



Spatial curvature as a distorted mapping of Euclidean space

Andrew Chubykalo¹, Augusto Espinoza and Victor Kuligin²

¹Unidad Académica de Física, Universidad Autónoma de Zacatecas, A.P. C-580, Zacatecas, México

²Physical faculty Department of an electronics, Voronezh State University, Russia

E-mail: achubykalo@yahoo.com.mx

Abstract. The article makes an attempt to understand the essence of the curvature of space-time in general relativity. To clarify the physical meaning, a mapping method is used, based on the analogy with the philosophical categories “phenomenon-essence-observer”. It is shown that curvilinear space can exist only inside the Euclidean space. It is also shown that the interpretation of the phenomenon called the “Big Bang” is incorrect.

Key words: The curvature of space; the mapping of spaces; the Big Bang.

1. Introduction

Space, like time, is not material for us in the *ordinary* sense (although, as for the *materiality of time*, see “The Mathematical Justification of a Possible Wave of the Nature of the Time Flow of Kozyrev” [1]). However, let us recall Engels’ statement, cited by Lenin in [2]: “*Space and time are not simple properties of matter, but the root forms of its being.*” Therefore, “*slowing down time*” and “*squeezing scale*” in SRT is nonsense from a materialistic point of view (although are possible spatial oscillations of time [1]!). To the classical properties of our space we always refer continuity, infinity, homogeneity, three-dimensionality and isotropy. Accordingly, we always identify space with the geometry of a three-dimensional space in which material bodies do not affect the properties of space. However, with the advent of general relativity, our notion of space and time had to be changed.

For more than one decade there has been a discussion of the consequences of general relativity. Some aspects of the theory of the “Big Bang”, “black holes”, “dark matter”, etc. not always meet common sense, logic and everyday ideas. There are reasons for this. It is hard to believe that a small inaccuracy in geometry, which originated in the early 19th century, will have a great impact on physics. The source of the error lies in the content of the concept of “curvature of space”. What is this “curvature of space” and how to measure it using the “circular and ruler” method? Let us try to understand.

We will only consider the geometry of space, and we will practically not discuss the content of the physical hypotheses that led to the birth of new concepts in physics.

2. Three-dimensional space

2.1 Introduction of curvature

We will consider, for *the sake of clarity*, the curvature in three-dimensional space. Three-dimensional space is clearly and easily perceived by man. Modern physicists and mathematicians construct curvilinear space in a simple way. Let, for example, there be some three-dimensional space free from material objects. Mathematicians define a second-rank metric tensor, g_{ik} , which describes the curvilinear properties of the original three-dimensional space. It seems that there are no *pitfalls* here.

However, if you look closely, you can see the hidden problem. The metric tensor g_{ik} introduced by us depends on the coordinates x, y, z of this space; $g_{ik}(x, y, z)$. It is logical to assume that the independent variables x, y, z belong to the flat (three-dimensional Euclidean) space that existed before the introduction of the metric tensor. Thus, the metric tensor is not introduced into “any free” space. The tensor is introduced into a *flat* Euclidean space.

Then the following circumstance holds. As soon as physicists begin to study and describe the properties of a curvilinear space with the metric g_{ik} , they “forget” about the existence of Euclidean space. This fundamental error appeared more than 200 years ago and turned into prejudice, and then into dogma.

2.2 Steps to the correct understanding

In mathematics, formal logic is the method of proof and the criterion for its verification. If in the arguments “disappear” the logical links, if instead of arguments we rely on intuition, then the evidence turns into an ordinary subjective opinion. In the previous section, with the introduction of $g_{ik}(x, y, z)$, there are no necessary links of reasoning. This deprives the procedure for constructing a curvilinear space by the method of specifying the metric tensor $g_{ik}(x, y, z)$ of important quality. There is a suspicion that in the three-dimensional space chosen by us there exists not only a *curvilinear* space of 3 dimensions, but also a flat three-dimensional Euclidean space *combined with it*.

Let us begin our analysis of this problem in successive steps.

Step 1. Let us start with the task. We chose three-dimensional space as an example for analysis. We intend to build a curvilinear space in it in a conventional way. This space must have a given metric tensor $g_{ik}(x, y, z)$.

Step 2. We still have no idea about this 3-dimensional space. We do not know: is there any curvature in the three-dimensional space that we have chosen? Here there is uncertainty, which we must resolve. We assume that the independent variables x, y, z belong to the three-dimensional (Euclidean) space, and they form the orthogonal axes of the Cartesian coordinates. We cannot consider space curvilinear. “Curvilinear space” we are only going to “introduce”. Later we confirm the correctness of this assumption.

Step 3. So, we have independent variables (x, y, z) in three-dimensional Euclidean space. For simplicity, we assume that the scale along these axes is the same. The axes form the orthogonal “grid” of the Euclidean space.

Step 4. Now we define the metric tensor $g_{ik}(x, y, z)$ on this Euclidean space. How to understand this *correctly*?

Euclidean space after the introduction of the metric tensor suddenly twisted? Have the introduced axes (x, y, z) now lost their straightness and orthogonality? If lost, then for what reason? This was due to our subjective desire and choice of the required metric tensor $g_{ik}(x, y, z)$? Of course, the three-dimensional Euclidean space is preserved. But now the curvilinear space “arranged” inside the Euclidean space, described by the metric tensor $g_{ik}(x, y, z)$. The components of this tensor are expressed in terms of the variables (x, y, z) of the Euclidean space. Such a result significantly affects the interpretation of physical phenomena, for example, in general relativity.

2.3 In search of a philosophical analogy

In the philosophy of physics there are such categories: “phenomenon-observer-essence.” In **physics**, the observer learns the essence by examining a set of phenomena. He perceives phenomena through the senses or devices. Information about the phenomena he receives through the “information carrier.” Such a carrier can be, for example, light or acoustic waves. These waves, delivering information to the observer, can distort its content due to propagation conditions.

Is it possible to introduce similar philosophical categories in the geometry of space? It is possible, if with certain limitations, to use the analogy carefully to analyze relationships and relationships in geometry. Here, philosophy, operating with common concepts, is similar in a certain sense to topology, which also relies on idealized, generalized concepts.

Analogy. Suppose that we have two independent three-dimensional Euclidean spaces: $E_A(x, y, z)$ and $E_B(u, v, w)$. Assume that with the help of some space transformation operator with its coordinates we can map the three-dimensional space $E_B(u, v, w)$ onto the interior of the space $E_A(x, y, z)$. The analogy is shown in Fig.1 We denote the transformation operator as \tilde{H}_{BA} .

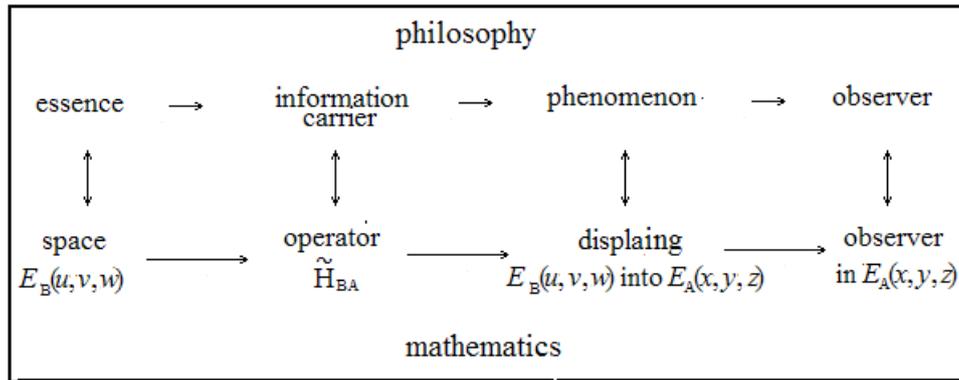


Fig. 1. Here the role of the displayed object (the analog of the “essence”) is played by the space $E_B(u, v, w)$. The role of the information carrier is played by the operator \tilde{H}_{BA} , which maps the space E_B into the space E_A . The role of the phenomenon is performed by the mapping. The role of the “observer”, which is registered by the phenomenon, is played by a hypothetical observer in the space $E_A(x, y, z)$. For us it is important that the space $E_B(u, v, w)$ is mapped into $E_A(x, y, z)$ as a curvilinear space¹.

Let us expand the operator \tilde{H}_{BA} :

$$u = H_u(x, y, z), \quad v = H_v(x, y, z), \quad w = H_w(x, y, z).$$

¹The mapping process resembles a conformal mapping in the theory of a complex variable.

Let $u = \text{const}$, $v = \text{const}$, $w = \text{const}$ inside the space E_B . In the space E_B , these surfaces form three families of orthogonal planes. In the space E_A we obtain a family of “curvilinear” surfaces, which in the general case may not be orthogonal.

At present, there is no need to impose any special requirements on the operator \tilde{H}_{BA} . The operator realizes the mapping in one direction and the requirement of “one-to-one” for the transformation is not needed. It is important for us that the map be smooth and twice differentiable. We denote the mapping of the space E_B into the space E_A as HE_{BA} .

Note. In principle, we could map the space $E_B(u, v, w)$ not to the whole volume of the space $E_A(x, y, z)$, but to a part of the space. For example, we could map $E_B(u, v, w)$ to the interior of a sphere of radius R that belongs to the space $E_A(x, y, z)$.

Thus, we see that a curvilinear map of the space $E_B(u, v, w)$ appeared inside the Euclidean space E_A , that is, $HE_{BA}(x, y, z)$. Let the metric tensor of this curvilinear space be $g_{mn}(x, y, z)$.

If the metric tensor g_{ik} , introduced earlier in section 1, is equal to the metric tensor g_{mn} ($g_{ik} = g_{mn}$), then we can assume that both curvilinear spaces are equivalent. They can differ only in linear terms, the second partial derivatives of which are zero. These terms do not have a fundamental character for us, since the curvature of space does not depend on them.

Thus, our hypothesis about the three-dimensional Euclidean space into which the curvilinear space described by the metric tensor $g_{ik}(x, y, z)$ is mapped can be considered consistent.

2.4 First Conclusions

The first conclusions contradict the existing ideas.

Any three-dimensional space is Euclidean. In it, you can always enter a Cartesian coordinate system with orthogonal axes.

- Generalization. Any N -dimensional space is initially Euclidean. It can not be curvilinear. [3]
- Curvilinear space can not exist independently. It can only be built *inside* Euclidean space. Remove the screen in the cinema and you will not see the movie. Remove the Euclidean space, then the curvilinear space disappears!
- The curvature of space is not an absolute value. Curvature is a relative concept. The curvature of the space is determined with respect to the Euclidean space, *within which* there exists a given curvilinear space.
- These conclusions can be extended to spaces with different number of dimensions $N = 2, 3, 4 \dots$ Let us quote from [3]: “Physical space forms a four-dimensional “surface” in the **flat** N -dimensional space. Each point x^μ in the surface determines a definite point y^n in the N -dimensional space. Each coordinate y^n is a function of the four x 's; say $y^n(x)$. The equations of the surface would be given by eliminating the four x 's from the $Ny^n(x)$'s. There are $N - 4$ such equations.”

3. On space and time

The above arguments and conclusions can easily be generalized to the case of Euclidean spaces of N dimensions. We consider the space of four dimensions ($N = 4$), extending the conclusions obtained to it. If we represent the fourth variable as an imaginary coordinate ($x_4 = ict$), then we obtain a pseudo-Euclidean space $E(\mathbf{r}, t)$. Formally, it differs little from the four-dimensional Euclidean space. The Minkowski space can be considered in a similar way. But first let us figure out *what the fourth dimension is*. We know the geometric figures of three genera: one dimension – a line, two dimensions – a plane, three dimensions – a body. A line is a trace from the motion of a point (having zero dimensions) in space, the surface is a trace from the motion of the line, the body is a trace from the motion of the surface. That is, every figure of the higher dimension is a trace from the movement of the figure of the lower dimension. But he asks: why did the number “three” suddenly become the limit? Why not to admit that there can exist figures with a large number of dimensions? Time as a fourth dimension? Nothing hinders to consider it that way, although we would violate the *general* principle leading to the formation of figures of a higher dimension.

Time is an indispensable condition for the perception of *any* space, an addition to *any* number of dimensions. Recognizing the space-time complex for the four-dimensional phenomenon, which is quite often done, we immediately face the question of four-dimensional space – as a derivative of the motion of a cube in space. In this space, apart from the perpendiculars defining the length, width and height, there must also be a fourth perpendicular, which is perpendicular to each of the three above-mentioned. It is impossible to imagine this new direction for us, three-dimensional creatures, but it is possible to calculate such a figure mathematically, if we accept the initial assumption as qualified. In this case, a completely harmonious, consistent four-dimensional geometry is obtained. However, as for the *physical* four-dimensional space and the four-dimensional physical objects (creatures) contained in it, it is proved not directly, but through the admissibility of two-dimensional physical objects (creatures), so this permissibility becomes a key, *fundamental* issue.

In Ouspensky’s book “The Fourth Dimension”, first published in 1914 in Russian (English translation, see [4]), there is one very important argument, for some reason *ignored* by modern authors. We consider it appropriate, to give here this reasoning in sufficient detail:

First, the book describes how the world appears to a two-dimensional creature, how it would react to the fact that we suddenly “picked up” someone from his fellows from the plane or, on the contrary, put something unexpected on it ... “All this for a flat creature it would be pure magic ... “. Further – the traditional transition (on the basis of *these* arguments - emphasize!) To the question of what the world of four dimensions should be. And in the general flow of reasoning - a small “private” remark in passing, as something insignificant. The two-dimensional creatures “cannot have a real being, having only two dimensions. They must certainly *have at least the smallest third dimension*. Otherwise they will be only imaginary figures, existing only in someone's mind, which do not really exist” [4].

Wow! Any “even the smallest” thickness of flat creatures makes them *one hundred percent three-dimensional*, that is, according to the terminology accepted by the author “non-planar” creatures, and all the chatter about the impressions of two-dimensional insects from our mysterious interference into their peaceful life loses meaning. So, it makes no sense to transfer the differences between two-dimensional and three-dimensional spaces into four-dimensional space. And how can it be otherwise? All authors who describe the reality of the four-dimensional world mistakenly attribute to two-dimensional creatures the ability to see a “circle” on the surface of the lake, the “outline of the body” of the bathing, “elliptical shape”, “circular creatures,” etc. All these forms are found only when viewed from above (from a three-dimensional space!), And not from the side, from which only lines should be visible. But ... should it? Lines can be something real, can be seen only when they have “at least the smallest” thickness. But in the two-dimensional space it cannot be *at all*, otherwise the two-dimensional space will immediately turn into a three-dimensional one, even if the “height” is only one atom thick, as P. Ouspensky offers a compromise [4]. The question is why we need to garden about the possibility of the *real* existence of any N -dimensional spaces if it is explained from the very

beginning that, according to all the reasoning given to this, the two-dimensional space that is original for these theoretical constructions cannot exist as a real physical space. Two dimensions of it are real only for those who look at the plane from the outside. "Inside" such a space is really only *one* dimension. "Flat creatures" would have to admire the lines of "zero thickness", possible only in the imagination. They themselves would also be "zero height". The plane *itself does not exist* for a two-dimensional world, it is necessary to penetrate this world "from the side" of the plane, but there are no "sides" for two-dimensional figures. This is again the concept of three-dimensional space. Whatever one may say, it turns out that there can be neither *real* "two-dimensional world" nor "plane creatures". These are only abstractions. It is instructive that if one disagrees with this, then it turns out that it can not exist as our *real*, three-dimensional space. P. Ouspensky imperceptibly for himself comes to this. To become real, two-dimensional creatures, according to him, should have "at least a small" third dimension. Accordingly, to be real, three-dimensional beings must possess "at least a very small extension in the fourth dimension, otherwise it will be only an imaginary figure, the projection of a four-dimensional body onto a three-dimensional space, like a cube drawn on paper." However, according to mathematical formulas, four-dimensional space is also not the limit. The four-dimensional supercube can be stretched in a direction perpendicular to all four mutual perpendiculars of the four-dimensional space. About the five-dimensional supercube can be said the same thing, and so on ad infinitum!

And one more blatant contradiction for some reason persistently dispenses with the supporters of the fourth dimension. According to their reasoning, creatures living in a higher space in terms of the number of dimensions can easily penetrate the step below, remove, rearrange. On the contrary, it does not work. That is why we have not yet penetrated into the four-dimensional space and we do not know how to use it. But we must at the same time easily penetrate into two-dimensional space, for him, it is we who are "super-creatures"! No, something does not work out a logical chain of reasoning, with the help of which *four-dimensional* is derived from the assumption of *two-dimensional* space. It turns out: *either* really, physically, beyond our imagination, *they are not at all*, or we for a four-dimensional being are just as unattainable as for us "flat creatures". It seems that the first "*either*" is justified. Let us note that, for example, Engels sharply criticized those mathematicians and physicists who "forget that all so-called pure mathematics deals with abstractions, that *all* its quantities are, strictly speaking, imaginary quantities, and that all abstractions, taken to the extreme, turn into nonsense or in its opposite" [5].

We do not set ourselves the goal of giving a new interpretation of general relativity. This is a difficult question. We want to identify some of the problems that arise from the "neighborhood" of Euclidean space-time alongside curvilinear space-time. The development and use of non-Euclidean and multidimensional geometry, discussions of multidimensional spaces, the introduction of the concept of "curvature of space" into everyday life, etc., began to inculcate in an involuntary way the notion of time and space as something quite independent, almost "controlling" "in them" of matter. But there is no reason to question the conclusions of the classics of materialism, and that there is nothing in the world except moving and interacting matter, that the concept of matter is the original, and "both forms of the existence of matter without matter are nothing, empty representations, abstractions existing only in our head" [5].

The abstract geometric "space" in this respect is just as much any real space as the number is matter. The "curvature of space" is so difficult to imagine, apparently, precisely because in its explanation it usually goes not from matter, but from the same in all directions and parts of its geometric space. Not matter exists in space, and space is formed by matter. *Where there is no matter, there is nothing, including space*. At the same time, we would like to emphasize the concept of *interaction* when explaining the essence of such categories as time and space. Roughly speaking, space is nothing more than a measure of the interaction of matter, its unity in interaction, the order of location of simultaneously existing objects. Only interacting elements of matter form a single space. The time is the sequence, synchrony, consistency, interdependence, direct or indirect, of all regular interactions (and yet, for the sake of objectivity, we refer the reader to [1], where we are talking about the so-called *substantial or material* time, theoretically predicted and experimentally discovered by Kozyrev, see the references in [1]). The properties of the *physical* space are determined not by the eternal and immutable laws of geometry, but by the *actual* interaction of matter. In geometry, a straight line is a mental line, taking as a sample a ray of light, the shortest distance between two points. In the real expanses of the

universe, as already proved, there is nothing to take for a sample – the rays of light also interact with the surrounding material world and are bent near large masses.

We, of course, will rely on geometry, while taking into account its requirements taking into account the objective reality.

4. Pseudo-Euclidean space

Thus, suppose that we have a pseudo-Euclidean space $E_A(\mathbf{r}, t)$ in which there exists a curvilinear map of some pseudo-Euclidean space $E_B(\mathbf{u}, \tau)$, which we denoted as $HE_{BA}(\mathbf{r}, t)$. The mapping can be time-dependent and time-dependent. Its two dimensions are real.

We will show, for example, what conclusions result from the method of mapping in the analysis of the “Big Bang”. Assume that the curvilinear map is inside the sphere of an infinite 4-radius, which is located in $E_A(\mathbf{r}, t)$. Suppose that the operator of the mapping \tilde{H}_{BA} allows us to reduce the radius of this sphere in time to zero. The curved display of HE_{BA} will “shrink” to the point before the “inevitable Big Bang”. In the framework of general relativity, the following assertion holds. Space, time and matter “stick together” at an infinitesimal point. Around is a strange “emptiness” that does not have spatial dimensions and time.

- From the space mapping position, such a statement is not correct. The “point” is not in the “emptiness”. It is always in the initial space-time continuum $E_A(\mathbf{r}, t)$, since the space-time $E_A(\mathbf{r}, t)$ is not “deformed” by the mapping operator.
- Now we will talk about material objects that have mass and inertia. Assume that material objects belong to $E_A(\mathbf{r}, t)$. On the one hand, according to general relativity, the curvature of space and material gravitational objects have a mutual relationship. On the other hand, the 4-space $E_A(\mathbf{r}, t)$ and the material objects in it do not depend on the operator. Consequently, in the case of “contraction” of the curvilinear 4-space, the mutual connection between the curvature and the gravitating masses is lost. The space-time in the mapping “contracts” together with its curvature, and the material objects in $E_A(\mathbf{r}, t)$ remain unchanged. Hence it follows that the material objects belonging to $E_A(\mathbf{r}, t)$ should not exist in $E_A(\mathbf{r}, t)$ in principle. They exist in $E_B(\mathbf{u}, \tau)$ and must be “transported” to $E_A(\mathbf{r}, t)$ from $E_B(\mathbf{u}, \tau)$ together with the “curvature”!
- Recall that the mapped objects and the mapped curvilinear 4-space are phenomena. The operator \tilde{H}_{BA} deforms material objects and “dresses” them in a “curvilinear space-time shell” only when $E_B(\mathbf{u}, \tau)$ is mapped into $E_A(\mathbf{r}, t)$.
- Thus, all the “deported” inertial material bodies from $E_B(\mathbf{u}, \tau)$ to $E_A(\mathbf{r}, t)$ are a “mapping” of some real “prototypes” existing in $E_B(\mathbf{u}, \tau)$. We - people are not an exception and we have our own “prototypes”.
- Can you imagine that you are a “distorted display” of your “undistorted prototype”, which roams somewhere far in $E_B(\mathbf{u}, \tau)$? Unlike you, it cannot be “squeezed into a point”, i.e. he is not subject to the action of the operator and, accordingly, to the impact of the “Big Bang”!

We see that even at the first stage of the rethinking of the phenomena of physics, many difficulties will arise in interpreting the phenomena of general relativity. We will no longer go beyond the limits of geometry and discuss these questions. There are a lot of strange and obscure things going beyond common sense.

We can extend our conclusions to a two-dimensional space (plane). Let us reproduce these conclusions for the plane:

1. Any two-dimensional space is always initially Euclidean. There are no geometric methods (“compass-ruler-pencil”) for measuring the internal relative curvature of different sections of the plane. Any two-dimensional space is always initially Euclidean. Любое двумерное пространство всегда изначально является Евклидовым.
2. A curvilinear two-dimensional space on a plane cannot exist independently. A curvilinear space on a plane can only exist as a *non-rectilinear* mapping of another Euclidean space.

4. Conclusion

Analyzing the way of constructing a curvilinear space, we discovered an “old” error of geometries. At scientists at construction of curvilinear spaces the initial Euclidean space as though “disappears”, “is lost”. This circumstance did not allow us to give a correct and logically rigorous interpretation of the phenomena within the framework of general relativity. Explanations of phenomena within the framework of general relativity resemble the stories of science fiction writers. Already now we can say that most of the explanations of phenomena in the framework of modern GTR, Cosmology, Astrophysics (see, for example, [6]), etc. They are not correct and need a thorough revision. This is the “price” of an old 200-year-old error in geometry.

References

- [1] Chubykalo, A. and Espinoza, A.(2014)*International Journal of Physics and Astronomy*2(3) 1-10.
- [2] Lenin, V. I.(2002)*Materialism and Empirio-Criticism: Critical Comments on a Reactionary Philosophy*. Univ. Press of the Pacific.
- [3] Dirac, P.(1975)*General Theory of relativity*. John Wiley & Sons, Inc
- [4] Ouspensky, P. D.(2005)*The Fourth Dimension*. Kessinger Publishing, LLC, ISBN13: 9781425349356.
- [5] Engels, F.(1964)*Dialectics of Nature*.3rd ed.Progress Publishers.
- [6] Burke, W.(1980)*Spacetime, Geometry, Cosmology*. Mill Valley, California.