}) = k\mathbf{R}$, then the above equation becomes:

$$\Delta E_P = \frac{1}{2}\,k \sum_{n=1}^{N} \int_{R_{n-1}}^{R_n} dR^2 = \frac{1}{2}\,k \sum_{n=1}^{N} (R_n^2 - R_{n-1}^2) = \frac{1}{2}\,k\,h^2\,N^2 = \frac{1}{2}\,MgH \tag{37}$$

with the definitions: $R_n = nh$, $mg = kh$, $M = Nm$, $H = Nh$

The variation of the gravitational energy of the loaded mass m is given by:

$$\Delta E_S = \int_0^{\mathbf{R}(T)} \mathbf{F}_S(\mathbf{R}) \cdot d\mathbf{R} = \sum_{n=1}^{N} \int_{R_{n-1}}^{R_n} \mathbf{F}_S(\mathbf{R}) \cdot d\mathbf{R} \tag{38}$$

The gravitational force $\mathbf{F}_S(\mathbf{R}) = nm\mathbf{g}$ is a constant force, therefore, we have:

$$\Delta E_S = mg\sum_{n=1}^{N} \int_{R_{n-1}}^{R_n} dR = mg\sum_{n=1}^{N} (R_n - R_{n-1}) = \frac{1}{2}\,mgh\,N(N+1) = \frac{1}{2}MgH(1 + \frac{1}{N}) \tag{39}$$

Therefore, the variation of energy dissipated in the form of heat is equal to:

$$\Delta E_J = \Delta E_S - \Delta E_P = \frac{MgH}{2N} \tag{40}$$

Finally, by increasing the number N of steps of the process, we decrease the energy loss as demonstrated by *Gupta* [26].

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